Advances in Dynamical Systems and Applications.
ISSN 0973-5321, Volume 12, Number 1, (2017) pp. 9-14

http://www.ripublication.com

Internal Blood Pressure Equation in Terms of Multivariable Aleph Function

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Abstract

Our purpose in this paper is to consider an equation of Internal Blood Pressure. We obtain internal blood pressure equation in terms of multivariable aleph function. We also point out some important special cases of the main result.

Keywords: Multivariable Aleph function, Internal Blood Pressure equation.

AMS subject classification: 33C60.

1. INTRODUCTION

The Aleph($\aleph$)-function introduced by Sudland et al. [10,11] and multivariable Aleph function of several complex variables generalizes the multivariable I-function recently study by C.K. Sharma and Ahmad [8] as:

$$\aleph[z_1,\ldots,z_r]$$
$$= \aleph^{0,n:m_1,n_1,\ldots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\ldots;p_{i(r)},q_{i(r)},\tau_{i(r)};R^{(r)}}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left|\begin{matrix}\left[\left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,n}\right],\left[\tau_i\left(a_{ji};\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{n+1,p_i}\right]: \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,\left[\tau_i\left(b_{ji};\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{m+1,q_i}\right]:\end{matrix}\right.\right.$$

$$\left[\left(c_j^{(1)};\gamma_j^{(1)}\right)_{1,n_1}\right],\left[\tau_{i^{(1)}}\left(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)}\right)_{n_1+1,p_{i^{(1)}}}\right];\dots;\left[\left(c_j^{(r)};\gamma_j^{(r)}\right)_{1,n_r}\right],\left[\tau_{i^{(r)}}\left(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)}\right)_{n_r+1,p_{i^{(r)}}}\right]$$

$$\left[\left(d_j^{(1)};\delta_j^{(1)}\right)_{1,m_1}\right],\left[\tau_{i^{(1)}}\left(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)}\right)_{m_1+1,q_{i^{(1)}}}\right];\dots;\left[\left(d_j^{(r)};\delta_j^{(r)}\right)_{1,m_r}\right],\left[\tau_{i^{(r)}}\left(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)}\right)_{m_r+1,q_{i^{(r)}}}\right]\Bigg]$$

$$=\frac{1}{(2\pi i)^r}\int_{L_1}\dots\int_{L_r}\Omega(h_1,\dots,h_r)\prod_{k=1}^{r}\emptyset_k(h_k)z_k^{h_k}\;dh_1\dots dh_r \qquad \dots(1.1)$$

Where

$$\Omega(h_1,\dots,h_r)=\frac{\prod_{j=1}^{n}\Gamma\left(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}h_k\right)}{\sum_{i=1}^{R}\tau_i\prod_{j=1}^{q_i}\Gamma\left(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}h_k\right)\prod_{j=n+1}^{p_i}\Gamma\left(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}h_k\right)}$$

and

$$\emptyset_k(h_k)=\frac{\prod_{j=n+1}^{m_k}\Gamma\left(d_j^{(k)}-\delta_j^{(k)}h_k\right)\prod_{j=1}^{n_k}\Gamma\left(1-c_j^{(k)}+\gamma_j^{(k)}h_k\right)}{\sum_{i^{(k)}=1}^{R^{(k)}}\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma\left(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}h_k\right)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma\left(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}h_k\right)} \qquad \dots(1.2)$$

$$k=1,\dots,r \qquad \dots(1.3)$$

The integration path $L_1,L_2,..,L_r=L_{i\gamma\infty}$, $(\gamma\in\Re)$ extends from $\gamma-i\infty$ to $\gamma+i\infty$ and is such that the poles of $\Gamma\left(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}h_k\right), j=1,\dots n$ do not coincide with the poles of $\Gamma\left(b_j+\sum_{k=1}^{r}\beta_j^{(k)}h_k\right)$, $j=1,\dots m$. The parameters $p_{i^{(k)}},q_{i^{(k)}}$ are non-negative integers satisfying the condition $0\le n\le p_{i^{(k)}}, 0\le m\le q_{i^{(k)}}$, $\tau_{i^{(k)}}>0$ for $i=1,\dots R$. The parameters $\alpha_j^{(k)},\beta_j^{(k)},\alpha_{ji}^{(k)},\beta_{ji}^{(k)}>0$ and $a_j,b_j,a_{ji},b_{ji}\in\mathbb{C}$. The empty product in (1.2) is interpreted as unity. The existence conditions for the defining integral (1.1) are given below:

$$\Theta_i^{(k)}>0\,,|arg(z_k)|<\frac{\pi}{2}\Theta_i^{(k)} \qquad \dots(1.4)$$

$$\Theta_i^{(k)}\ge 0\,,|arg(z_k)|<\frac{\pi}{2}\Theta_i^{(k)}\,,\Re\{\xi_i^{(k)}\}+1<0 \qquad \dots(1.5)$$

$$\Theta_i^{(k)}=\sum_{j=1}^{n}\alpha_j^{(k)}+\sum_{j=1}^{m}\beta_j^{(k)}-\tau_i\left(\sum_{j=n+1}^{p_{i^{(k)}}}\alpha_{ji}^{(k)}+\sum_{j=m+1}^{q_{i^{(k)}}}\beta_{ji}^{(k)}\right)+\sum_{j=1}^{n_k}\gamma_j^{(k)}+\sum_{j=1}^{m_k}\delta_j^{(k)}$$

$$-\tau_{i^{(k)}}\left(\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}+\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}\right) \qquad \dots(1.6)$$

$$\xi_i^{(k)} = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j + \tau_i \left(\sum_{j=m+1}^{q_{i(k)}} b_{ji} - \sum_{j=n+1}^{p_{i(k)}} a_{ji} \right) + \frac{1}{2}\left(p_{i^{(k)}} - q_{i^{(k)}}\right) \quad \ldots (1.7)$$

With $i = 1, \ldots, R;\ \ k = 1, \ldots, r;\quad i^{(k)} = 1, \ldots, R^{(k)}$

The Internal Blood Pressure is given by the following equation [7, p.77]:

$$\mathrm{V} \propto \mathrm{P} \quad \ldots (1.8)$$

from which we get the following differential equation

$$\frac{dV}{dP} = h;\ \ V \to 0, P \to 0 \quad \ldots (1.9)$$

Here P be the Internal Blood Pressure in Blood vessel having volume V at any time.

2. MAIN THEOREM

Theorem: Let P be the internal blood pressure in blood vessel having volume V at any time and $(P_1, P_2, \ldots, P_r)$ and $(V_1, V_2, \ldots, V_r)$ be the partial change in internal pressure and volume, with $V > (V_1, V_2, \ldots, V_r),\ P > (P_1, P_2, \ldots, P_r)$ and $\Theta_i^{(k)}$, $\xi_i^{(k)}$ given by (1.6), (1.7) and conditions

(i) $\Theta_i^{(k)} > 0$, $|arg(z_k)| < \frac{\pi}{2}\Theta_i^{(k)}$,

(ii) $\Theta_i^{(k)} \geq 0$, $|arg(z_k)| < \frac{\pi}{2}\Theta_i^{(k)}$, $\Re\{\xi_i^{(k)}\} + 1 < 0$.

Then

$$\aleph^{0,n+1:m_1,n_1,\ldots,m_r,n_r}_{p_i+1,q_i+1,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\ldots;p_{i(r)},q_{i(r)},\tau_{i(r)};R^{(r)}} \begin{bmatrix} z_1 \\ \vdots \\ z_r \end{bmatrix}$$

$$\left| \begin{matrix} \left[\left(a_j; \alpha_j^{(1)}, \ldots, \alpha_j^{(r)}\right)_{1,n}\right], (V; V_1, V_2, \ldots, V_r), \left[\tau_i\left(a_{ji}; \alpha_j^{(1)}, \ldots, \alpha_j^{(r)}\right)_{n+1,p_i}\right]: \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \left[\tau_i\left(b_{ji}; \beta_j^{(1)}, \ldots, \beta_j^{(r)}\right)_{m+1,q_i}\right] (1+V; V_1, V_2, \ldots, V_r): \end{matrix} \right.$$

$$\left. \begin{matrix} \left[\left(c_j^{(1)}; \gamma_j^{(1)}\right)_{1,n_1}\right], \left[\tau_{i^{(1)}}\left(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)}\right)_{n_1+1,p_{i^{(1)}}}\right]; \ldots; \left[\left(c_j^{(r)}; \gamma_j^{(r)}\right)_{1,n_r}\right], \left[\tau_{i^{(r)}}\left(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)}\right)_{n_r+1,p_{i^{(r)}}}\right] \\ \left[\left(d_j^{(1)}; \delta_j^{(1)}\right)_{1,m_1}\right], \left[\tau_{i^{(1)}}\left(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)}\right)_{m_1+1,q_{i^{(1)}}}\right]; \ldots; \left[\left(d_j^{(r)}; \delta_j^{(r)}\right)_{1,m_r}\right], \left[\tau_{i^{(r)}}\left(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)}\right)_{m_r+1,q_{i^{(r)}}}\right] \end{matrix} \right]$$

$$= h\, \aleph^{0,n+1:m_1,n_1,\ldots,m_r,n_r}_{p_i+1,q_i+1,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\ldots;p_{i(r)},q_{i(r)},\tau_{i(r)};R^{(r)}} \begin{bmatrix} z_1 \\ \vdots \\ z_r \end{bmatrix}$$

$$\left|\begin{matrix}\left[\left(a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)}\right)_{1,n}\right],(P;P_1,P_2,\dots,P_r),\left[\tau_i\left(a_{ji};\alpha_j^{(1)},\dots,\alpha_j^{(r)}\right)_{n+1,p_i}\right]: \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots,\left[\tau_i\left(b_{ji};\beta_j^{(1)},\dots,\beta_j^{(r)}\right)_{m+1,q_i}\right](1+P;P_1,P_2,\dots,P_r):\end{matrix}\right.$$

$$\left.\begin{matrix}\left[\left(c_j^{(1)};\gamma_j^{(1)}\right)_{1,n_1}\right],\left[\tau_{i^{(1)}}\left(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)}\right)_{n_1+1,p_{i^{(1)}}}\right];\dots;\left[\left(c_j^{(r)};\gamma_j^{(r)}\right)_{1,n_r}\right],\left[\tau_{i^{(r)}}\left(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)}\right)_{n_r+1,p_{i^{(r)}}}\right] \\ \left[\left(d_j^{(1)};\delta_j^{(1)}\right)_{1,m_1}\right],\left[\tau_{i^{(1)}}\left(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)}\right)_{m_1+1,q_{i^{(1)}}}\right];\dots;\left[\left(d_j^{(r)};\delta_j^{(r)}\right)_{1,m_r}\right],\left[\tau_{i^{(r)}}\left(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)}\right)_{m_r+1,q_{i^{(r)}}}\right]\end{matrix}\right]$$

$$+k\,\aleph^{0,n:m_1,n_1,\dots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\dots;p_{i(r)},q_{i(r)},\tau_{i(r)};R^{(r)}}\begin{bmatrix}z_1\\ \vdots\\ z_r\end{bmatrix}$$

$$\left|\begin{matrix}\left[\left(a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)}\right)_{1,n}\right],\left[\tau_i\left(a_{ji};\alpha_j^{(1)},\dots,\alpha_j^{(r)}\right)_{n+1,p_i}\right]: \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots,\left[\tau_i\left(b_{ji};\beta_j^{(1)},\dots,\beta_j^{(r)}\right)_{m+1,q_i}\right]:\end{matrix}\right.$$

$$\left.\begin{matrix}\left[\left(c_j^{(1)};\gamma_j^{(1)}\right)_{1,n_1}\right],\left[\tau_{i^{(1)}}\left(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)}\right)_{n_1+1,p_{i^{(1)}}}\right];\dots;\left[\left(c_j^{(r)};\gamma_j^{(r)}\right)_{1,n_r}\right],\left[\tau_{i^{(r)}}\left(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)}\right)_{n_r+1,p_{i^{(r)}}}\right] \\ \left[\left(d_j^{(1)};\delta_j^{(1)}\right)_{1,m_1}\right],\left[\tau_{i^{(1)}}\left(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)}\right)_{m_1+1,q_{i^{(1)}}}\right];\dots;\left[\left(d_j^{(r)};\delta_j^{(r)}\right)_{1,m_r}\right],\left[\tau_{i^{(r)}}\left(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)}\right)_{m_r+1,q_{i^{(r)}}}\right]\end{matrix}\right]$$

... (2.1)

Where h is proportional constant.

Proof : Integrating (1.9), we have

$$V = hP + k \qquad \text{... (2.2)}$$

Where k is integral constant

Or

$$\frac{\Gamma(1+V)}{\Gamma(V)} = h\frac{\Gamma(1+P)}{\Gamma(P)} + k \qquad \text{... (2.3)}$$

Replacing P = P +$(P_1h_1, P_2h_2, \dots, P_rh_r)$ and V = V +$(V_1h_1, V_2h_2, \dots, V_rh_r)$ (since as volume increases Internal Blood Pressure will also increase) in (2.3) and multiplying both sides by

$$\frac{1}{(2\pi i)^r}\Omega^{0,n:m_1,n_1,\dots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\dots;p_{i(r)},q_{i(r)},\tau_{i(r)};R^{(r)}}(h_1,\dots,h_r)\prod_{k=1}^{r}\phi_k(h_k)z_k^{h_k}$$

further integrating with respect to $h_1, \dots, h_r$ in the direction of contour from $\gamma - i\infty$ to $\gamma + i\infty$ and with the help of (1.1), we get (2.1).

3. SPECIAL CASES

Corollary 3.1 Let the condition of Main theorem be satisfied, set $r = 1$ then multivariable Aleph function of r- variable convert in Aleph function of one variable defined by sudland [10] now the equation (2.1) reduces to following result which is obtained by Chaurasia and Gill [4].

$$\aleph^{m+1,n}_{p_i+1,q_i+1,\tau_i;R}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n},(V,V_1),\left[\tau_i(a_{ji},\alpha_{ji})_{n+1,p_i;R}\right]\\(1+V,V_1),(b_j,\beta_j)_{1,m},\left[\tau_i(b_{ji};\beta_{ji})_{m+1,q_{i;R}}\right]\end{matrix}\right.\right]$$

$$= h\,\aleph^{m+1,n}_{p_i+1,q_i+1,\tau_i;R}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n},(P,P_1),\left[\tau_i(a_{ji},\alpha_{ji})_{n+1,p_i;R}\right]\\(1+P,P_1),(b_j,\beta_j)_{1,m},\left[\tau_i(b_{ji};\beta_{ji})_{m+1,q_{i;R}}\right]\end{matrix}\right.\right]$$

$$+k\,\aleph^{m,n}_{p_i,q_i,\tau_i;R}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n},\left[\tau_i(a_{ji},\alpha_{ji})_{n+1,p_i;R}\right]\\(b_j,\beta_j)_{1,m},\left[\tau_i(b_{ji};\beta_{ji})_{m+1,q_{i;R}}\right]\end{matrix}\right.\right] \quad \ldots(3.1)$$

Corollary 3.2 If we take $r = 1$, $\tau_i = 1, i = 1, \ldots, R$ in the integral (2.1) we get the I-function introduced by V.P. Sexena [6], let the condition of main theorem be satisfied, we arrive at the following result

$$I^{m+1,n}_{p_i+1,q_i+1;R}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n},(V,V_1);(a_{ji},\alpha_{ji})_{n+1,p_i;R}\\(1+V,V_1),(b_j,\beta_j)_{1,m};(b_{ji};\beta_{ji})_{m+1,q_{i;R}}\end{matrix}\right.\right]$$

$$= h\,I^{m+1,n}_{p_i+1,q_i+1;R}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n},(P,P_1);(a_{ji},\alpha_{ji})_{n+1,p_i;R}\\(1+P,P_1),(b_j,\beta_j)_{1,m};(b_{ji};\beta_{ji})_{m+1,q_{i;R}}\end{matrix}\right.\right]$$

$$+k\,I^{m+1,n}_{p_i+1,q_i+1;R}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i;R}\\(b_j,\beta_j)_{1,m};(b_{ji};\beta_{ji})_{m+1,q_{i;R}}\end{matrix}\right.\right] \quad \ldots(3.2)$$

Corollary 3.3 If we set $r = 1$, $\tau_i = 1, i = 1, \ldots, R$ and $R = 1$ in the integral (2.1) we get the Fox's H-function [3], let the condition of main theorem be satisfied, we arrive at the following result, We have a known result obtained by Srivastava [9]

$$H^{m+1,n}_{p+1,q+1}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,p};(V,V_1)\\(1+V,V_1),(b_j,\beta_j)_{1,q}\end{matrix}\right.\right]$$

$$= h\,H^{m+1,n}_{p+1,q+1}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(P,P_1)\\(1+P,P_1);(b_j,\beta_j)_{1,m}\end{matrix}\right.\right] + k\,H^{m,n}_{p,q}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n}\\(b_j,\beta_j)_{1,m}\end{matrix}\right.\right] \quad \ldots(3.3)$$

REFERENCES

[1] Buschman, R.G., and Srivastava, H.M., 1990, “The H-Function associated with a certain class of Feynman integrals,” J. Phys. A. Math. Gen., 23, pp. 4707-4710.

[2] Chaurasia, V.B.L., 2004, “Equation of internal blood pressure and the H-function,” Acta Ciencia Indica, 30M, No.4, pp. 719-720.

[3] Fox, C., 1961, “The G and H-function as symmetrical fourier kernels,” Trans. Amer. Math. Soc., 98, pp. 395-429.

[4] Gill, V., and Chaurasia, V.B.L., 2013, “Aleph-function and equation of internal blood pressure,” Acta universitatis, 35, pp. 211-214.

[5] Mathai, A.M., and Saxena, R.K., 1978, “The H-function with applications in statistics and other disciplines,” Wiley Eastern, New Delhi.

[6] Saxena, V.P., 1982 “Formal solution of certain new pair of dual integral equations involving H-functions”, Proc. Nat. Acad. Sci. India Sect. A 51, 366-375.

[7] Saxena, V.P., 1989, “Introductory Bio-Mathematics,” Vishwa Prakashan, New Delhi.

[8] Sharma, C.K., and S.S. Ahmad, 1994, S.S., “On the multivariable I-function,” Acta ciencia Indica Math, 20(2), pp. 113-116.

[9] Srivastava, S.S., 1999, “An advanced study of generalized hypergeometric functionns with their applications,” Ph.D. Thesis, A.P.S. University, Rewa.

[10] Sudland, N., Baulmann, B., and Nonnenmacher, T.F., 1998, “Open problem: who knows about the Aleph-function?” Fract. Calc. Appl. Anal. 1(4), pp. 401-402.

[11] Sudland, N., Baulmann, B., and Nonnenmacher, T.F., 2001, “Fractional Driftless Fokker-Planck Equation with Power Law Diffusion Coefficients,” Computer Algebra in Scientific computing (CASCKonstanz 2001), Springer, Berlin, pp. 513-525.