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Consequences of Fetching Analytic Function from Its Real or Imaginary Part using *Milne-Thomson* Method: A Critical Analysis

Ashu Vij

Assistant Professor
P.G. Department of Mathematics
D.A.V.College, Amritsar -143001

Abstract

In this paper, a critical analysis of *Milne-Thomson* method is made for finding the analytic function when real or imaginary part of function is given. The Milne-Thomson method is used for finding the harmonic conjugate of a function. It has been examined that in some cases, after obtaining the analytic function using Milne Thompson method from a real or imaginary part , the analytic function does provide the same real or imaginary part from which the function was earlier obtained. The problem and the consequences of using this method is discussed with a suitable example.

Keywords: Analytic function, Milne Thompson method, harmonic conjugate.

INTRODUCTION

Complex analysis is one of the classical branches in mathematics with its origin in the 18th century. Augustin Cauchy (1789-1857) and Bernhard Riemann (1826-1866) are considered as founders of complex analysis[1] . In contrast with purely analytic approach of Cauchy, Riemann emphasized on its geometric aspect. The term "complex analysis" is an extremely useful and beautiful part of mathematics and forms the basis of many techniques employed in many branches of mathematics and physics. A complex variable w (dependent) is said to be function of complex variable z (independent) if for each value of $z = x + iy$ in a certain region R, there correspond one or more definite values of w. To indicate a function of complex variable we use the notation $w = f(z)$. Since w is a complex variable, so it is written as $w = f(z) = u(x,y) + iv(x,y)$

Also $u(x, y) = \text{Re } f(z)$, and $v(x, y) = \text{Im } f(z)$.

For example, the monomial function $f(z) = z^3$ is written as

$$z^3 = (x + i\,y)^3 = (x^3 - 3xy^2) + i\,(3x^2 y - y^3),$$

So $\text{Re } z^3 = x^3 - 3xy^2$, $\text{Im } z^3 = 3x^2 y - y^3$.

ANALYTIC FUNCTION

The complex function of a complex variable which possess derivative whenever function is defined is said to be analytic. More precisely, a function f of a complex variable z is analytic at a point z_0 if its derivative exist not only at z_0 but also at each point z in some neighborhood of z_0.[2]

Also the term Holomorphic function is used frequently having identical meaning.

According to Serge Lang[3], a function f(z) is analytic at z_0 if there exists a power series

$$f(z) = \sum_{n=1}^{\infty} a_n (z - z_0)^n$$

and some $r > 0$ such that the series converges absolutely for $|z - z_0| < r$

HARMONIC FUNCTION OR POTENTIAL FUNCTION

Any function of x and y say $u(x, y)$, having continuous partial derivatives of first and second order and also satisfying Laplace equation $\Delta u = 0$ is called a Harmonic function.

There is profound connection between harmonic functions i.e. solutions of the Laplace equation of two variables and complex functions viz. the real and imaginary parts of a complex analytic function are automatically harmonic. This result can be written as in the following theorem:

Theorem: The real and imaginary parts u and v of an analytic function of $f(z)$ are harmonic.

Harmonic conjugate

Given a harmonic function u we say that another harmonic function v is its harmonic conjugate if the complex-valued function $f = u + i\,v$ is analytic.

Consequences of Construction of Analytic Function from Real or Imaginary Part using Milne-Thompson method

Let $f(z) = u + iv$ be an analytic function where both u and v are conjugate functions. If one of these is given then one can find other using Milne Thompson method[4]. The Milne-Thomson method is widely used for finding the harmonic conjugate of a function upto an imaginary constant. It has been examined that in some cases, after obtaining the analytic function using Milne Thompson method from a real or imaginary part, the analytic function does provide the same real or imaginary part from which the function was earlier obtained. This fact may put a major question on authenticity of this method and areas where this method is used or there may be some condition on the analytic function which can be obtained from its real or imaginary part using the above said method.

Now an example is given to support the above claim.

NUMERICAL EXAMPLE

Consider an unknown analytic function whose imaginary part is

$$v = e^{-x}(x\cos y + \sin y) \qquad (1)$$

Now $\frac{\partial v}{\partial x} = - e^{-x}(x\cos y + \sin y) + e^{-x}(\cos y)$

$= e^{-x}(\cos y - x\cos y - \sin y)$

And $\frac{\partial v}{\partial y} = e^{-x}(-x\sin y + \cos y)$

According to Milne Thompson method,

$f'(z) = v_y(z,0) + i\, v_x(z,0)$

$= e^{-z} - i\, e^{-z}(z - 1) = e^{-z}(1 + i) - i\, e^{-z} z$

On integration,

$f(z) = (1+i)\int e^{-z}dz - i\int ze^{-z}dz$

$= (1+i)e^{-z}/(-1) - i[z\, e^{-z}/(-1) - e^{-z}/(-1)] + C$

$$= e^{-z}(iz - 1) + C \qquad (2)$$

where C is an arbitrary complex constant.

So we have obtained an analytic function from its imaginary part using Milne Thompson method up to an complex constant.

Now let us try the reverse procedure i.e. try to find imaginary part from the analytic function f(z) obtained above. For this we separate real and imaginary part of the function f(z) by substituting z=x+iy in the function f(z) given by (2).

So $f(z)= e^{-x-iy}(ix -y-1) + C$

$=e^{-x}(\cos y-i\sin y)(ix-y-1)+C$

So its imaginary part is

$v = e^{-x} [x\cos y + (y+1)\sin y]$

upto an imaginary constant.

This imaginary part v is not same as given in equation (1) .

This fact lead to a contradiction and of course put a serious consequence on Milne Thompson method.

CONCLUSION

The above stated example put a question mark on Milne Thompson method either on itself or there is some need to put some constraint on application of this method. As this method is most widely used for finding analytic function from a real or imaginary part so there is some need to see where it fails and why it is not applicable here in this particular example. I shall be highly thankful if some more examples of this type are found which do not comply with Milne Thompson method.

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