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(τ_i, τ_j)-ρ-Closed Sets in Biopological Spaces

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Abstract.

In this paper, to introduce and study new classes of sets called ρ-closed sets and ρ_s-closed sets, ρ-open sets and ρ_s-open sets in bitopological spaces. Some Properties and Characterizations of such closed and open sets are investigated.

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1. Introduction

In 1963, J.C. Kelly[14] defined a bitopological space (X, τ_1, τ_2) to be a set X equipped with two topologies τ_1, τ_2 on X and he initiated a systematic study of bitopological space. The study of generalized closed sets in a bitopological space was initiated by Levine in [15]and the concept of $T_{1/2}$ spaces was introduced. Various authors, like I. Arockiarani[2], S.P. Arya and T.M. Nour[3], R. Devi [8] and Y. Gnanambal[12] and have turned attention to the various concepts of topology by considering bitopological spaces instead of topological spaces. In 1996, H.Maki, J. Umehara and T. Noiri[16] introduced the classes of pregeneralized closed sets and used them to obtain properties ofpre-$T_{1/2}$ spaces. The modified forms of generalized closed sets and generalized continuitywere studied by K. Balachandran, P. Sundaram and H. Maki [4]. In 2008, S. Jafari, T. Noiri, N. Rajesh and M.L. Thivagar [13] introduced the concept of $\tilde{g}$-closed sets and their properties. In 2014, O. Uma Maheswari, A. Vadivel and D. Sivakumar[19] introduced the concept of (τ_i, τ_j)-$\#rg$-closed sets and their properties. In 2012, C. Devamanoharan, S. Pious Missier and S.Jafari[7] introducedρ-closed sets in topological spaces.In this paper, we introduce new classes of sets called (τ_i, τ_j)-ρ-closed sets inbitopological spaces.

Before entering into our work we recall the following definitions, which are due to various authors.

2. Preliminaries

Throughout this paper (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) will always denote bitopological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of (X, τ_1, τ_2), $Cl(A)$, $Int(A)$ and $D[A]$ denote the closure, the interior and the derived set of A, respectively.

Definition 2.1. Let a subset A of a space (X, τ) is called

1. *Regular open [18] if* $A = \tau_i\text{-}int(\tau_j\text{-}cl(A))$ *and regular closed if* $A = \tau_j\text{-}cl(\tau_i\text{-}int(A))$.
2. π*-open [22] if it is the finite union of* (τ_i, τ_j)*-regular open sets.*
3. *Regular semiopen [6] if there is a regular open set U such that* $U \subseteq A \subseteq cl(U)$.

Definition 2.2. *Let* (X, τ_1, τ_2) *be a bitopological space and* $A \subseteq X$.

1. (τ_i, τ_j)*-Preopen [9] if* $A \subseteq \tau_i\text{-}int(\tau_j\text{-}cl(A))$ *and preclosed if* $\tau_j\text{-}cl(\tau_i\text{-}int(A)) \subseteq A$.
2. (τ_i, τ_j)*-Semi-open [9] if* $A \subseteq \tau_j\text{-}cl(\tau_i\text{-}int(A))$ *and semi-closed if* $\tau_i\text{-}int(\tau_j\text{-}cl(A)) \subseteq A$.
3. (τ_i, τ_j)*-*α*-open [9] if* $A \subseteq \tau_i\text{-}int(\tau_j\text{-}cl(\tau_i\text{-}int(A)))$ *and* α*-closed if* $\tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl(A))) \subseteq A$.
4. (τ_i, τ_j)*-Semi preopen [9] if* $A \subseteq \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl(A)))$ *and semi preclosed if* $\tau_i\text{-}int(\tau_j\text{-}cl(\tau_i\text{-}int(A))) \subseteq A$.

The Pre-interior of A, denoted by pint(A), is the union of all preopen subsets of A.
The Pre-closure of A, denoted by Pcl(A), is the intersection of all Preclosed sets containing A.

Lemma 2.3([1]).*For any subset A of X, the following relations hold.*

1. $Scl(A) = A \cup int(cl(A))$.
2. $\alpha cl(A) = A \cup cl(int(cl(A)))$.
3. $Pcl(A) = A \cup cl(int(A))$.
4. $Spcl(A) = A \cup int(cl(int(A)))$.

Definition 2.4. *Let* (X, τ) *be a topological space. A subset* $A \subseteq X$ *is said to be*

1. $\hat{g}$*-closed [20] if* $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is semi open in X.*
2. **g-closed [21] if* $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* $\hat{g}$*-open in X.*
3. *#g-semi closed (briefly #gs-closed)[21] if* $scl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is *g-open in X.*
4. $\tilde{g}$*-closed set [21] if* $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is #gs-open in X.*

The complements of the above mentioned sets are called their respective open sets.

Definition 2.5. *Let* (X, τ_1, τ_2) *be a topological space. A subset* $A \subseteq X$ *is said to be*

1. (τ_i, τ_j)*-generalized closed (briefly g-closed)[10] if* $\tau_j\text{-}cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_i*-open in X.*
2. (τ_i, τ_j)*-generalized preclosed (briefly gp-closed)[11] if* $\tau_j\text{-}Pcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_i*-open in X.*

3. (τ_i, τ_j)-*generalized preregular closed (briefly gpr-closed)[12] if* τ_j-*Pcl*(A) ⊆U *whenever* A ⊆U *and* U*is* τ_i-*regular open in X.*
4. (τ_i, τ_j)-*pregeneralized closed (briefly pg-closed)[11] if* τ_j-*Pcl*(A) ⊆U *whenever* A⊆U *and* U *is* τ_i-*preopen in X.*
5. (τ_i, τ_j)-*g**-*preclosed (briefly g*p-closed)[11] if* τ_j-*Pcl*(A) ⊆U *whenever* A⊆U *and* U *is* τ_i-*g-open in X.*
6. (τ_i, τ_j)-*generalized semi-preclosed (briefly gsp-closed)[9] if* τ_j-*spcl*(A) ⊆U *whenever* A⊆U *and* U *is* τ_i-*open in X.*
7. (τ_i, τ_j)-π*gp-closed [17] if* τ_j-*Pcl*(A) ⊆U *whenever* A⊆U *and* U *is* τ_i-π-*open in X.*
8. (τ_i, τ_j)-*rwclosed [5] if* τ_j-*cl*(A) ⊆U *whenever* A⊆U *and* U *is* τ_i-*regular semi open in X*
9. (τ_i, τ_j)-#*rg-closed [19] if* τ_j-*cl*(A) ⊆U *whenever* A⊆U *and* U *is* τ_i-*rw-open in X*

3 Basic Properties of ρ-Closed Sets

We introduce the following definition.

Definition 3.1. *A subset A of a space* (X, τ_1, τ_2) *is said to be* (τ_i, τ_j)-ρ-*closed in* (X, τ_1, τ_2) *if* τ_2-*Pcl*(A) ⊆τ_1-*Int*(U) *whenever* A⊆U *and* U *is* τ_1-˜*g-open in* (X, τ_1, τ_2).

Theorem 3.2. *Every* τ_i-*open and* τ_j-*preclosed subset of* (X, τ_1, τ_2) *is* (τ_i, τ_j)-ρ-*closed.*

Proof. Let A be an τ_i-open and τ_j-Preclosed subset of (X, τ_1, τ_2). Let A⊆U and U be τ_1-˜g-open in X. Then τ_2-*Pcl*(A) = A= τ_1-*Int*(A) ⊆τ_1-*Int*(U). Hence A is (τ_i, τ_j)-ρ-closed.
The converse of the above theorem need not be true as it is seen from the following example.

Example 3.3. *Let* X = {a, b, c} *and* τ_1= {ϕ, {a}, {c, a}, X} *and* τ_2 = {ϕ, {a}, {b}, {a, b}, X}. *Then the set* A= {a, b} *is*(τ_i, τ_j)-ρ-*closed but it is neither* τ_i-*open set nor* τ_j-*preclosed set in* (X, τ_1, τ_2).

Theorem 3.4. *Every* (τ_i, τ_j)-ρ-*closed set is* (τ_i, τ_j)-*gp-closed.*

Proof. Let A be any ρ-closed set in X. Let A⊆U and U be τ_1-open in X. Every open set is ˜g-open and thus A is ρ-closed. Therefore τ_2-*Pcl*(A) ⊆τ_1-*Int*(U) = U. Hence A is (τ_i, τ_j)-gp-closed.
The converse of the above theorem need not be true as it is seen from the following example.

Example 3.5. *Let* X = {a, b, c, d}, τ_1= {ϕ, {c}, {a, b}, {a, b, c}, X} and τ_2 = {ϕ, {b, c}, {a, b, c}, {b, c, d}, X}. *Then the set* A = {a} *is*(τ_i, τ_j)-*gp-closed but not* (τ_i, τ_j)-ρ-*closed in* (X, τ_1, τ_2).

Theorem 3.6. *Every (τ_i, τ_j)-ρ-closed set is (τ_i, τ_j)-gpr-closed.*

Proof. Let A be any ρ-closed set. Let $A \subseteq U$ and U be τ_1-regular open. Observe that every regular open set is open and every open set is τ_1-$\tilde{}$g-open and therefore A is ρ-closed. It follows that τ_2-$Pcl(A) \subseteq \tau_1$-$Int(U) = U$. Hence A is (τ_i, τ_j)-gpr-closed.
The converse of the above theorem need not be true as it is seen from the following example.

Example 3.7. In *example 3.3,* (τ_i, τ_j)-ρ-*closed set:*$\{\phi$, {b}, $\{c\}$, $\{a, b\}$, $\{a, c\}$, {b, c}, $X\}$, (τ_i, τ_j)-gpr *closed set: P(X),*
Then the set $A = \{a\}$ *is* (τ_i, τ_j)-*gpr closed but not* (τ_i, τ_j)-ρ-*closed in* (X, τ_1, τ_2).

Theorem 3.8. *Every*(τ_i, τ_j)-ρ-*closed set is*(τ_i, τ_j)-*gsp-closed.*

Proof. Let A be any (τ_i, τ_j)-ρ-closed set. Let $A \subseteq U$ and U beτ_i-open. Since every τ_i-open set is τ_i-$\tilde{}$g-open and thus A is ρ-closed. Therefore τ_2-$Pcl(A) \subseteq \tau_1$-$Int(U) = U$ and so τ_2-$Spcl(A) \subseteq U$. Hence A is(τ_i, τ_j)-gsp-closed.
The converse of this theorem need not be true as it is seen from the following example.

Example 3.9. In Example 3.5, (τ_i, τ_j)-ρ-*closed set:* $\{\phi$, {c}, {d}, $\{a, b\}$, $\{a$, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, $X\}$, (τ_i, τ_j)-gsp-*closed set: P(X), Then the set A*= $\{a, c\}$ *is* (τ_i, τ_j)-*gsp-closed but not* (τ_i, τ_j)-ρ-*closed in* (X, τ_1, τ_2).

Theorem 3.10. *Every* (τ_i, τ_j)-ρ-*closed set is* (τ_i, τ_j)-π*gp-closed.*

Proof. Let A be any (τ_i, τ_j)-ρ-closed set. Let $A \subseteq U$ and U be τ_i-π-open. Since every τ_i-π-open set isτ_i-open and everyτ_I-open set is τ_i-$\tilde{}$g-open and therefore A is(τ_i, τ_j)-ρ-closed. This means thatτ_j-$Pcl(A) \subseteq \tau_i$-$Int(U) = U$. Hence A is (τ_i, τ_j)-πgp-closed.
The converse of the above theorem need not be true as it is seen from the following example.

Example 3.11. *Let* $X = \{a, b, c\}$ *and* $\tau_1 = \{\phi, \{a\}, \{a, c\}, X\}$, $\tau_2 = \{\phi$, {a}, {b}, {a, b}, $X\}$.
(τ_i, τ_j)-ρ-*closed set:*$\{\phi$, {b}, {c}, $\{a, b\}$, $\{a$, c}, {b, c}, $X\}$, (τ_i, τ_j)-πgp-*closed set: P(X). Then the set* $A = \{a\}$ *is*(τ_i, τ_j)-π*gp-closed but not* (τ_i, τ_j)-ρ-*closed in* (X, τ_1, τ_2).

Remark 3.12. (τ_i, τ_j)-ρ-*closedness and* (τ_i, τ_j)-*pre closedness are independent concepts as we illustrate by means of the following examples.*

Example 3.13. *As in Example 5, the set A*= $\{a, b\}$ *is* (τ_i, τ_j)-ρ-*closed but not* (τ_i, τ_j)-*Preclosed in* (X, τ_1, τ_2).

Remark 3.14. (τ_i, τ_j)-*ρ-closed sets are independent concepts of* (τ_i, τ_j)-*semi-closed sets and* (τ_i, τ_j)-*Semi-Preclosed sets as we illustrate by means of the following example.*

Example 3.15. *Let X* = {*a, b, c, d*} *and* τ_1= {ϕ, {*b*}, {*c*}, {*b, c*}, {*b, c, d*}, *X*} and τ_2= *{ϕ, {a}, {b}, {a, b}, X}.* (τ_i, τ_j)-*ρ closed set:*{ϕ, {a}, {*a, b*}, {*a,* c}, {a, d}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, *X*}, (τ_i, τ_j)-Semi *closed set:*{ϕ, {c}, {d}, {a, c}, {b, c}, {c, d}, {a, c, d}, {b, c, d}, X}, (τ_i, τ_j)-Semi pre *closed set:*{ϕ, {a}, {c}, {d}, {a, b}, {a, c}, {a, d}, {c, d}, {a, c, d}, {b, c, d}, X}
Then the set A = {*a, b, c*} *is* (τ_i, τ_j)-*ρ-closed but neither* (τ_i, τ_j)-*semi-closed nor* (τ_i, τ_j)-*semi-pre-closed. The set B* = {*c*} *is both* (τ_i, τ_j)-*semi-closed and* (τ_i, τ_j)-*semi-preclosed but not* (τ_i, τ_j)-*ρ-closed.*

Remark 3.16. (τ_i, τ_j)-*ρ-closedness and*(τ_i, τ_j)-*g-closedness are independent concepts as we illustrate by means of the following examples.*

Example 3.17. *Let X* = {*a, b, c, d, e*}. Letτ_1= {ϕ, {*a, b*}, {*a, b, d*}, {*a, b, c, d*}, {*a, b, d, e,* }, *X*} and τ_2= {ϕ, {*b*}, {d, *e*}, {*b*, d, e}, {*a*, c, *d, e,* }, *X*}. (τ_i, τ_j)-*ρ*-closed set: {ϕ, {c}, {d}, {e}, {a, b}, {a, c}, {a, d}, {a, e}, {*b, c*}, {b, d}, {b, e}, {c, d}, {c, e}, {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {b, c, d}, {b, c, e}, {a, b, c, d}, {a, b, c, e}, {a, c, d, e}, {b, c, d, e}, *X*}, (τ_i, τ_j)-g-closed set: {ϕ, {*b*}, {*c*}, {a, c}, {*b, c*}, {c, e}, {a, b, c}, {a, c, e}, {b, c, e}, {c, d, e}, {a, b, c, e}, {a, c, d, e}, {b, c, d, e}, *X*}
Then the set A= {*a, c, d*} *is*(τ_i, τ_j)-*ρ-closed but not* (τ_i, τ_j)-*g-closed in* (*X*, τ_1, τ_2).

Remark 3.18. (τ_i, τ_j)-*ρ-closedness and* (τ_i, τ_j)-*pg-closedness are independent concepts as we illustrate by means of the following example.*

Example 3.19. In Example 3.17.
(τ_i, τ_j)-ρ-closed set: {ϕ, {c}, {d}, {e}, {a, b}, {a, c}, {a, d}, {a, e}, {*b, c*}, {b, d}, {b, e}, {c, d}, {c, e}, {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {b, c, d}, {b, c, e}, {a, b, c, d}, {a, b, c, e}, {a, c, d, e}, {b, c, d, e}, *X*} and (τ_i, τ_j)-pg-closed set: P(X)-{{d, e}, {a, d, e}, {b, d, e}, {a, b, d, e}, {b, c, d, e}}
Then the set A = {b} *is* (τ_i, τ_j)-*pg-closed but not* (τ_i, τ_j)-*ρ-closed in* (*X*, τ_1, τ_2) *and the set B* = {*b*, c, d, e} *is* (τ_i, τ_j)-*ρ-closed but not* (τ_i, τ_j)-*pg-closed in* (*X*, τ_1, τ_2).

Remark 3.20. (τ_i, τ_j)-*ρ-closedness and* (τ_i, τ_j)-*g*p-closedness are independent concepts as we illustrate by means of the following example.*

Example 3.21. In Example 3.17.
(τ_i, τ_j)-ρ-closed set: {ϕ, {c}, {d}, {e}, {a, b}, {a, c}, {a, d}, {a, e}, {*b, c*}, {b, d}, {b, e}, {c, d}, {c, e}, {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {b, c, d}, {b, c, e}, {a, b, c, d}, {a, b, c, e}, {a, c, d, e}, {b, c, d, e}, *X*} and (τ_i, τ_j)-g*p-closed set: P(X)-{{d, e}, {a, d, e}, {b, d, e}, {a, b, d, e}}
Then *the set B* = {*a*} *is* (τ_i, τ_j)-*g*p-closed but not* (τ_i, τ_j)-*ρ-closed in* (*X*, τ_1, τ_2).

Remark 3.22. *(τ_i, τ_j)-ρ-closedness and (τ_i, τ_j)-α-closedness are independent concepts as we illustrate by means of the following examples.*

Example 3.23. *As in Example 3.3*
(τ_i, τ_j)-ρ-closed set: $\{\phi$, $\{b\}$, $\{c\}$, {a, b}, {a, c}, $\{b, c\}$, $X\}$, (τ_i, τ_j)-α-closed set: $\{\phi$, $\{b\}$, $\{c\}$, {a, c}, $\{b, c\}$, $X\}$,
Then set A= {a, b} *is (τ_i, τ_j)-ρ-closed but not (τ_i, τ_j)-α-closed in (X, τ_1, τ_2).*

Remark 3.24. *(τ_i, τ_j)-ρ-closedness and (τ_i, τ_j)-# rg-closedness are independent concepts as we illustrate by means of the following examples.*

Example 3.25. Let X = {a, b, c}, $\tau_1 = \{\phi, \{a\}, \{b\}$, {a, b}, $X\}$, $\tau_2 = \{\phi, \{a\}$, {a, c}, $X\}$
(τ_i, τ_j)-ρ-closed set: $\{\phi$, $\{b\}$, $\{c\}$, {a, c}, $X\}$, (τ_i, τ_j)-# rg-closed set: $\{\phi$, $\{b\}$, {a, c}, $\{b, c\}$, $X\}$,
Then set A= {c} *is (τ_i, τ_j)-ρ-closed but not (τ_i, τ_j)-# rg-closed in (X, τ_1, τ_2) and B = {b, c}is (τ_i, τ_j)-# rg-closed but not (τ_i, τ_j)-ρ-closed in (X, τ_1, τ_2)*

Definition 3.26. *A subset A of (X, τ_1, τ_2) is said to be ρ_s-closed in (X, τ_1, τ_2) if τ_2-$Pcl(A) \subseteq \tau_1$-$Int(\tau_2$-$cl(U))$ whenever $A \subseteq U$ and U is τ_1-$\tilde{g}$-open in (X, τ_1, τ_2).*

Theorem 3.27. *Every(τ_i, τ_j)-ρ-closed set is(τ_i, τ_j)-ρ_s-closed set.*

Proof. Let *A* be any ρ-closed set. Let $A \subseteq U$ and *U* beτ_1-$\tilde{g}$-open in *X*. Since *A* is (τ_i, τ_j)-ρ-closed, τ_2-$Pcl(A) \subseteq \tau_1$-$Int(U) \subseteq \tau_1$-$Int(\tau_2$-$cl(U))$. Hence *A* is(τ_i, τ_j)-ρ_s-closed.
The converse of the above theorem need not be true as it is seen from the following example.

Example 3.28. In example 3.3
(τ_i, τ_j)-ρ-closed set: $\{\phi$, $\{b\}$, $\{c\}$, {a, b}, {a, c}, {b, c}, $X\}$, (τ_i, τ_j)-ρ_s-closed set: $\{\phi$, {a}, $\{b\}$, $\{c\}$, {a, b}, {a, c}, {b, c}, $X\}$,
Then set A= {a} is (τ_i, τ_j)-ρ_s-closed set but not (τ_i, τ_j)-ρ-closed set *in* (X, τ_1, τ_2).

Remark 3.29. *From the above discussions and known results should be accompanied by a reference we have the following implications A→ B (A=B) represents A implies B but not conversely (A and B are independent of each\other →. See Figure 1.*

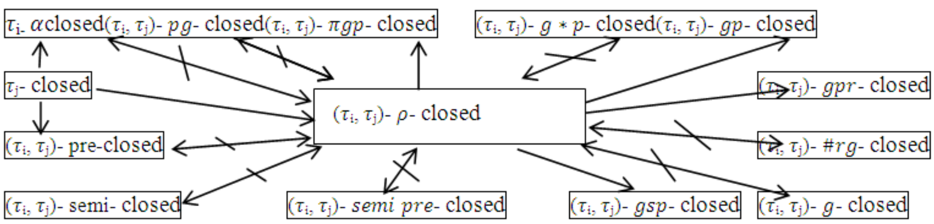

Figure 1: Implications.

4 ρ-Open Sets and ρ_s-Open Sets

Definition 4.1.

1. *A subset A of* (X, τ_1, τ_2) *is said to be* ρ*-open in* (X, τ_1, τ_2) *if its complement* $X - A$ *is* ρ*-closed in* (X, τ_1, τ_2).
2. *A subset A of* (X, τ_1, τ_2) *is said to be* ρ_s*-open in* (X, τ_1, τ_2) *if its complement* $X - A$ *is* ρ_s*-closed in* (X, τ_1, τ_2).

Theorem 4.2. *Let* (X, τ_1, τ_2) *be a topological space and* $A \subseteq X$.

1. *A is an* (τ_i, τ_j)-ρ*-open set if and only if* $\tau_2\text{-}cl(K) \subseteq \tau_1\text{-}pint(A)$ *whenever* $K \subseteq A$ *and K is* $\tilde{g}$*-closed.*
2. *A is an* (τ_i, τ_j)-ρ_s*-open set if and only if* $\tau_2\text{-}cl(\tau_1\text{-}int(K)) \subseteq \tau_1\text{-}pint(A)$ *whenever* $K \subseteq A$ *and K is* $\tilde{g}$*-closed.*
3. *If A is* (τ_i, τ_j)-ρ*-open, then A is* (τ_i, τ_j)-ρ_s*-open.*

Proof.

1. **Necessity.** Let A be an (τ_i, τ_j)-ρ-open set in (X, τ_1, τ_2). Let $K \subseteq A$ and K be τ_2-$\tilde{g}$-closed. Then $X - A$ is (τ_i, τ_j)-ρ-closed and it is contained in the τ_1-$\tilde{g}$-open set $X - K$.

Therefore $\tau_2\text{-}Pcl(X - A) \subseteq \tau_1\text{-}Int(X - K)$, $X\text{-}\tau_1\text{-}pint(A) \subseteq X - \tau_2\text{-}cl(K)$, Hence $\tau_2\text{-}cl(K) \subseteq \tau_1\text{-}pint(A)$.

Sufficiency. If K is $\tilde{g}$-closed set such that $\tau_2\text{-}cl(K) \subseteq \tau_1\text{-}pint(A)$ whenever $K \subseteq A$. It follows that $X - A \subseteq X - K$ and $X\text{-}\tau_1\text{-}pint(A) \subseteq X - \tau_2\text{-}cl(K)$. Therefore $\tau_2\text{-}Pcl(X - A) \subseteq \tau_1\text{-}Int(X - K)$.
Hence $X - A$ is (τ_i, τ_j)-ρ-closed and A becomes an ρ-open set.

2. **Necessity:** Let A be a (τ_i, τ_j)-ρ_s-open set in (X, τ_1, τ_2). Let $K \subseteq A$ and K be τ_2-$\tilde{g}$-closed. Then $X - A$ is (τ_i, τ_j)-ρ_s-closed and is contained in the τ_1-$\tilde{g}$-open set $X-K$. Therefore $\tau_2\text{-}Pcl(X-A) \subseteq \tau_1 Int(\tau_2\text{-}cl(X-K))$ and so $X\text{-}\tau_1\text{-}pint(A) \subseteq \tau_1\text{-}int(X\text{-}\tau_1\text{-}int(K)) = X - \tau_2\text{-}cl(\tau_1\text{-}int(K))$. Hence $\tau_2\text{-}cl(\tau_1\text{-}int(K) \subseteq \tau_1\text{-}pint(A)$.

Sufficiency: If K is $\tilde{g}$-closed set such that $\tau_2\text{-}cl(\tau_1\text{-}int(K)) \subseteq \tau_1\text{-}pint(A)$ whenever $K \subseteq A$, it follows that. $X - A \subseteq X - K$, $X - \tau_1\text{-}pint(A) \subseteq X\text{-}\tau_2\text{-}cl(\tau_1\text{-}int(K))$ and $\tau_2\text{-}Pcl(X - A) \subseteq X\text{-}\tau_2\text{-}cl(\tau_1\text{-}int(K)) = \tau_1\text{-}int(\tau_2\text{-}cl(X - K))$. Hence $X - A$ is (τ_i, τ_j)-ρ_s-closed and A becomes an (τ_i, τ_j)-ρ_s-open set.

3. Let A be an τ_i-ρ-open. Let also $K \subseteq A$ and K be $\tilde{g}$-closed. Since A is τ_i-ρ-open we have $\tau_2\text{-}cl(K) \subseteq \tau_1\text{-}pint(A)$. Therefore $\tau_2\text{-}cl(\tau_1\text{-}int(K) \subseteq pint(A)$. Hence by 2, A is τ_i-ρ_s-open.

Theorem 4.3. *If* $\tau_1\text{-}pint(A) \subseteq B \subseteq A$ *and A is* τ_i- ρ*-open then B is* τ_i- ρ*-open.*

Proof. If $\tau_1\text{-}pint(A) \subseteq B \subseteq A$, then $X - A \subseteq X - B \subseteq X\text{-}\tau_1\text{-}pint(A)$ that is $X - A \subseteq X - B \subseteq \tau_2\text{-}Pcl(X - A)$. Observe that $X - A$ is τ_j-ρ-closed and $X - B$ is τ_j - ρ-closed and hence B is τ_I - ρ -open.

Theorem 4.4. *If A⊆K isτ_i - ρ-closed then τ_2-Pcl(A) −A is τ_i −ρ_i -open.*

Proof. Let A be a ρ-closed. Then τ_2 -$Pcl(A)$−A contains no nonempty ˜g-closed set. Therefore ∅ = K ⊆τ_2-$Pcl(A)$ −A and ∅ = K is ˜g-closed. Clearly τ_2-$cl(K)$ ⊆τ_1-$pint(\tau_2$-$Pcl(A)$ −$A)$. Hence by Theorem 4.2, τ_2-$Pcl(A)$ −A is τ_i−ρ-open.

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