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Dynamical behaviour of an exploited fish species obeying modified logistic growth function with taxation as a control instrument

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Abstract

In this paper we discuss and analyse a mathematical model to study the dynamical behaviour of an exploited fish species which obeys the *modified logistic growth function* [1]. Fishing is permitted after imposing tax per unit harvested biomass by the Government or private agencies in order to control over exploitation. The steady states of the dynamical system are determined. The local stability for the non-trivial steady states is discussed. The global stability of the non-trivial interior equilibrium is also studied. It is also examined whether the system possesses any limit cycle. All the results are illustrated with the help of four numerical examples.

Keywords: modified logistic growth function, steady states, variational matrix, local stability, limit cycle, global stability.

1. INTRODUCTION:

Fish is a major renewable resource for the human community. However, some of the fish species are likely to become extinct due to excessive harvesting. So the Government or the private agencies have to monitor and regulate the over exploitation of the species. Various techniques in regulating fisheries have been discussed by

Anderson and Lee [2], Sutinen and Anderson [3] and others. Taxation, license fees, lease of property rights, seasonal harvesting etc. are usually considered as possible governing instrument in fishery regulation. Among these, taxation is superior to other control policies because of its flexibility. As described by Clark ([1], Art.4.6, p-116), "Economists are particularly attracted to taxation, partly because of its flexibility and partly because many of the advantages of a competitive economic system can be better maintained under taxation than under other regulatory methods".

A single species fishery model using taxation as a control measure was first discussed by Clark [1]. Chaudhury and Johnson [4] extended that model using a realistic catch-rate function. Ganguly and Chaudhury [5] made a capital theoretic study of a single species with taxation as control policy. Pradhan and Chaudhury [6] studied a fully dynamic reaction model of fishery consisting of two competing fish species with taxation as a control instrument. Ray and Pradhan [7] developed a dynamic reaction model of an inshore-offshore fishery with taxation as a control instrument. In all these models growth functions of the species are considered as simple logistic growth function $rx\left(1-\frac{x}{k}\right)$. In this growth function it is assumed that the natural growth rate of the species is directly proportional to the population density. Nobody considered the modified logistic growth function $rx^{\alpha}\left(1-\frac{x}{k}\right)$ $(\alpha > 0)$ as the growth rate of the species for its complexity. For $\alpha = 1$ the modified growth function becomes the simple logistic growth function.

In this paper we consider that the growth rate of the species obeys the modified logistic growth rate. It is also assumed that the regulatory agencies regulate the fishery by imposing the tax per unit harvested biomass for controlling over exploitation. Different ranges of tax are determined for which the non trivial steady states of the dynamical system exist. The fishing effort considered here is a dynamic variable depending on the net revenue to be earned by the fishermen. The existence of the steady states of the system is discussed. The local stability of the non trivial steady states is studied. The global stability of the non trivial interior steady state is also studied. It is also examined whether the system possesses any limit cycle. Four numerical examples are given as an illustration.

2. THE MODEL:

Let at any time t, $x(t)$ be the population density of a particular fish species which obeys the modified logistic growth function $f(x) = rx^{\alpha}\left(1-\frac{x}{k}\right)$. Here α is a positive constant, *r* is the intrinsic growth rate of the fish species and *k* is the carrying capacity of the environment in which the species live. When $\alpha = 1$, the growth function becomes the simple logistic growth function [1] which is used by many researchers in different fishery models.

For $0 < \alpha < 1$, the growth rate is low and for $\alpha > 1$, the growth rate is high compared to the growth rate which is simply proportional to the population density.

Now, $f^{/}(x) = rx^{\alpha-1}\left\{\alpha - \left(\frac{\alpha+1}{k}\right)x\right\} = 0 \Rightarrow x = \frac{k\alpha}{\alpha+1}$.

Therefore, $f^{/}(x) > 0$ for $0 < x < \frac{k\alpha}{\alpha+1}$ and $f^{/}(x) < 0$ for $\frac{k\alpha}{\alpha+1} < x < k$.

Thus the growth rate of the species is increasing in the interval $\left(0, \frac{k\alpha}{\alpha+1}\right)$ and decreasing in the interval $\left(\frac{k\alpha}{\alpha+1}, k\right)$. The extreme value of the population is $\frac{k\alpha}{\alpha+1}$ where the growth rate is maximum. Following figures are the growth curves for different values of α when we assume the intrinsic growth rate of the population is $r = 5$ and the environmental carrying capacity $k = 100$ in appropriate units.

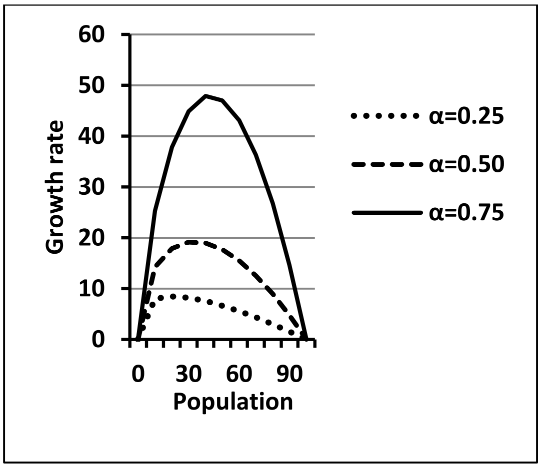

Fig. 1: Growth curves for 0 $< \alpha < 1$

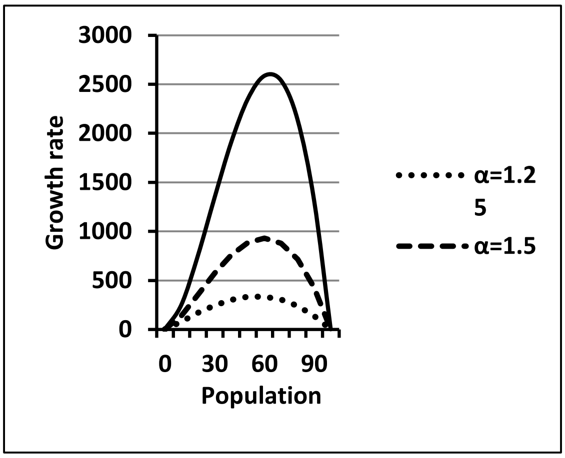

Fig. 2: Growth curves for α ≥ 1

Now, we assume that fishing is allowed to harvest this species after imposing a suitable tax by the regulatory agencies to control over exploitation of the species. Let the regulatory agencies impose the tax τ (> 0) per unit biomass of the harvested fish. $\tau < 0$ implies the subsidy given to the fishermen when fishing runs in loss. But in the present social scenario, we assume that it is not possible to give the subsidy to the loser fishermen. It is assumed that if the fishing runs in loss, then fishing will be stopped.

Let $E(t)$ be the effort for harvesting the fish species at any time *t*.

The net economic revenue to the fishermen (perceived rent) is $\{q(p-\tau)x - c\}E$, where p is the market price per unit biomass of the harvested fish, q is the catchability coefficient and *c* is the cost per unit effort for harvesting the species.

Considering $E(t)$ as the dynamic variable i.e. time dependent variable, we have

$\frac{dE(t)}{dt} = \lambda\{q(p-\tau)x - c\}E$, where λ is the stiffness parameter measuring the effort and the perceived rent.

Therefore, we have the following dynamical system:

$$\left.\begin{aligned} \frac{dx}{dt} &= rx^{\alpha}\left(1-\frac{x}{k}\right) - qEx, \quad \alpha > 0 \\ \frac{dE}{dt} &= \lambda\{q(p-\tau)x - c\}E \end{aligned}\right\} \tag{1}$$

3. DYNAMICAL BEHAVIOUR:

A) Existence of steady states:

The dynamical system (1) has the following steady states:

i) $P_0 = (0,0)$ is the trivial equilibrium point which always exists,

ii) $P_1 = (k,0)$ is the boundary (axial) equilibrium point which always exists as the fish species reach to the maximum population level, i.e. the carrying capacity of the environment if there is no harvesting,

iii) $P_2 = (x_1, E_1)$ is the non trivial interior equilibrium point which exists under certain condition.

Here, $x_1 = \frac{c}{q(p-\tau)}$ (2)

and $E_1 = \frac{1}{q}\left\{rx_1^{\alpha-1}\left(1-\frac{x_1}{k}\right)\right\}$ (3)

Now, $E_1 > 0$ iff $x_1 < k$ i.e. $\tau < p - \frac{c}{kq}$ (4)

Let $\tau_{max} = p - \frac{c}{kq}$. (5)

Therefore, for existence of the non trivial interior steady state $P_2(x_1, E_1)$, the regulatory agencies have to select the tax τ per unit biomass of the harvested fish such that

$0 < \tau < \tau_{max}$.

When $\tau = \tau_{max}$, then $E_1 = 0$, i.e. the equilibrium becomes the boundary (axial) equilibrium.

B) Local stability analysis for non-trivial steady states:

The variational matrix of the system (1) is $V(x,E) = (J_{ij})_{2\times 2}$ $(i,j = 1,2)$.

Where $J_{11} = \frac{\partial}{\partial x}\left(\frac{dx}{dt}\right) = r\alpha x^{\alpha-1}\left(1-\frac{x}{k}\right) - \frac{rx^{\alpha}}{k} - qE,$ (6)

$J_{12} = \frac{\partial}{\partial E}\left(\frac{dx}{dt}\right) = -qE,$ (7)

$J_{21} = \frac{\partial}{\partial x}\left(\frac{dE}{dt}\right) = \lambda q(p-\tau)E$ (8)

and $J_{22} = \frac{\partial}{\partial E}\left(\frac{dE}{dt}\right) = \lambda\{q(p-\tau)x - c\}.$ (9)

Now, $V(k,0) = \begin{pmatrix} -rk^{\alpha-1} & 0 \\ 0 & \lambda\{q(p-\tau)k - c\} \end{pmatrix}$.

Therefore, the characteristic roots of $V(k,0)$ are $-rk^{\alpha-1}(<0)$ and $\lambda\{q(p-\tau)k-c\}$.

If $\tau < p-\frac{c}{kq}=\tau_{max}$ then $\lambda\{q(p-\tau)k-c\}>0$ and then $P_1(k,0)$ is a saddle point i.e. $P_1(k,0)$ is a unstable equilibrium point.

Since $\tau < p-\frac{c}{kq}=\tau_{max}$ is the necessary and sufficient condition for the existence of the non trivial interior equilibrium point $P_2(x_1,E_1)$, so when the non trivial interior equilibrium point exists then the boundary equilibrium point is unstable.

If $\tau > \tau_{max}$, then $\lambda\{q(p-\tau)k-c\}<0$ and so the boundary equilibrium $P_1(k,0)$ is asymptotically stable node.

In reality, it is true that when the regulatory agency imposes a tax $\tau > \tau_{max}$, the harvesting effort will be zero and then the boundary equilibrium will be automatically stable.

Now we discuss about the stability of the non trivial interior equilibrium point $P_2(x_1,E_1)$.

$$V(x_1,E_1)=\begin{pmatrix} J_{11}(x_1,E_1) & J_{12}(x_1,E_1) \\ J_{21}(x_1,E_1) & J_{22}(x_1,E_1) \end{pmatrix}$$

Trace of $V(x_1,E_1)=J_{11}(x_1,E_1)+J_{22}(x_1,E_1)$ and

$\det V(x_1,E_1)=J_{11}(x_1,E_1)J_{22}(x_1,E_1)-J_{12}(x_1,E_1)J_{21}(x_1,E_1)$,

where $J_{11}(x_1,E_1)=r\alpha x_1^{\alpha-1}\left(1-\frac{x_1}{k}\right)-\frac{rx_1^{\alpha}}{k}-qE_1$ by (6). (10)

$J_{22}(x_1,E_1)=0$ by (9), $J_{12}(x_1,E_1)=-qE_1$ by (7) and $J_{21}(x_1,E_1)=\lambda q(p-\tau)E_1$ by (8).

Therefore, $\det V(x_1,E_1)=\lambda q^2(p-\tau)E_1^2>0$.

So product of the eigen values of the matrix $V(x_1,E_1)$ is positive.

We now discuss the following two cases:

Case I: $0<\alpha\le 1$.

$$\begin{aligned}\text{Trace of } V(x_1,E_1) &= J_{11}(x_1,E_1)\\ &= r\alpha x_1^{\alpha-1}\left(1-\frac{x_1}{k}\right)-\frac{rx_1^{\alpha}}{k}-qE_1 \quad \text{by (10)}\\ &= r\alpha x_1^{\alpha-1}\left(1-\frac{x_1}{k}\right)-\frac{rx_1^{\alpha}}{k}-rx_1^{\alpha-1}\left(1-\frac{x_1}{k}\right), \text{ from } \frac{dx}{dt}=0.\\ &= rx_1^{\alpha-1}\left(1-\frac{x_1}{k}\right)(\alpha-1)-\frac{rx_1^{\alpha}}{k}\le 0.\end{aligned}$$

Therefore, if $0<\alpha\le 1$, then sum of the eigen values of $V(x_1,E_1)$ is negative and the product of the eigen values of $V(x_1,E_1)$ is positive. So the eigen values of $V(x_1,E_1)$ are either both real and negative or complex conjugate with negative real parts.

So the non trivial interior equilibrium point $P_2(x_1,E_1)$ is either a stable node or stable focus when $0<\alpha\le 1$.

Case II: $\alpha>1$.

Trace of $V(x_1, E_1) = J_{11}(x_1, E_1)$

$$= r\left\{(\alpha-1)x_1^{\alpha-1} - \frac{(\alpha-1)x_1^{\alpha}}{k} - \frac{x_1^{\alpha}}{k}\right\}$$

$$= rx_1^{\alpha-1}\left\{\alpha - 1 - \frac{\alpha x_1}{k}\right\}$$

Therefore, Trace of $V(x_1, E_1) < 0$ iff $\frac{\alpha x_1}{k} > \alpha - 1$.

Now, $\frac{\alpha x_1}{k} > \alpha - 1 \Rightarrow \tau > p - \frac{c\alpha}{kq(\alpha-1)}$ by (2).

But the interior equilibrium $P_2(x_1, E_1)$ exists only when $\tau < p - \frac{c}{kq}$.

Since the product of the eigen values of $V(x_1, E_1)$ is positive, so when $\alpha > 1$, the non trivial interior equilibrium $P_2(x_1, E_1)$ is locally asymptotically stable, either a stable node or a stable focus iff $p - \frac{c\alpha}{kq(\alpha-1)} < \tau < p - \frac{c}{kq}$. (11)

If $\tau < min\left\{p - \frac{c}{kq}, p - \frac{c\alpha}{kq(\alpha-1)}\right\} = p - \frac{c\alpha}{kq(\alpha-1)}$, then $J_{11}(x_1, E_1) > 0$

i.e. trace of $V(x_1, E_1) > 0$. So the non trivial interior equilibrium point $P_2(x_1, E_1)$ is unstable.

Thus in order to existence of the stable non trivial interior equilibrium point $P_2(x_1, E_1)$ of the system (1), the regulatory agencies should follow the following criterion:

i) When $0 < \alpha \leq 1$, then the non trivial interior equilibrium $P_2(x_1, E_1)$ exists if the agencies impose the tax τ such that $\tau < p - \frac{c}{kq}$ and it is always asymptotically stable.

ii) When $\alpha > 1$, then the non trivial interior equilibrium $P_2(x_1, E_1)$ exists and is asymptotically stable if the regulatory agencies impose the tax τ such that
$p - \frac{c\alpha}{kq(\alpha-1)} < \tau < p - \frac{c}{kq}$.

If $\tau < p - \frac{c\alpha}{kq(\alpha-1)}$, then the non trivial interior equilibrium point exists but it is unstable.

C) Limit Cycles:

We now examine the existence of limit cycles of the dynamical system (1) by using Bendixon-Dulac criterion [8].

We define a Dulac function $B(x, E) = x^{-\alpha}E^{-1}$.

Now, $\frac{\partial}{\partial x}\left(B\frac{dx}{dt}\right) + \frac{\partial}{\partial E}\left(B\frac{dE}{dt}\right)$

$$= \frac{\partial}{\partial x}\left\{r\left(1 - \frac{x}{K}\right)E^{-1} - qx^{1-\alpha}\right\} + \frac{\partial}{\partial E}[\lambda\{q(p-\tau)x - c\}x^{-\alpha}]$$

$$= -\frac{rE^{-1}}{k} - q(1-\alpha)x^{-\alpha} < 0 \text{ if } 0 < \alpha \leq 1.$$

If $\alpha > 1$, then $\frac{\partial}{\partial x}\left(B\frac{dx}{dt}\right) + \frac{\partial}{\partial E}\left(B\frac{dE}{dt}\right) = q(1-\alpha)x^{-\alpha} - \frac{rE^{-1}}{k} < 0$ if $rx^{\alpha} > kq(\alpha-1)E$.

Therefore, $\frac{\partial}{\partial x}\left(B\frac{dx}{dt}\right)+\frac{\partial}{\partial E}\left(B\frac{dE}{dt}\right)<0\ \forall(x,E)\in\Omega$
where $\Omega=\{(x,E): rx^{\alpha}>kq(\alpha-1)E, x>0, E>0\}$.
From the above results it is clear that, if $0<\alpha\le 1$, then the system (1) does not possess any limit cycle in the region $R_2^+=\{(x,E): x>0, E>0\}$. If $\alpha>1$, then the system (1) does not possess any limit cycle in the region Ω.

D) Global stability:

The dynamical system (1) has a unique positive non trivial interior equilibrium point $P_2(x_1,E_1)$ provided the condition (4) holds. We now examine the global stability of $P_2(x_1,E_1)$ of the system (1). For the fixed environmental carrying capacity k, $x_1\le k$ and the effort of harvesting $E_1\le E_{max}$, where E_{max} is the maximum effort given by the fishermen. So the solution (x_1,E_1) of the dynamical system (1) is uniformly bounded in the finite region W which is a sub set of $R_2^+=\{(x,E): x>0, E>0\}$.
We now define a Lyapunov function [9] as follows:
$L(x,E)=x-x_1-x_1 ln\left(\frac{x}{x_1}\right)+d\left\{E-E_1-E_1 ln\left(\frac{E}{E_1}\right)\right\}$, where d is a suitable positive constant to be determined in the subsequence steps. $L(x,E)$ is a positive definite function in the region W except at $P_2(x_1,E_1)$ where it vanishes.
Here $L(x_1,E_1)=0$ and $\lim_{(x,E)\to(0,0)} L(x,E)=\lim_{(x,E)\to(\infty,\infty)} L(x,E)=\infty$.
The time derivative of $L(x,E)$ along the solution of (1) is

$$\frac{d}{dt}\{L(x,E)\}=\left(\frac{x-x_1}{x}\right)\frac{dx}{dt}+d\left(\frac{E-E_1}{E}\right)\frac{dE}{dt}$$

$$=(x-x_1)\left\{rx^{\alpha-1}\left(1-\frac{x}{k}\right)-qE\right\}+d\lambda(E-E_1)\{q(p-\tau)x-c\}$$

$$=(x-x_1)\left\{rx^{\alpha-1}\left(1-\frac{x}{k}\right)-qE-r{x_1}^{\alpha-1}\left(1-\frac{x_1}{k}\right)+qE_1\right\}$$
$$+d\lambda(E-E_1)\{q(p-\tau)x-c-q(p-\tau)x_1+c\}$$

$$=(x-x_1)\left\{r(x^{\alpha-1}-x_1^{\alpha-1})-\frac{r}{k}(x^{\alpha}-x_1^{\alpha})\right\}-q(x-x_1)(E-E_1)$$
$$+d\lambda q(p-\tau)(E-E_1)(x-x_1)$$

$$=r(x-x_1)(x^{\alpha-1}-x_1^{\alpha-1})-\frac{r}{k}(x-x_1)(x^{\alpha}-x_1^{\alpha})\ \text{ for } d=\frac{1}{\lambda(p-\tau)}>0.$$

$$=\frac{r}{k}(x-x_1)(kx^{\alpha-1}-kx_1^{\alpha-1}-x^{\alpha}+x_1^{\alpha})<0\ \forall(x,E)\in\Omega_1\cup\Omega_2 \text{ where}$$

$\Omega_1=\{(x,E): x<x_1, x^{\alpha}-kx_1^{\alpha-1}<x_1^{\alpha}-kx_1^{\alpha-1}\}$ and
$\Omega_2=\{(x,E): x>x_1, x^{\alpha}-kx_1^{\alpha-1}>x_1^{\alpha}-kx_1^{\alpha-1}\}$.

Also $\frac{d}{dt}\{L(x,E)\}=0$ at $P_2(x_1,E_1)$.
Hence by Lassalle's invariance principle [10], $P_2(x_1,E_1)$ is globally asymptotically stable for all $(x,E)\in\Omega_1\cup\Omega_2$.

4. NUMERICAL EXAMPLES:

i) Let $\boldsymbol{\alpha = 0.5}$, $r = 5$, $k = 100$, $q = 0.01$, $\lambda = 1$, $p = 30, c = 20$ in appropriate units.

Now, $p - \frac{c}{kq} = 10$.

If the regulatory agencies impose the tax τ such that $0 \le \tau < 10$, then the non trivial interior equilibrium point $P_2(x_1, E_1)$ of the dynamical system (1) exists and for different values of τ there exists different steady states. Since $0 < \alpha < 1$, so by case-I these steady states are asymptotically stable. For $\tau = 10$, the boundary equilibrium point $(100,0)$ exists which is unstable. For different values of tax τ in the interval $[0,10]$, the steady states and their nature are given in table-I.

Table-I: Steady states for different tax levels and their nature for $\boldsymbol{\alpha = 0.5}$

tax τ	x_1	E_1	Nature of the steady states
0	66.67	20.41	stable
1	68.97	18.68	stable
2	71.43	16.72	stable
3	74.07	15.06	stable
4	76.92	13.16	stable
5	80.00	11.18	stable
6	83.33	9.13	stable
7	86.96	6.99	stable
8	90.91	4.77	stable
9	95.24	2.44	stable
10	100.00	0	**unstable**

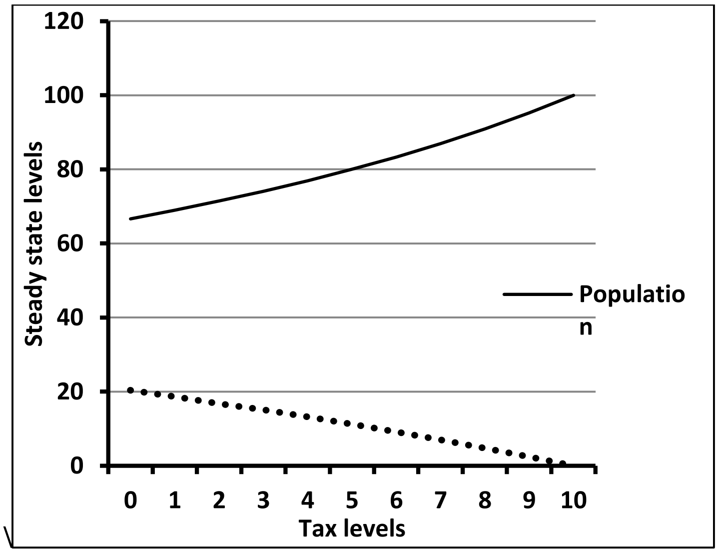

Fig.3: Variation of steady states for $\boldsymbol{\alpha = 0.5}$ and $\boldsymbol{\tau \in [0, 10]}$

From the above table and figure it is clear that for the increase of the tax level, the steady state levels of the species increase while the steady state of effort levels decrease. In reality it is true that when the regulatory agencies impose a higher tax, the effort level will decrease and as a result the steady state level of the species will increase.

ii) Let $\boldsymbol{\alpha} = \mathbf{1}$, $r = 5$, $k = 100$, $q = 0.01$, $\lambda = 1$, $p = 30, c = 20$ in appropriate units.
This is the example of a system when the species obeys the simple logistic growth function. The dynamical behaviour of the system is same as example (i) and we have the following table and figure.

Table-II: Steady states for different tax levels and their nature for $\boldsymbol{\alpha} = \mathbf{1}$

tax τ	x_1	E_1	Nature of the steady states
0	66.67	166.65	stable
10	68.97	155.15	stable
20	71.43	142.85	stable
30	74.07	129.65	stable
40	76.92	115.40	stable
50	80.00	100.00	stable
60	83.33	83.35	stable
70	86.96	65.20	stable
80	90.91	45.45	stable
90	95.24	23.80	stable
100	100.00	0	**unstable**

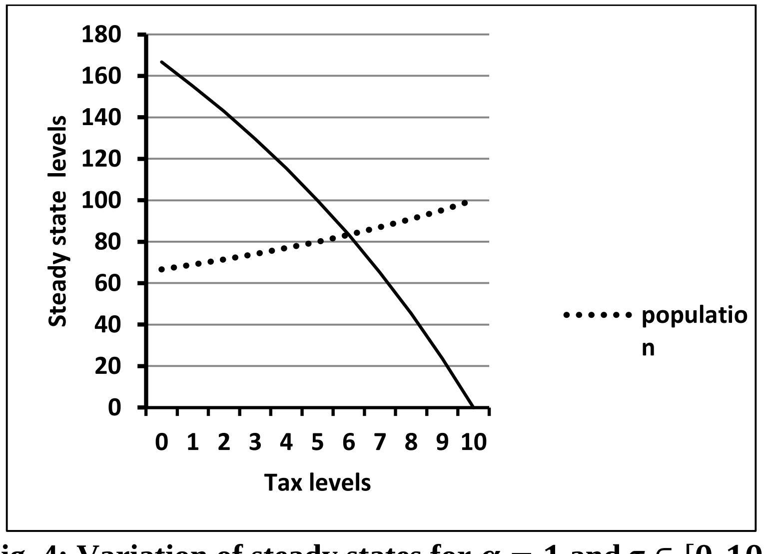

Fig. 4: Variation of steady states for $\boldsymbol{\alpha} = \mathbf{1}$ and $\boldsymbol{\tau} \in [\mathbf{0}, \mathbf{10}]$.

Explanation of the table and the figure is same as in example (i).
iii) Let $\boldsymbol{\alpha = 1.5}$, $r = 5$, $k = 100$, $q = 0.01$, $\lambda = 1$, $p = 30, c = 20$ in appropriate units.
Here $p - \frac{c}{kq} = 10$ and $p - \frac{c\alpha}{kq(\alpha-1)} = -30$.
If the regulatory agencies impose the tax τ such that $max(0, -30) \le \tau < 10$, i.e. $0 \le \tau < 10$ then the non trivial interior equilibrium point $P_2(x_1, E_1)$ of the dynamical system (1) exists and for different values of τ there exists different steady states. Since $\alpha > 1$, so by case-II these steady states are asymptotically stable. For $\tau = 10$, the boundary equilibrium point $(100,0)$ exists which is unstable. For different values of tax τ in the interval $[0,10]$, the steady states and their nature are given in table-III.

Table-III: Steady states for different tax levels and their nature for $\boldsymbol{\alpha = 1.5}$

tax τ	x_1	E_1	Nature of the steady states
0	66.67	1360.73	stable
1	68.97	1288.49	stable
2	71.43	1207.31	stable
3	74.07	1115.82	stable
4	76.92	1012.82	stable
5	80.00	894.43	stable
6	83.33	760.86	stable
7	86.96	608.01	stable
8	90.91	433.35	stable
9	95.24	232.27	stable
10	100.00	0	**unstable**

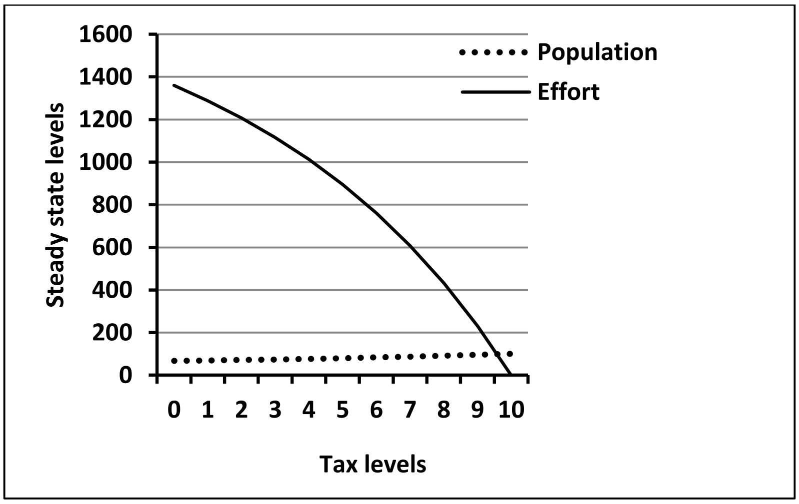

Fig.5: Variation of steady states for $\boldsymbol{\alpha = 1.5}$ and $\boldsymbol{\tau \in [0, 10]}$.

iv) Let $\boldsymbol{\alpha = 3.5}$, $r = 5$, $k = 100$, $q = 0.01$, $\lambda = 1$, $p = 30, c = 20$ in appropriate units.
Here $p - \frac{c}{kq} = 10$ and $p - \frac{c\alpha}{kq(\alpha-1)} = 2$.
In this case if the regulatory agencies impose the tax τ such that $0 \leq \tau \leq 10$, then the non trivial steady states exist but the steady states are asymptotically stable if $2 < \tau \leq 10$ by case II. If $\tau \in [0,2]$, the steady states are unstable. For different values of tax τ in the interval $[0,10]$, the steady states and their nature are given in table-IV.

Table-IV: Steady states for different tax levels and their nature for $\boldsymbol{\alpha = 3.5}$

tax τ	x_1	E_1	Nature of the steady states
0	66.67	6048273.98	**unstable**
1	68.97	6129179.56	**unstable**
2	71.43	6160016.42	**unstable**
3	74.07	6121789.37	stable
4	76.92	5988306.18	stable
5	80.00	5724334.02	stable
6	83.33	5283346.07	stable
7	86.96	4597761.33	stable
8	90.91	3581484.14	stable
9	95.24	2106810.26	stable
10	100.00	0	**unstable**

In the figure-6 we take one scale equal to 1000 units for the steady states of effort levels.
From all the examples we see that when α increases, the effort level is also increases but the steady state levels of the species remain unchanged. For $\alpha > 1$, the effort levels are very high. The biological interpretation of this phenomenon is that for the higher value of α, the growth rate $rx^{\alpha}\left(1 - \frac{x}{k}\right)$ of the fish species is also very high and so the fishermen give greater effort to earn more capital.

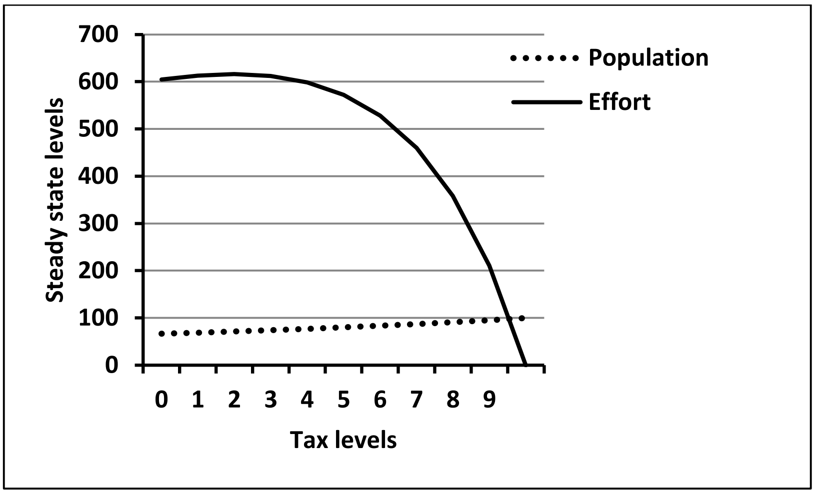

Fig.6: Variation of steady states for $\alpha = 3.5$ and $\tau \in [0, 10]$.

5. CONCLUSION:

In this paper we considered that the fish species obeys the modified logistic growth function $rx^{\alpha}\left(1-\frac{x}{k}\right)$ $(\alpha > 0)$ instead of simple logistic growth function $rx\left(1-\frac{x}{k}\right)$ which has been frequently used by many researchers. Here we give emphasis on taxation to control the over exploitation of the fish species. For different values of α we find the suitable ranges of tax for which the steady states exist and are stable.

REFERENCES

[1]. C.W. Clark, *Mathematical Bioeconomics*: *The Optimal Management of Renewable Resources*, John Willy and Sons, New York, 1976.

[2]. Anderson, L.G. and Lee, D.R., *Optimal governing instrument, operation level and enforcement in natural resource regulation: the case of the fishery*, American Journal of Agricultural Economics, 68, 1986, 678-690.

[3]. Sutinen, J.G. and Anderson, P., *The economics of fisheries law enforcement, Land Economics*, 61(4), 1985, 387-397.

[4]. Chaudhury K.S. and Johnson, T., *Bioeconomic dynamics of a fishery modeled as an S-system.* Mathematical Biosciences, 99, 1990, 231-249.

[5]. Ganguly S. and Chaudhury K.S., *Regulation of a single species fishery by taxation. Ecological Modelling*, 82, 1995,51-60.

[6]. Pradhan T. and Chaudhury K.S., *A dynamic reaction model of a two species fishery with taxation as a control instrument: a capital theoretic analysis*. Ecological Modelling, 121, 1999,1-6.

[7]. Ray J. and Pradhan T., *A dynamic reaction model of an inshore-offshore fishery with taxation as a control instrument*. IOSR Journal of Mathematics (IOSR-JM), 10, 2014,13-20.

[8]. Osuna O. and Villasenor G., *On the Dulac functions*, Qualitative Theory of Dynamical System, 10 (1), 2011, 43-49.

[9]. Salle J. La and Lefschetz S., *Stability by Liapunov's Direct Method with Application*, Academic Press, New York, 1961.

[10]. Hale J.K., *Ordinary Differential Equations*, Wiley Interscience, New York, 1969.