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A Mathematical Model for Unemployment-Taking an Action without Delay

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Abstract

The present work described and analyzed a mathematical model for unemployment using system of dynamic differential equations. In this model we analyzed an effect of the action of the government and private sector to control unemployment without any delay. Also, we observed the effect of attempt of self-employment made by unemployed persons. The model is described by a system of differential equations and also found the non-negative equilibrium point of the system to check the stability. At last Numerical simulation is given to compare with analytical result.

Keywords: Employed persons, unemployed persons, self employment, newly created vacancies, present jobs.

1. INTRODUCTION

Unemployment is the serious problem for the whole world. Unemployment effects economically, socially and many times mentally to the person. This problem is not limited to the particular person but it catches the whole family and slowly-slowly the whole country, especially young generation. Although they are full of life with so many dreams, they get depressed from long period of unemployment.

Nikolopoulos and Tzanetis ([6]) developed and analyzed a model for a housing allocation of homeless families due to natural disaster. Based on this concept Misra and Singh ([1,2]) presented a nonlinear mathematical model for unemployment. G.N.Pathan and P.H.Bhathawala ([7]) developed a mathematical model for unemployment with effect of self employment based on concept of above papers. N.Sirghi, M.Neamtu, and D.Deac presented in ([3]) a nonlinear dynamic model using four variables: Number of unemployed persons, number of employed persons, number of present jobs in the market and number of newly created vacancies. In ([4]) M. Neamtu presented a model for unemployment based on some concept of ([2,3]) with adding a new variable number of immigrants.

Using concept of these papers we developed a dynamic mathematical model for unemployment with four variables: Number of unemployed persons, number of employed persons, number of present jobs in the market, number of newly created vacancies and we introduce an impact of self employment with the assumption that government and privet sector both tried to create new vacancies without any delay.

The paper is organized as follows: Section 2 describes Model for unemployment, Section 3 describes an equilibrium analysis, Section 4 describes the stability of equilibrium point, Numerical simulation describes in section 5 and Conclusion is given in section 6.

2. MATHEMATICAL MODEL:

In this process we assume that all entrants of the category unemployment are fully qualified to do any job at any time *t*. Number of unemployed persons, $U(t)$ increases with constant rate a_1. The rate of movement from unemployed class to employed class is jointly proportional to $U(t)$ and $(P(t)+V(t)-E(t))$. Where $P(t)$ denoted the present jobs in the market available by government and private sector. Government and privet sector try to create new vacancies without delay denoted by $V(t)$ and number of employed persons denoted $E(t)$, Migration as well as death of unemployed persons is proportional to their number with the rate a_3, many times employed person leave the job because of dissatisfaction or fired from their job and joint unemployed class with the rate a_4. Unemployed persons have to create chances for self employment to survive. Unemployed person who start their own independent work and become self employed is proportional to its number with the rate a_5. a_6 is the rate of death and retirement of employed person. The variation in the present job is proportional to c_1 and depreciation rate in present jobs is c_2. α and β are rate of newly created vacancies and diminution of newly created vacancies.

$$\frac{dU}{dt} = a_1 - a_2U(P+V-E) - a_3U + a_4E - a_5U \qquad \text{\_\_\_\_(1)}$$

$$\frac{dE}{dt} = a_2U(P+V-E) - a_4E + a_5U - a_6E \qquad \text{\_\_\_\_(2)}$$

$$\frac{dP}{dt} = c_1U - c_2P \qquad \text{\_\_\_\_(3)}$$

$$\frac{dV}{dt} = \alpha U - \delta V \qquad \text{\_\_\_\_(4)}$$

Lemma: The set $\Omega = \{(U,E,P,V) : 0 \le U + E \le \frac{a_1}{\gamma}, 0 \le P \le \frac{c_1 a_1}{c_2\gamma}, 0 \le V \le \frac{\alpha a_1}{\gamma\delta}\}$,where $\gamma = \min(a_3, a_6)$ is a region of attraction for the system (1) – (4) and it attracts all solutions initiating in the interior of the positive octant.

Proof:

From equation (1) – (2) we get,

$$\frac{d}{dt}(U(t)+E(t)) = a_1 - a_3U(t) - a_6E(t)$$

Which gives

$$\frac{d}{dt}(U(t)+E(t)) \le a_1 - \gamma(U(t)+E(t))$$

Where $\gamma = \min(a_3, a_6)$.

By taking limit supremum

$$\lim_{t\to\infty}\sup(U(t)+E(t)) \le \frac{a_1}{\gamma}$$

from (3) we have

$$\frac{dP}{dt} = c_1U(t) - c_2P(t)$$

$$\therefore \frac{dP}{dt} \le \frac{c_1a_1}{\gamma} - c_2P(t)$$

By taking limit supremum which leads to,

$$\lim_{t\to\infty}\sup P(t) \le \frac{c_1a_1}{c_2\gamma}$$

from (4) we have

$$\frac{dV}{dt} = \alpha U(t) - \delta V(t)$$

$$\therefore \frac{dV}{dt} \le \frac{\alpha a_1}{\gamma} - \delta V(t)$$

By taking limit supremum which leads to,

$$\lim_{t\to\infty} \sup V(t) \le \frac{\alpha a_1}{\delta\gamma}$$

This proves the lemma.

3. EQUILIBRIUM ANALYSIS

The model system (1) - (4) has only one non negative equilibrium point $E_0(U^*, E^*, P^*, V^*)$ which obtained by solving the following set of algebraic equations.

$$a_1 - a_2U(P+V-E) - a_3U + a_4E - a_5U = 0 \qquad \text{\_\_\_\_(5)}$$

$$a_2U(P+V-E) - a_4E + a_5U - a_6E = 0 \qquad \text{\_\_\_\_(6)}$$

$$c_1U - c_2P = 0 \qquad \text{\_\_\_\_(7)}$$

$$\alpha U - \delta V = 0 \qquad \text{\_\_\_\_(8)}$$

Taking an addition of equation (5) and (6)

$$a_1 - a_3U - a_6E = 0$$

$$\therefore E = \frac{a_1 - a_3U}{a_6} \qquad \text{\_\_\_\_(9)}$$

From (7)

$$P = \frac{c_1U}{c_2} \qquad \text{\_\_\_\_(10)}$$

From (8)

$$V = \frac{\alpha U}{\delta} \qquad \text{\_\_\_\_(11)}$$

$$\therefore\ P + V - E = \frac{aa_6U - a_1}{a_6} \qquad \_\_\_\_(12)$$

Where $a = \frac{\alpha}{\delta} + \frac{a_3}{a_6} + \frac{c_1}{c_2}$

Put values of equation (9) and (12) in (5) we get,

$$A_0U^2 - A_1U - A_2 = 0 \qquad \_\_\_\_(13)$$

Where,

$A_0 = aa_2a_6$, $A_1 = a_1a_2 - a_3(a_4 + a_6) - a_5a_6$,

$A_2 = a_1(a_4 + a_6)$.

From equation (13)

$$h(U) = A_0U^2 - A_1U - A_2 \qquad \_\_\_\_(14)$$

$+ \quad - \quad -$

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Since A_i, *i =0, 1, 2 all* are positive and number of changes in signs of equation (14) is only one. So, by Descart's rule equation (14) has only one positive solution say $U*$. So, we get the non-negative equilibrium point of model with coordinates:

$$E* = \frac{a_1 - a_3U*}{a_6}$$

$$P* = \frac{c_1U*}{c_2}$$

$$V* = \frac{\alpha U*}{\delta}$$

So, $E_0(U*, E*, P*, V*)$ is required non negative solution of the Model.

4. STABILITY ANALYSIS

To check the local stability of equilibrium point $E_0(U*, E*, P*, V*)$ we calculate the variational matrix M of the model system (1) – (4) corresponding to $E_0(U*, E*, P*, V*)$

$$M = \begin{bmatrix} -q_{11} & q_{12} & -l_2 & -l_2 \\ q_{21} & -q_{22} & l_2 & l_2 \\ c_1 & 0 & -c_2 & 0 \\ \alpha & 0 & 0 & -\delta \end{bmatrix}$$

Where

$l_1 = a_2(P+V-E)$, $l_2 = a_2U$, $q_{11} = l_1 + a_3 + a_5$,

$q_{12} = l_2 + a_4$, $q_{21} = l_1 + a_5$, $q_{22} = l_2 + a_4 + a_6$

The characteristic equation of above matrix is

$$\lambda^4 + d_1\lambda^3 + d_2\lambda^2 + d_3\lambda + d_4 = 0 \qquad \text{\_\_\_\_\_(15)}$$

Where

$d_1 = q_{11} + q_{22} + c_2 + \delta$,

$d_2 = q_{11}(q_{22} + c_2 + \delta) + q_{22}(c_2 + \delta) + c_2\delta - q_{12}q_{21} + c_1l_2 + l_2\alpha$,

$d_3 = q_{11}[q_{22}(c_2 + \delta) + c_2\delta] + c_2q_{22}\delta - q_{12}[q_{21}(c_2 + \delta) + c_1l_2 + l_2\alpha]$

$\quad + c_1l_2(q_{22} + \delta) + l_2\alpha(q_{22} + c_2)$,

$d_4 = q_{11}c_2q_{22}\delta - q_{12}[q_{21}c_2\delta + l_2(c_1\delta + \alpha c_2)] + c_1q_{22}l_2\delta + l_2q_{22}c_2\alpha$

Since, d_1, d_2, d_3, d_4 are positive then all coefficients of equation (15) are positive and some algebraic manipulation convey that $d_1d_2 > d_3$ and

$d_1d_2d_3 > d_3^{\,2} + d_1^{\,2}d_4$. So, by Routh Hurwitz criteria all roots of equation (15) are negative or having a negative real part. Therefore equilibrium point $E_0 = (U^*, E^*, P^*, V^*)$ is locally asymptotically stable.

5. NUMERICAL SIMULATION

For the Numerical simulation using MATLAB 7.6.0 we consider the following data,

$a_1 = 5000$, $a_2 = 0.02$, $a_3 = 0.004$, $a_4 = 0.01$, $a_5 = 0.07$, $a_6 = 0.06$,

$c_1 = 0.07$, $c_2 = 0.02$, $\alpha = 0.04$, $\delta = 0.001$,

The equilibrium values of the model are:

$U^* = 1916$, $P^* = 6706$, $E^* = 83205$, $V^* = 76640$.

The eigenvalues of the variational matrix corresponding to the equilibrium point $E_0 = (U^*, P^*, E^*, V^*)$ of model system (1) - (4) are: -41.1738, -0.0079, $-0.0617 \pm 0.0486i$. All eigenvalues are either negative or having a negative real part. So, equilibrium $E_0 = (U^*, P^*, E^*, V^*)$ is locally asymptotically stable.

Using above data, Fig.1 and Fig. 2 represent the graph of variations in the number of unemployed persons with respect to time with difference values of a_2 and a_5 respectively. Fig.1 shows that if rate of unemployed persons to join employed class is increases than number of unemployed person decreases. Fig.2 indicates that if rate of self employment goes higher than number of unemployed person goes lower. It also shows that for lower unemployed rate needs very high self employment rate.

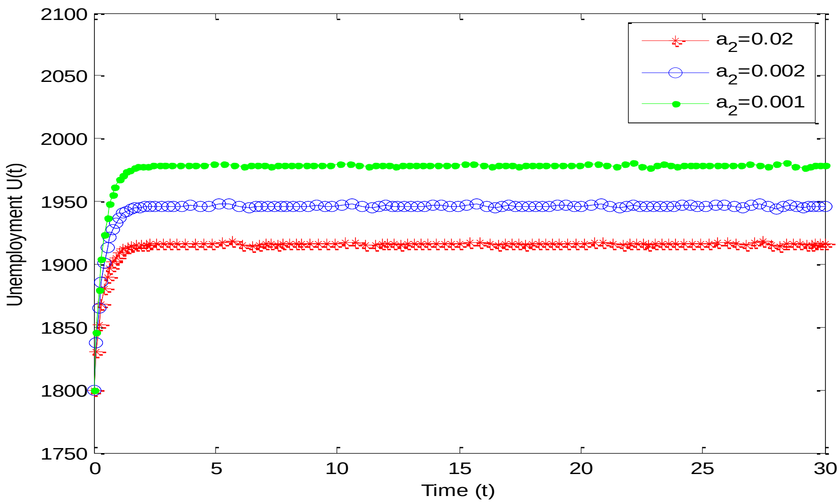

Fig.1

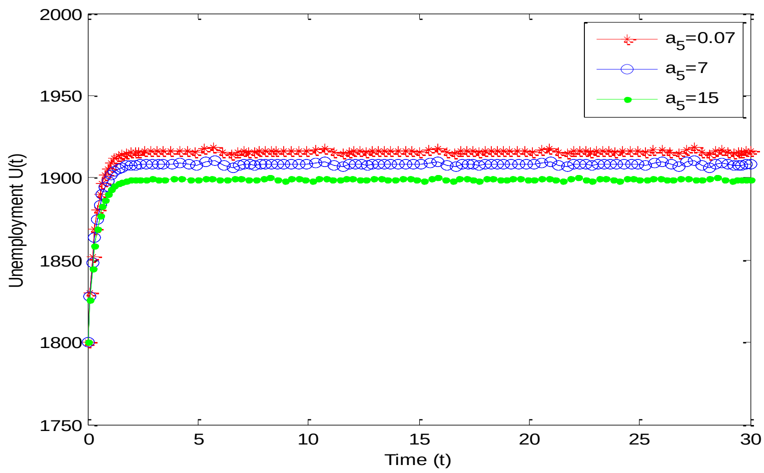

Fig. 2

CONCLUSION

The paper proposed and analyzed a nonlinear mathematical model for unemployment using four dynamic variables: Number of unemployed persons, number of present jobs in the market, number of employed persons and newly created vacancies. We find that equilibrium point is locally asymptotically stable. Theoretical calculation is verified by Numerical simulation which is done using MATLAB 7.6.0

From above calculations and graphs we can see that to decrease number of unemployed person the rate of unemployed persons who joined employed class should be higher, that means the government and private sector both try to create higher number of new vacancies. We also observe that if present jobs varies then in this situation rate of self employed should be higher to reduce unemployed. From fig. 2 it can be seen that despite difference of a_5 (rate of self employment) is very high, graph of unemployed person with respect to time is very close. It suggests that if present job varies then rate of self employed should be high to control unemployment.

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