Advances in Dynamical Systems and Applications.
ISSN 0973-5321, Volume 12, Number 1, (2017) pp. 115-122

http://www.ripublication.com

A Few Contributions to the Cases of Optimization Models through Mathematical Analysis: An Empirical Study

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Abstract

Quality and Reliability have assumed greater importance in the present days where competitiveness is the key element. Reliability considerations are playing an increased role in almost all-engineering disciplines. The effort in engineering is concerned with designing and constructions of systems/products with improved performance. In this direction demand for cost -effective system, which performs better, is always increasing on a higher note. Rapid progress in science and technology has made many of the day's engineering systems more versatile and powerful. The increasing level of sophistication in Hi-tech industrial processes leads to reliability related problems. Thus, reliability problems not only continue to exist but are likely to require more complex solutions. The reliability problems not only of the system depend on performance of its constituent components. Hence in the recent time mathematical and statistical models were developed for evaluating system reliability and related measures. Thus, the system reliability analysis and modeling is one of the interesting areas of the reliability analyses. Most of the modeling and solutions considered in the literature of reliability was confined to individual / intrinsic failure category. In practice, we observe that probability of failure of one, two or more order of magnitude is predicted and the concept of common cause failures identified to be one of the most dominant causes of failures in many real line applications. This research paper describes an attempt to develop a modified optimum design redundant system viz., N-modular redundant and TMR systems with a specific emphasis and involvement of failure rate and time of operation. Thus, in this research investigation, an alternative optimum designing of redundant systems is attempted by suitably defining a system cost function. In this paper, discuss a few Contributions to the Cases of Optimization Models through Mathematical Analysis.

Keywords: Quality and Reliability, Mathematical and statistical models, intrinsic failure, Mathematical Analysis.

I. INTRODUCTION

Reliability is a popular concept that has been celebrated for years as a commendable attribute of a person or an artifact. Reliability Engineering aims to develop methods and tools to evaluate and demonstrate reliability, maintainability, availability and safety of components, sub systems and systems. The increasing level of sophistication in high-tech industrial process makes the reliability problems continues to exist and requires ever more complex solutions. The reliability analyses play a key role in planning, designing, testing, manufacturing, acceptance and use of product obtaining for effectiveness of the product. The theory of reliability is a vital part of quality control. The reliability theory provides us an optimistic, perspective on the opportunities for life extension of the component.

Generally, reliability is defined as "The probability that the unit/product perform its intended function adequately for a given period of time under the stated operating conditions or environment." The definition emphasizes the four elements (1) probability (2) intended function (3) time and (4) operating (environment) conditions.

If "X" indicates time till the failure or life, which is always a random variable, then the probability that it does not fail before time 't' in a given environment is the reliability, i.e., $R(t) = P(X > t)$. Therefore, reliability is always a function of time t. It also depends on environmental conditions under which it is operating. Since R (t) is a probability
measure, it always satisfies the condition.

$$0 < R(t) < 1 \text{ and } R(t) = 0 \text{ if } t \to \infty \; R(t) = 1 \text{ if } t = 0.$$

Reliability is quality measurement, which is under the influence of time and environment unlike quality, which is degree of conformation alone not considering the time length and environment of operation. Therefore, reliability and quality slightly differ. Quality is associated simply with manufacturing ability where as reliability is associated with fields performance and design.

II. DESIGN FOR RELIABILITY

The system designer is encountered with several problems while planning and designing the system with reasonable level of reliability. Therefore, a thorough reliability analysis needs to be attended to at the design stage itself. The various means of increasing the system reliability and the constraints associated with them must be known. A number of techniques are available to enhance the system reliability. Some of the important techniques are:

- Parts improvement method
- Effective and creative design
- System simplification
- Use of over-rated components
- Structural redundancy
- Maintenance and repair

The method of parts improvement sometimes leads to higher cost, while the method of creative design involves thinking of design engineer and moreover does not completely eliminate failures. Simplification method while attempting to do away with complexity sometimes may end up with over simplification and leads to poor quality. The over rated components method again leads to over cost. However structural redundancy provides a very effective means of improving system reliability, augmenting alternative paths at the component/subsystem level and improves the reliability considerably when over rated components are not available. Maintenance and repair certainly seem to be the best wherever possible, to increase the reliability. This approach combined with redundancy will give maximum reliability i.e., nearing to one.

Some important reliability measures in reliability analysis:

Reliability measures quantify the effectiveness of the system. Some of the most commonly used measures in Reliability theory are:

Reliability Function [R (T)]:

The probability that the system fails between 0 and t given by cumulative distribution function

$$F(t) = P[T \leq t]$$

where, T is non-negative random variable, which is the time to failure of a component.

The reliability function of a system at time at time t, $t \leq 0$ is

$$R(t) = P[T > t]$$
$$= 1 - F(t)$$

Mean Time Between Failures (MTBF):

Mean Time Between Failure (MTBF) is one of the important measures in the Reliability theory. Mean time between failures is the expected value of time to failure of the components or system, i.e., it is an average time taken by any two failures of the system. Mean Time Between Failure (MTBF) is also known, as Mean Time To Failure (MTTF) when the components in the system are cannot repaired. Mean Time

Between Failure is usually the mathematical expected value of time to failure of the system, which is given by

$$MTTF = E(T) = \int_0^\infty t\, f(t)\, dt = \int_0^\infty R(t)\, dt$$

$$MTBF = E(T) = \int_0^\infty t.f(t)\, dt = \int_0^\infty A(t)\, dt$$

where f (t) is failure density function,
R (t) is Reliability function
A (t) is an Availability function

The redundancy approach in the system design is highly appropriate in almost all the types of systems because of its merit over the other methods. The redundancy approach is good because of the following reasons:

- Desired level of reliability can be achieved under resource flexibility
- Improvement in reliability per unit resource is optimum when compared to any other approach
- Redundancy requires less skill on the part of the design engineer
- It is quick method of solution
- This method can be the best choice in case of the failure of all other approaches for improvement of reliability

The development of miniaturization techniques has made possible the application of redundancy rather conveniently in space vehicles also, thus allowing an increase in their life. A few examples of such systems or sub-systems where redundancy is extensively used are:

- Interconnected power systems
- Protective systems for nuclear reactors
- Aircraft propulsion systems
- Satellite communication systems
- Ignition systems for rocket engines
- Temperature control systems for space vehicles
- Data-processing systems

Non-Series-Parallel Configuration:

A complex system on simplification can produce a non-series-parallel configuration. Bridge configuration comes under this category. The Reliability of the below configuration can be evaluated using logic diagram and appropriate probability rules.

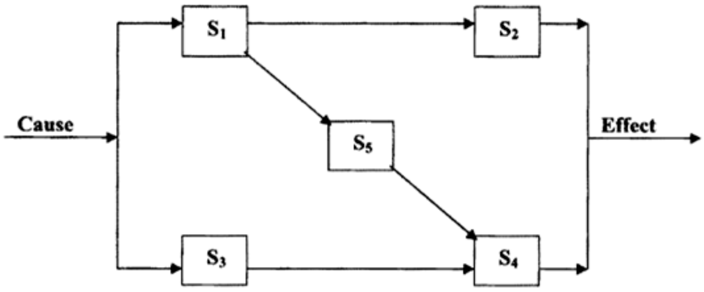

Fig.1: Complex configuration

A structure is called complex when the Reliability block diagram either cannot be reduced to a series / parallel structure with independent elements or does not exist like other models. For any complex system, two steps have to be followed for obtaining the Reliability or unreliability expressions of the system.

Step1:

Enumeration of minimal path sets or cut sets.

Step2:

Reliability expression or unreliability expression is obtained from Minimal path sets or cut sets by using disjoint techniques.

Coherent Systems:

A Coherent System is defined by the following four conditions:

1. When a group of components in the system is failed causing the system to be failed, the occurrence of any additional failure or failures will not return the system to a successful condition.
2. When a group of components in the system is successful and the system is successful. The system will not fail if some of the failed components are returned to the successful condition.
3. When all the components in the system are successful the system is successful.
4. When all these conditions in the system are failed then the system is failed. If a system fulfils all these conditions then it is a Coherent System.

The schematic representation of the Coherent System is

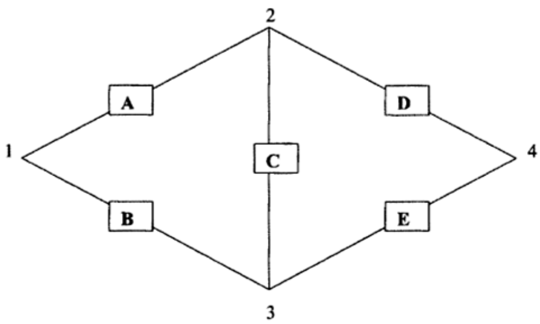

Fig.2: Coherent System

General network systems include Bridge Network, non-series-parallel structures and other complex system configurations. Unspecialized Systems where the structure is not explicit and the molecules of the system are not necessarily physically connected. Each of these systems-structures can be further classified using a general framework.

Common Cause Failures (CCF):

In reliability analysis, most of the models assume the event of individual failure, which mean that the components in system will fail individually and singly, which are also known as intrinsic failures. The concept of common cause failure is identified to be one of the most dominant causes of failures in many real line applications. Thus, most of the techniques and methods that are considered in the literature of reliability would belong to individual / intrinsic failure category.

However, in practice we found that the probability of failure of one, two or more order of magnitude is predicted. The more examples come from the nuclear industry. In the recent times, there has been paramount interest in these failures and their effect on system reliability analysis. Some of the practical and typical examples of CCF failures are multiple loss of aircraft jet engines due to bird ingestion, loss of power station emergency fuel of water due to inadequate maintenance of valves, and multiple failures due to stress corrosion in redundant structures, common fuel supply failures which are leading to diesel generators failures, sudden appearance of high temperature computer chips.

Therefore, one of the most important modes of failures which severely degrade the actual operating reliability of the system is identified in recent times which is known is common cause failures. This is also sometimes referred to as common mode failures.

The objective of Reliability Theory is to establish and to study the characteristics of reliability density function and evaluate reliability indices. With the help of these indices one can think of achieving improvement in the reliability of the system or components. Industrial producers are interested in the following:

i. Development of methods for evaluation of reliability characteristics.
ii. Development of methods for ensuring optimum reliability.
iii. Development of methods for optimum design.
iv. Estimation of reliability indices given data from rare context.

This research paper work is basically oriented towards the point given in (ii) in this connection.

In reliability, one is concerned with designing an item to last as long as possible without failure. System reliability optimization has been attracting many researchers for the last four decades. The major focus of recent work is on the development of method and algorithm for redundancy allocation problem.

III. OVERVIEW OF RESEARCH WORK

The research work is an attempt to develop a modified optimum design redundant system viz., N-modular redundant and TMR systems with a specific emphasis and involvement of failure rate and time of operation.

Thus, the optimum design models of k-out-of-n: G redundant systems that are discussed in this thesis consider a suitable and an alternative system cost function rate for this purpose. This system cost function is a modified one from the earlier models discussed by Nakagawa, Bai-et-al and Pham by involving these two parameters, i.e., failure rate and time of operation exclusively. The earlier optimum models stated above have developed optimum component size n* by minimizing a defined system cost function. These models do not involve the influence of failure rate and time element (t) of the system. The system cost function defined by Nakagawa represents cost of individual component (C_1) and cost of failure of the system (C_2) and failure rate (λ). The policy of the Nakagawa model which suggests optimum components (n*) does not involve the impact of failure rate except cost of component (C_1) and system down cost (C_2).

Similarly, the optimum model discussed by Bai-et-al as an extension of the Nakagawa model involves the presence of lethal common cause failure which defines a system cost function as a function of cost of individual components (C_1), cost of failure of the system (C_2), individual component constant failure rate(λ_1) and constant common cause failure rate (λ_2). Even in this model the author does not consider the time of operation of the system. But obviously the system operation and reliability will always be influenced by the elements of failure rate (λ) and time of operation (t). Therefore, the designing of optimum redundant system shall invariably be involved with the failure rate and time operation of the system.

IV. CONCLUSION

Thus, in this research investigation, an alternative optimum designing of redundant systems is attempted by suitably defining a system cost function which is slightly different from the models already discussed. The models discussed in this research work determines the optimum redundant components (n*) under the influence of both failure rate and time of operation of system. The research work derives optimum redundant system as a function of failure rate (λ) and time operation (t). This paper deals with the mathematical modeling of redundant system with the above ingredients to improve the system performance with minimum cost. These mathematical models suggest the optimal redundancy considering the system average cost rate function. This research wok successfully realizes this and the modified model proposed in this research not only bridges the gap but interestingly the result prove that the present proposed models are improved compared to earlier model proposed by Nakagawa and Bai-et-al. The optimal redundancy suggested in this paper is highly cost effective compared to earlier researchers.

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