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Fixed Point Theorem In Fuzzy 3-Metric Space

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Abstract

In this paper we prove a common fixed point theorem for three mappings in fuzzy metric space, fuzzy 2-metric space and fuzzy 3-metric space.

Keywords: Fixed point, Fuzzy 2-metric space and Fuzzy 3-metric space.

1. INTRODUCTION:

We know that the fixed points that can be discussed are of two types. The first types deals with contraction and are referred to as Banach fixed point theorems. The second types deals with compact mappings and is more involved. Metric fixed point theorem plays very important role. Many authors proved fixed point theorem in various spaces like Banach space, G- metric space, Hilbert space, cone metric space, soft metric space etc. Wadkar et al. [17-18] proved fixed point theorems in soft b-metric space. They are also proved point in different spaces such as dislocated metric space, soft metric

space, dislocated soft metric space (see[15],[16], [19]). In [13] the concept of fuzzy sets was introduced by Zadeh. Deng [4], Ekland and at el. [5], Kaleva and Seikkala [8], Kramosil and Michalek [9] have introduced the concept of fuzzy metric spaces in different ways. Many authors have also studied the fixed point theory in these fuzzy metric spaces are [2-6] and for fuzzy mappings are [1], [7], [10], [11]. Recently Wenzhi [12] and many others initiated the study of Probabilistic 2-metric spaces (or 2-PM spaces). We know that 2-metric space is a real valued function of a point triples on a set X, whose abstract properties were suggested by the area function in Euclidean spaces. Now it is natural to expect 3-metric space which is suggested by the volume function. The method of introducing this is naturally different from 2-metric space theory.
In this paper we prove a common fixed point theorem for three mappings in fuzzy metric space. We extend this result to fuzzy 2 metric and fuzzy 3-metric spaces. This result is motivated by [14].

2. PRELIMINARIES:

Def.2.1: A binary operation *: [0, 1] × [0, 1] → [0, 1] is called a t-norm in ([0, 1], *) if for all a, b, c, d∈ [0, 1] following conditions are satisfied:

i. a*1 = a,
ii. a*b = b*a,
iii. a*b ≤ c*d whenever a ≤ c and b ≤ d,
iv. a*(b*c) = (a*b)*c.

Def.2.2: The 3-tuple (X, M, *) is called a fuzzy metric space (FM space) if X is an arbitrary set, * is a continuous t- norm and M is a fuzzy set in $X^2 \times [0,\infty]$ satisfying the following conditions for all x, y, z ∈X and s, t > 0

FM-1 $M(x,y,0)=0$,

FM-2 $M(x,y,t)=1$ for all t >0 if and only if x= y,

FM-3 $M(x,y,t)=M(y,x,t)$,

FM-4 $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$

FM-5 $M(x,y,.):[0,1]\to[0,1]$ is left continuous,

FM-6 $\lim_{t\to\infty} M(x,y,t)=1$

Lemma 2.1: M(x, y, z,.) is non decreasing for all x, y, z ∈ X.

Def.2.3: Let (X, M, *) is a fuzzy metric space.

i. A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ and denoted by $\lim_{n\to\infty} x_n = x$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$, for all t > 0.

ii. A sequence $\{x_n\}$ in X is called a Cauchy sequence if ∀ t> 0and p > 0,
$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1.$$

iii. If every Cauchy sequence in a fuzzy metric space is convergent then it is complete.

Lemma 2.2: Let $\{y_n\}$ be a sequence in fuzzy metric space with the condition (FM-6). If there exist a number $q \in (0,1)$ such that
$$M(y_{n+2}, y_{n+1}, qt) \ge M(y_{n+1}, y_n, t)$$
for all t > 0 and n= 1, 2... , then $\{y_n\}$ is a Cauchy sequence.

Lemma 2.3: If for all x, y in X, t > 0 and for a number q $\in (0,1)$
$M(x, y, qt) \ge M(x, y, t)$, then x = y.

Remark 2.1: Lemma 2.1, 2.2 and 2.3 hold for fuzzy 2 metric spaces and fuzzy 3 metric spaces also.

Def.2.4: A function M is continuous in fuzzy metric space iff when ever, $x_n \to x$, $y_n \to y$ then $\lim_{n\to\infty} M(x_n, y_n, t) = M(x, y, t)$, for each $t > 0$.

Def.2.5: Two mappings A and S on a fuzzy metric space X are said to weakly commuting if $M(ASx, SAx, t) \ge M(Ax, Sx, t)$, $\forall x \in X$ and $t > 0$.

Def.2.6: A binary operation *: [0, 1] × [0, 1] × [0, 1] → [0, 1] is called a continuous t- norm if ([0, 1], *) is an abelian, topological monoid with unit 1 such that $a_1 * b_1 * c_1 \le a_2 * b_2 * c_2$ whenever $a_1 \le a_2$, $b_1 \le b_2$, $c_1 \le c_2$, for all $a_1, b_1, c_1, a_2, b_2, c_2$ in [0, 1].

Def.2.7: The 3-tuple (X, M, *) is called a fuzzy 2- metric space if X is an arbitrary set, * is a continuous t- norm and M is a fuzzy set in $X^3 \times [0,\infty]$ satisfying the following conditions for all x, y, z, u∈ X and $t_1, t_2, t_3 > 0$.

FM'-1 M(x, y, z, 0) = 0,

FM'-2 M(x, y, z, t) = 1, ∀t > 0 iff x= y,

FM'-3 M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t),
(Symmetric about three variables)

FM'-4 $M(x, y, z, t_1 + t_2 + t_3) \ge M(x, y, u, t_1) * M(x, u, z, t_2) * M(u, y, z, t_3)$,
(This corresponds to tetrahedron inequality in 2-metric space)

The function value M(x, y, z, t) may be interpreted as the probability that the area of triangle is less than t.

FM'-5 M(x, y, z, .):[0, 1)→[0, 1] is left continuous.

Def.2.8: Let (X, M, *) is called a fuzzy 2 metric space:

i. A sequence $\{x_n\}$ in fuzzy 2 metric space X is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, a, t) = 1$, for all a in X and t > 0.

ii. A sequence $\{x_n\}$ in fuzzy 2 metric space X is called a Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_n, a, t) = 1$, for all a in X and t> 0, p > 0.

iii. A fuzzy 2 metric space is said to be complete every Cauchy sequence is convergent.

Def.2.9: A function M is continuous in fuzzy 2 metric space iff whenever $x_n \to x,\ y_n \to y$ then

$$\lim_{n\to\infty} M(x_n, y_n, a, t) = M(x, y, a, t),\ \text{for all } a \in X \text{ and } t > 0.$$

Def.2.10: Two mappings A and S on a fuzzy 2 metric space X are said to weakly commuting if

M (ASx, SAx, a, t) ≥ M (Ax, Sx, a, t), ∀x, a ∈ X and t > 0.

Def.2.11: A binary operation *: $[0, 1]^4 \to [0, 1]$ is called a continuous t- norm if ([0, 1], *) is an abelian topological monoid with unit 1 such that $a_1 * b_1 * c_1 * d_1 \le a_2 * b_2 * c_2 * d_2$ whenever $a_1 \le a_2, b_1 \le b_2,\ c_1 \le c_2,\ d_1 \le d_2$, for all $a_1, b_1, c_1, a_2, b_2, c_2$ and d_1, d_2 are in [0,1].

Def.2.12: The 3-tuple (X, M, *) is called a fuzzy 3- metric space if X is an arbitrary set, * is a continuous t- norm and M is a fuzzy set in $X^4 \times [0,\infty]$ satisfying the following conditions for all x, y, z, u, w ∈ X and $t_1, t_2, t_3, t_4 > 0$.

FM''-1 M(x, y, z, w, 0) = 0,

FM''-2 M(x, y, z, w, t) = 1, ∀t > 0 iff x= y,
(Only when three simplex ⟨x, y, z, w⟩ degenerate)

FM''-3 M(x, y, z, w, t) = M(x, w, z, y, t) = M(y, z, w, x, t)= M(z, w, x, y, t)=....
(Symmetric about three variables)

FM''-4 $M(x, y, z, w, t_1 + t_2 + t_3 + t_4) \ge M(x, y, z, u, t_1) * M(x, y, u, w, t_2) * M(x, u, z, w, t_3) * M(u, y, z, w, t_4)$.

FM''-5 M(x, y, z, w, .):[0, 1)→[0, 1] is left continuous.

Def.2.13: Let (X, M, *) is called a fuzzy 3 metric space:

i. A sequence $\{x_n\}$ in fuzzy 3 metric space X is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, a, b, t) = 1$, for all a, b in X and t > 0.

ii. A sequence $\{x_n\}$ in fuzzy 3 metric space X is called a Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_n, a, b, t) = 1$, for all a, b in X and t> 0, p > 0.

iii. A fuzzy 3 metric space is said to be complete if every Cauchy sequence is convergent.

Def.2.14: A function M is continuous in fuzzy 3 metric space iff whenever $x_n \to x,\ y_n \to y$ then

$$\lim_{n\to\infty} M(x_n, y_n, a,, b, t) = M(x, y, a, b, t)\text{, for all } a, b \in X\ ,\ t > 0 .$$

Def.2.15: Two mappings A and S on a fuzzy 3metric space X are said to weakly commuting if

M (ASx, SAx, a, b, t) ≥ M (Ax, Sx, a, b, t), ∀x, a, b∈ X and t > 0

Lemma 2.4: let (X, M, *) be a fuzzy 2 metric space. If there exist k ∈ (0, 1) such that M(x, y, z, kt) ≥ M (x, y, z, t), for all x, y, z ∈ X with z ≠ x, z ≠ y and t >0 then x =y.

3. Main Results

Theorem 3.1: Let (X, M, *) be a complete fuzzy metric space with the condition (FM6). Let S and T be continuous mappings of X in X, then S and T have common fixed point in X if there exist a continuous mapping A of X into S(X)⋂T(X) which commutes with S & T and

$$M\left(Ax, Ay, qt\right) \ge \min\left\{M\left(Sx, Ay, t\right), M\left(Tx, Ax, t\right), M\left(Ty, Ax, t\right)\right\} \tag{1}$$

for all $x, y, z \in X, t > 0$ and $0 < q < 1$. Then S, T and A have a unique common fixed point.

Proof: We define sequences $\{x_n\}$ such that $Ax_{2n} = Sx_{2n-1}$ and $Ax_{2n-1} = STx_{2n}$, $n = 1, 2...$, we shall prove that $\{Ax_n\}$ is a Cauchy sequence. For this put $x = x_{2n}$ and $y = x_{2n+1}$ in (1), we write

$$M\left(Ax_{2n}, Ax_{2n+1}, qt\right) \ge \min\left\{\begin{matrix} M\left(Sx_{2n}, Ax_{2n+1}, t\right), M\left(Tx_{2n}, Ax_{2n}, t\right), \\ M\left(Tx_{2n+1}, Ax_{2n}, t\right) \end{matrix}\right\}$$

$$\ge \min\left\{M\left(Ax_{2n+1}, Ax_{2n+1}, t\right), M\left(Ax_{2n-1}, Ax_{2n}, t\right), M\left(Ax_{2n}, Ax_{2n}, t\right)\right\}$$

$$\ge M\left(Ax_{2n-1}, Ax_{2n}, t\right)$$

$$\geq M\left(Ax_{2n-1}, Ax_{2n}, t/q\right)$$

Therefore $M\left(Ax_{2n}, Ax_{2n+1}, qt\right) \geq M\left(Ax_{2n-1}, Ax_{2n}, t/q\right)$

By induction $M\left(Ax_{2k}, Ax_{2m+1}, qt\right) \geq M\left(Ax_{2k-1}, Ax_{2m}, t/q\right)$

For every k and m in N. further if $2m+1 > 2k$ then

$$M\left(Ax_{2k}, Ax_{2m+1}, qt\right) \geq M\left(Ax_{2k-1}, Ax_{2m}, t/q\right)$$
$$\ldots$$
$$\geq M\left(Ax_0, Ax_{2m+1-2k}, t/q^{2k}\right) \tag{2}$$

If $2k > 2m+1$ then

$$M\left(Ax_{2k}, Ax_{2m+1}, qt\right) \geq M\left(Ax_{2k-1}, Ax_{2m}, t/q\right)$$
$$\ldots \tag{3}$$
$$\geq M\left(Ax_{2k-(2m+1)}, Ax_0, t/q^{2m+1}\right)$$

By simple induction with (2) and (3) we have

$$M\left(Ax_n, Ax_{n+p}, qt\right) \geq M\left(Ax_0, Ax_p, t/q^n\right)$$

For $n = 2k,\ p = 2m+1$ and by (FM-4)

$$M\left(Ax_n, Ax_{n+p}, qt\right) \geq M\left(Ax_0, Ax_1, t/2q^n\right) * M\left(Ax_1, Ax_p, t/2q^n\right) \tag{4}$$

If $n = 2k,\ p = 2m$ or $n = 2k+1,\ p = 2m$, for every positive integer p & n in N, by noting that $M\left(Ax_0, Ax_p, t/q^n\right) \to 1$ as $n \to \infty$. Thus $\{Ax_n\}$ is a Cauchy sequence. Since the space X is complete, there exist $z = \lim_{n\to\infty} Ax_n$ and $z = \lim_{n\to\infty} Sx_{2n-1} = \lim_{n\to\infty} Tx_{2n}$. It follows that $Az = Sz = Tz$ and

$$M\left(Az, A^2z, qt\right) \geq M\left(Az, AAz, qt\right)$$
$$\geq \min\left\{M\left(Sz, AAz, t\right), M\left(Tz, Az, t\right), M\left(TAz, Az, t\right)\right\}$$
$$\geq \min\left\{M\left(Sz, ATz, t\right), M\left(Az, Az, t\right), M\left(ATz, Az, t\right)\right\}$$
$$\geq \min\left\{M\left(Sz, ATz, t\right), M\left(Az, Az, t\right), M\left(ATz, Sz, t\right)\right\}$$
$$\geq M\left(Sz, ATz, t\right)$$
$$\geq M\left(Sz, AAz, t\right)$$
$$\geq M\left(Az, A^2z, t\right)$$
$$\ldots$$
$$\geq M\left(Az, A^2z, t/q^n\right)$$

Since $\lim_{n\to\infty} M\left(Az, A^2z, t/2q^n\right) = 1$ so $Az = A^2z$

Thus z is common fixed point of A, S & T.

For uniqueness, let w (w ≠ z) be another common fixed point of S, T and A by (1) we write

$$M\left(Az, Aw, qt\right) \geq \min\left\{M\left(Sz, Aw, t\right), M\left(Tz, Az, t\right), M\left(Tw, Az, t\right)\right\}$$

This implies $M\left(z, w, qt\right) \geq M\left(z, w, t\right)$.

Therefore by lemma 2.3, we write z = w. This completes the proof of the theorem 3.1. Now we prove theorem for fuzzy 2 metric spaces.

Theorem 3.2: Let (X, M, *) be a complete fuzzy 2- metric space. Let S &T be continuous mappings of X in X, then S & T have common fixed point in X if there exist a continuous mapping A of X into S(X)⋂T(X) which commutes with S & T and

$$M\left(Ax, Ay, a, qt\right) \geq \min\left\{M\left(Sx, Ay, a, t\right), M\left(Tx, Ax, a, t\right), M\left(Ty, Ax, a, t\right)\right\} \quad (5)$$

$\forall x, y, a \in X, t > 0$ & $0 < q < 1$. $\lim_{t\to\infty} M(x, y, z, t) = 1, \forall x, y, z \in X.$ (6)

Then S, T and A have a unique common fixed point.

Proof: We define sequences $\{x_n\}$ such that $Ax_{2n} = Sx_{2n-1}$ and $Ax_{2n-1} = STx_{2n}$, $n = 1, 2...$, we shall prove that $\{Ax_n\}$ is a Cauchy sequence. For this put $x = x_{2n}$ and $y = x_{2n+1}$ in (5), we write

$$M\left(Ax_{2n}, Ax_{2n+1}, a, qt\right) \geq \min\left\{\begin{matrix} M\left(Sx_{2n}, Ax_{2n+1}, a, t\right), M\left(Tx_{2n}, Ax_{2n}, a, t\right), \\ M\left(Tx_{2n+1}, Ax_{2n}, a, t\right) \end{matrix}\right\}$$

$$\geq \min\left\{\begin{matrix} M\left(Ax_{2n+1}, Ax_{2n+1}, a, t\right), M\left(Ax_{2n-1}, Ax_{2n}, a, t\right), \\ M\left(Ax_{2n}, Ax_{2n}, a, t\right) \end{matrix}\right\}$$

$$\geq M\left(Ax_{2n-1}, Ax_{2n}, a, t\right)$$

$$\geq M\left(Ax_{2n-1}, Ax_{2n}, a, t/q\right)$$

Therefore $M\left(Ax_{2n}, Ax_{2n+1}, a, qt\right) \geq M\left(Ax_{2n-1}, Ax_{2n}, a, t/q\right)$

By induction, for every k and m in N, we have

$$M\left(Ax_{2k}, Ax_{2m+1}, a, qt\right) \geq M\left(Ax_{2k-1}, Ax_{2m}, a, t/q\right).$$

Further if $2m+1 > 2k$ then

$$M\left(Ax_{2k}, Ax_{2m+1}, a, qt\right) \geq M\left(Ax_{2k-1}, Ax_{2m}, a, t/q\right)$$
$$\ldots$$
$$\geq M\left(Ax_0, Ax_{2m+1-2k}, a, t/q^{2k}\right) \qquad (7)$$

If $2k > 2m+1$ then

$$M\left(Ax_{2k}, Ax_{2m+1}, a, qt\right) \geq M\left(Ax_{2k-1}, Ax_{2m}, a, t/q\right)$$
$$\ldots$$
$$\geq M\left(Ax_{2k-(2m+1)}, Ax_0, a, t/q^{2m+1}\right) \qquad (8)$$

By simple induction with (7) and (8) we have

$$M\left(Ax_n, Ax_{n+p}, a, qt\right) \geq M\left(Ax_0, Ax_p, a, t/q^n\right)$$

For $n = 2k$, $p = 2m+1$ and by (FM-4)

$$M\left(Ax_n, Ax_{n+p}, a, qt\right) \geq M\left(Ax_0, Ax_p, Ax_1, a, t/3q^n\right) * M\left(Ax_0, Ax_1, a, t/3q^n\right)$$
$$* M\left(Ax_1, Ax_p, a, t/3q^n\right) \qquad (9)$$

If $n = 2k$, $p = 2m$ or $n = 2k+1$, $p = 2m$, for every positive integer p and n in N, by noting that $M\left(Ax_0, Ax_p, a, t/q^n\right) \to 1$ as $n \to \infty$. Thus $\{Ax_n\}$ is a Cauchy sequence. Since the space X is complete, there exists $z = \lim_{n\to\infty} Ax_n$ and $z = \lim_{n\to\infty} Sx_{2n-1} = \lim_{n\to\infty} Tx_{2n}$. It follows that $Az = Sz = Tz$ and

$$M\left(Az, A^2z, a, qt\right) \geq M\left(Az, AAz, a, qt\right)$$
$$\geq \min\left\{M\left(Sz, AAz, a, t\right), M\left(Tz, Az, a, t\right), M\left(TAz, Az, a, t\right)\right\}$$
$$\geq \min\left\{M\left(Sz, ATz, a, t\right), M\left(Az, Az, a, t\right), M\left(ATz, Az, a, t\right)\right\}$$
$$\geq \min\left\{M\left(Sz, ATz, a, t\right), M\left(Az, Az, a, t\right), M\left(ATz, Sz, a, t\right)\right\}$$
$$\geq M\left(Sz, ATz, a, t\right)$$
$$\geq M\left(Sz, AAz, a, t\right)$$
$$\geq M\left(Az, A^2z, a, t\right)$$
$$\ldots$$
$$\geq M\left(Az, A^2z, a, t/q^n\right)$$

Since $\lim_{n\to\infty} M\left(Az, A^2z, a, t/q^n\right) = 1$, so $Az = A^2z$

Thus z is common fixed point of A, S & T.

For uniqueness, let w (w ≠ z) be another common fixed point of S, T and A. By (5) we write

$$M\left(Az,Aw,a,qt\right)\geq \min\left\{M\left(Sz,Aw,a,t\right),M\left(Tz,Az,a,t\right),M\left(Tw,Az,a,t\right)\right\}$$

This implies $M\left(z,w,a,qt\right)\geq M\left(z,w,a,t\right)$.

Therefore by lemma 2.3, we write z = w. This completes the proof of the theorem 3.2. Now we prove theorem 3.1 for fuzzy 3 metric spaces.

Theorem 3.3: Let(X, M, *) be a complete fuzzy 3- metric space. Let S and T be continuous mappings of X in X, then S and T have common fixed point in X if there exist a continuous mapping A of X into S(X)⋂T(X) which commutes with S & T and

$$M\left(Ax,Ay,a,b,qt\right)\geq \min\left\{\begin{matrix}M\left(Sx,Ay,a,b,t\right),M\left(Tx,Ax,a,b,t\right),\\ M\left(Ty,Ax,a,b,t\right)\end{matrix}\right\} \tag{10}$$

$\forall x,y,a,\mathrm{b}\in X, t>0,\ 0<q<1.$ $\lim_{t\to\infty}M(x,y,z,w,t)=1\ \forall w,x,y,z\in X.$ (11)

Then S, T and A have a unique common fixed point.

Proof: we define sequences $\{x_n\}$ such that $Ax_{2n}=Sx_{2n-1}$ and $Ax_{2n-1}=STx_{2n}$, $n=1,2..$, we shall prove that $\{Ax_n\}$ is a Cauchy sequence. For this suppose $x=x_{2n}$ and $y=x_{2n+1}$ in (10), we write

$$M\left(Ax_{2n},Ax_{2n+1},a,b,qt\right)\geq \min\left\{\begin{matrix}M\left(Sx_{2n},Ax_{2n+1},a,b,t\right),M\left(Tx_{2n},Ax_{2n},a,b,t\right),\\ M\left(Tx_{2n+1},Ax_{2n},a,b,t\right)\end{matrix}\right\}$$

$$\geq \min\left\{\begin{matrix}M\left(Ax_{2n+1},Ax_{2n+1},a,b,t\right),M\left(Ax_{2n-1},Ax_{2n},a,b,t\right),\\ M\left(Ax_{2n},Ax_{2n},a,b,t\right)\end{matrix}\right\}$$

$$\geq M\left(Ax_{2n-1},Ax_{2n},a,b,t\right)$$

$$\geq M\left(Ax_{2n-1},Ax_{2n},a,b,t/q\right)$$

Therefore $M\left(Ax_{2n},Ax_{2n+1},a,b,qt\right)\geq M\left(Ax_{2n-1},Ax_{2n},a,b,t/q\right)$

By induction for every k and m in N

$$M\left(Ax_{2k},Ax_{2m+1},a,b,qt\right)\geq M\left(Ax_{2k-1},Ax_{2m},a,b,t/q\right)$$

Further if $2m+1>2k$ then

$$M\left(Ax_{2k},Ax_{2m+1},a,b,qt\right)\geq M\left(Ax_{2k-1},Ax_{2m},a,b,t/q\right)$$

$$\cdots$$

$$\geq M\left(Ax_0,Ax_{2m+1-2k},a,b,t/q^{2k}\right) \tag{12}$$

If $2k>2m+1$ then

$$M\left(Ax_{2k}, Ax_{2m+1}, a, b, qt\right) \geq M\left(Ax_{2k-1}, Ax_{2m}, a, b, t/q\right)$$
$$\cdots$$
$$\geq M\left(Ax_{2k-(2m+1)}, Ax_0, a, b, t/q^{2m+1}\right) \quad (13)$$

By simple induction with (12) and (13) we have

$$M\left(Ax_n, Ax_{n+p}, a, b, qt\right) \geq M\left(Ax_0, Ax_p, a, b, t/q^n\right)$$

For $n = 2k$, $p = 2m+1$ and by (FM-4)

$$\begin{aligned} M\left(Ax_n, Ax_{n+p}, a, b, qt\right) \geq\ & M\left(Ax_0, Ax_p, a, Ax_1, t/4q^n\right) \\ & *M\left(Ax_0, Ax_p, Ax_1, b, t/4q^n\right) \\ & *M\left(Ax_0, Ax_1, a, b, t/4q^n\right) \\ & *M\left(Ax_1, Ax_p, a, b, t/4q^n\right) \end{aligned} \quad (14)$$

If $n = 2k$, $p = 2m$ or $n = 2k+1$, $p = 2m$, for every positive integer p and n in N, by noting that $M\left(Ax_0, Ax_p, a, b, t/q^n\right) \to 1$ as $n \to \infty$.

Thus $\{Ax_n\}$ is a Cauchy sequence. Since the space X is complete, there exists $z = \lim_{n\to\infty} Ax_n$ and $z = \lim_{n\to\infty} Sx_{2n-1} = \lim_{n\to\infty} Tx_{2n}$

It follows that $Az = Sz = Tz$ and

$$\begin{aligned} M\left(Az, A^2z, a, b, qt\right) &\geq M\left(Az, AAz, a, b, qt\right) \\ &\geq \min\left\{M\left(Sz, AAz, a, b, t\right), M\left(Tz, Az, a, b, t\right), M\left(TAz, Az, a, b, t\right)\right\} \\ &\geq \min\left\{M\left(Sz, ATz, a, b, t\right), M\left(Az, Az, a, b, t\right), M\left(ATz, Az, a, b, t\right)\right\} \\ &\geq \min\left\{M\left(Sz, ATz, a, b, t\right), M\left(Az, Az, a, b, t\right), M\left(ATz, Sz, a, b, t\right)\right\} \\ &\geq M\left(Sz, ATz, a, b, t\right) \\ &\geq M\left(Sz, AAz, a, b, t\right) \\ &\geq M\left(Az, A^2z, a, b, t\right) \\ &\ldots\ldots \\ &\geq M\left(Az, A^2z, a, b, t/q^n\right) \end{aligned}$$

Since $\lim_{n\to\infty} M\left(Az, A^2z, a, b, t/q^n\right) = 1$, so $Az = A^2z$.

Thus z is common fixed point of A, S & T.

For uniqueness, let w (w ≠ z) be another common fixed point of S, T and A. By (10) we write

$$M\left(Az, Aw, a, b, qt\right) \geq \min\begin{Bmatrix} M\left(Sz, Aw, a, b, t\right), M\left(Tz, Az, a, b, t\right), \\ M\left(Tw, Az, a, b, t\right) \end{Bmatrix}$$

This implies $M\left(z, w, a, b, qt\right) \geq M\left(z, w, a, b, t\right)$

Therefore by lemma 2.3, we write z = w. This completes the proof of the theorem 3.3.

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