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Mathematical Analysis of a Motivated Stage Artist to be a Film Artist

[1]Nita H. Shah, [2]Foram A. Thakkar and [3]Bijal M. Yeolekar

[1, 2, 3]Department of Mathematics, Gujarat University,
Ahmedabad, 380009, Gujarat, India
Corresponding Author: [1]Nita H. Shah

Abstract

A mathematical model for the journey of exposed stage artist is subject to successful-unsuccessful and professional training is formulated and analysed. A stable model free equilibrium with endemic equilibrium associated to threshold which is less than unity is discussed. The unique endemic equilibrium when threshold is greater than unity is computed. A non-linear Lyapunov function is used to discuss global stability. The failure artists are nurtured by professional trainers (in terms of control) to be a successful film artist. Numerical Simulation of the model is carried out which suggest that using professional training the success rate of film artist is higher.

Keywords: Mathematical model, Threshold, Lyapunov function, Professional training, Simulation

1. INTRODUCTION

After career options for youngsters, modelling today has become one of the most inspiring as well as rewarding profession which attracts both male and female equally. This glamorous world of modelling offers varieties of opportunities to travel and meet various sections of people. Though, this field has no significance for education qualification, it requires lots of hard work, dedication and perseverance to be successful in building their physical attributes and personal qualities which is highly important in this field.

A motivated stage artist will always expose as they are willing to make their career in this field. But an exposed artist may or may not be successful at certain stage. The dream of becoming film artist can be fulfilled by both successful as well as unsuccessful modellers. Unsuccessful modellers can take professional training which includes courses in acting techniques, movements, voice, etc. which will help them a lot. No one has to give up anywhere. Though these individuals are not able to get a big role directly in film but they can reach there by getting hands on acting experience during college, taking wide range of roles such as characters in comedies, dramas, student film makers movies etc. which will provide them a lot of experience which will advance them to become film artist. To become a successful film actor, one needs to take advice from their mentor, practise and learn in such a way that you become an exclusive personality. Though this journey is very long and tough, all they need to have is skill, passion and patience. They should be prepared for rejections, failed auditions and a lot of sacrifice has to be given which could range from anywhere between quitting your current position to packing up and moving towards a new city which is suitable for their acting.

According to David H. Lawrence Xvii, who is an actor, voice talent, storytelling coach and technologist says that if an artist don't get mad skills in all three things: the art, the commerce and the science of acting, their chances for success rapidly evaporates. Also, he added that all one need to have an actor is "Live life. Get your heart broken. Enjoy successes and failures. Suffer with rejection. Be elated when you win. Live…life".

In this paper, we will analyse how a motivated stage artist becomes a film artist using *SEIR* model. Table-1 consisting of notations and its description along with its parametric values is described in Section 2. Local and global stability of the system is included in Section 3.1 and 3.2 respectively. Section 4 and 5 consists of optimal control model and numerical simulation of it respectively.

2. MATHEMATICAL MODEL

Here, we formulate a mathematical model for the analysis of motivated stage artist to become a film artist. The notations along with its parametric values are shown in Table-1.

Table 1: Notations and its Parametric Values

Notations		Parametric Values
N	Sample size	100
S	Number of motivated stage artist	8
E_S	Number of exposed stage artist	6
M_S	Number of successful stage modellers	2
M_U	Number of unsuccessful stage modellers	4
T_A	Number of small screen artist	3
F_A	Number of film artist	2
B	New recruitment	4
α_1	Rate at which exposed model becomes successful	0.3
α_2	Rate at which exposed model becomes unsuccessful	0.5
α_3	Social induced give up rate	0.1
η_1	Rate at which unsuccessful modellers becomes successful	0.2
η_2	Rate at which unsuccessful modellers becomes small screen artist	0.7
β	Rate at which exposed individuals comes in contact with the successful modellers	0.5
β_1	Rate at which exposed individuals comes in contact with the unsuccessful modellers	0.1
β_2	Rate at which exposed individuals comes in contact with the small screen artist	0.2
μ	Rate at which model artist give up	0.2
γ	Rate at which successful modellers become small screen artist	0.02
θ	Rate at which successful modellers become film artist	0.4
δ	Rate at which small screen artist becomes film artist	0.03
u_1	Rate at which unsuccessful modeller are professionally trained to be a part of M_S compartment	[0,1]

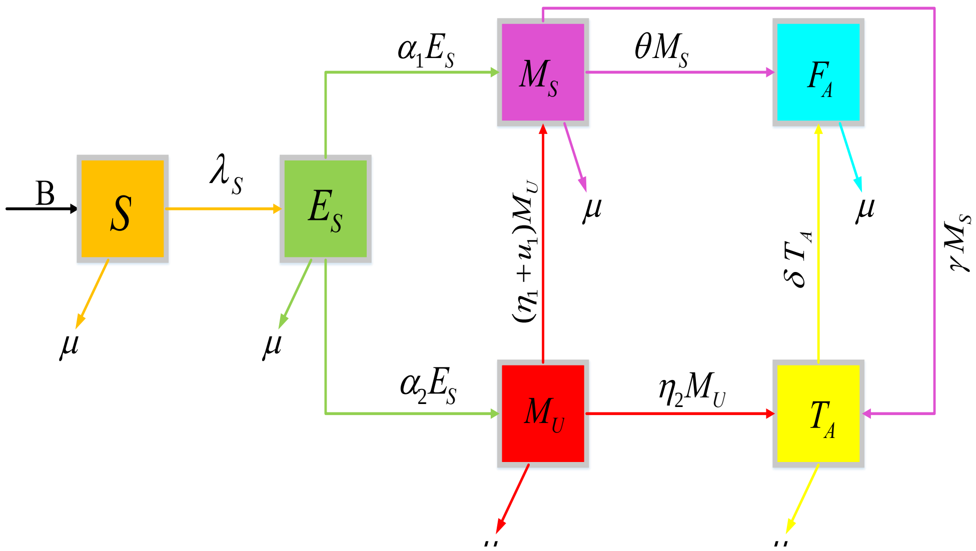

Figure 1: Dynamics of an artist to be a film artist

Motivated stage artist (S) gets exposed to modelling by coming in contact with the effective contact rate (χ) with the successful modellers (M_S), unsuccessful modellers (M_U) and small screen artist (T_A) to become a film artist (F_A). Unsuccessful modellers are to be given control in terms of professional training (u_1), through which they can move to M_S compartment at the rate η_1. It is not necessary that the unsuccessful modellers can go nowhere and do nothing. Only few of them give up, whereas other continue with their work and become small screen artist (T_A) and then by their hard work they move to F_A compartment at the rate δ. Successful modeller becomes film artist at the rate θ and γ is the rate at which one opts for small screen artist.

Now, the model from the above figure 1 is described by the system of following non-linear ordinary differential equations.

$$\frac{dS}{dt} = B - \chi S - \mu S$$
$$\frac{dE_S}{dt} = \chi S - \alpha_1 E_S - \alpha_2 E_S - \mu E_S$$
$$\frac{dM_S}{dt} = \alpha_1 E_S + (\eta_1 + u_1)M_U - \theta M_S - \gamma M_S - (\mu + \alpha_3)M_S$$
$$\frac{dM_U}{dt} = \alpha_2 E_S - (\eta_1 + u_1)M_U - \eta_2 M_U - (\mu + \alpha_3)M_U \qquad (1)$$
$$\frac{dT_A}{dt} = \eta_2 M_U + \gamma M_S - \delta T_A - \mu T_A$$
$$\frac{dF_A}{dt} = \delta T_A + \theta M_S - \mu F_A$$

where, $\chi = \frac{\beta(M_S + \beta_1 M_U + \beta_2 T_A)}{N}$ is the effective contact rate.

with $S + E_S + M_S + M_U + T_A + F_A = N$ and $S > 0,\ E_S, M_S, M_U, T_A, F_A \geq 0$

Adding the above set of systems of equations (1) we get,

$$\frac{d}{dt}\left(S + E_S + M_S + M_U + T_A + F_A\right) = B - \mu\left(S + E_S + M_S + M_U + T_A + F_A\right) - \alpha_3\left(M_S + M_U\right) \geq 0$$

This gives, $\limsup_{t\to\infty}\left(S + E_S + M_S + M_U + T_A + F_A\right) \leq \frac{B}{\mu}$

Thus, the feasible region for (1) is,

$$\Lambda = \left\{\left(S + E_S + M_S + M_U + T_A + F_A\right) / S + E_S + M_S + M_U + T_A + F_A \leq \frac{B}{\mu}, S > 0; E_S, M_S, M_U, T_A, F_A \geq 0\right\}$$

Now, to find the equilibrium of system (1), we set the rates in (1) to zero:

$$B - \chi S - \mu S = 0$$
$$\chi S - \alpha_1 E_S - \alpha_2 E_S - \mu E_S = 0$$
$$\alpha_1 E_S + (\eta_1 + u_1)M_U - \theta M_S - \gamma M_S - (\mu + \alpha_3)M_S = 0$$
$$\alpha_2 E_S - (\eta_1 + u_1)M_U - \eta_2 M_U - (\mu + \alpha_3)M_U = 0$$
$$\eta_2 M_U + \gamma M_S - \delta T_A - \mu T_A = 0$$
$$\delta T_A + \theta M_S - \mu F_A = 0 \qquad (2)$$

On solving these set of equations (2) we get the model free equilibrium point.

Therefore, Model free equilibrium point of this model is $E_0 = (\frac{B}{\mu}, 0, 0, 0, 0, 0)$.

Now, before calculating the other equilibrium point we need to calculate the basic reproduction number to know the motion of artists in the system using the next generation matrix method. The next generation matrix method is the spectral radius of matrix FV^{-1} where F and V are the Jacobian matrices of $\Im$ and v evaluated with respect to each compartment at an equilibrium state.

Let $X = (E_S, M_S, M_U, T_A, F_A, S)$

Then $\frac{dX}{dt} = \Im(X) - v(X)$

where $\Im(X)$ denotes the rate of new stage artist in the compartment and $v(X)$ denotes the transmission of stage artist from one compartment to other which is follows

$$\Im(X) = \begin{bmatrix} \chi S \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } v(X) = \begin{bmatrix} \alpha_1 E_S + \alpha_2 E_S + \mu E_S \\ -\alpha_1 E_S - (\eta_1 + u_1) M_U + \theta M_S + \gamma M_S + (\mu + \alpha_3) M_S \\ -\alpha_2 E_S + (\eta_1 + u_1) M_U + \eta_2 M_U + (\mu + \alpha_3) M_U \\ -\eta_2 M_U - \gamma M_S + \delta T_A + \mu T_A \\ -\delta T_A - \theta M_S + \mu F_A \\ -B + \chi S + \mu S \end{bmatrix}$$

Now, the derivative of $\Im$ and v calculated at an equilibrium point E_0 gives matrices F and V of order 6×6 defined as

$$F = \left[\frac{\partial \Im_i(E_0)}{\partial X_j} \right] \quad \text{and} \quad V = \left[\frac{\partial v_i(E_0)}{\partial X_j} \right] \text{ ; for } i, j = 1,2,3,4,5,6$$

So,

$$F = \begin{bmatrix} 0 & \frac{\beta B}{\mu N} & \frac{\beta \beta_1 B}{\mu N} & \frac{\beta \beta_2 B}{\mu N} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V=\begin{bmatrix} \alpha_1+\alpha_2+\mu & 0 & 0 & 0 & 0 & 0 \\ -\alpha_1 & \theta+\gamma+\mu+\alpha_3 & -(\eta_1+u_1) & 0 & 0 & 0 \\ -\alpha_2 & 0 & \eta_1+u_1+\eta_2+\mu+\alpha_3 & 0 & 0 & 0 \\ 0 & -\gamma & -\eta_2 & \delta+\mu & 0 & 0 \\ 0 & -\theta & 0 & -\delta & \mu & 0 \\ 0 & \dfrac{\beta B}{\mu N} & \dfrac{\beta\beta_1 B}{\mu N} & \dfrac{\beta\beta_2 B}{\mu N} & 0 & \mu \end{bmatrix}$$

Thus, the basic reproduction number $\left(R_0\right)$ calculated at an equilibrium point E_0 is

$$R_0=\frac{ART\alpha_2+AST\alpha_1+VQT\alpha_2+CQ\alpha_2\eta_2+CR\alpha_2\gamma+CS\alpha_1\gamma}{TQSP}$$

where,

$$A=\frac{\beta B}{\mu N},$$
$$V=\frac{\beta\beta_1 B}{\mu N},$$
$$C=\frac{\beta\beta_2 B}{\mu N},$$
$$P=\alpha_1+\alpha_2+\mu,$$
$$Q=\theta+\gamma+\mu+\alpha_3,$$
$$R=\eta_1+u_1,$$
$$S=\eta_1+u_1+\eta_2+\mu+\alpha_3,$$
$$T=\delta+\mu$$

We solve set of equations (2) to find a positive (modelling existence) equilibrium E^* of system (1),

We find that

$$S^* = \frac{B}{\chi + \mu}$$

$$E_S^* = \frac{\chi B}{P(\chi + \mu)}$$

$$M_S^* = \frac{\chi B(\alpha_1 S + \alpha_2 R)}{PQS(\chi + \mu)}$$

$$M_U^* = \frac{\alpha_2 \chi B}{PS(\chi + \mu)}$$

$$T_A^* = \frac{\chi B(\alpha_1 \gamma S + \alpha_2 \eta_2 Q + \alpha_2 \gamma R)}{TQSP(\chi + \mu)}$$

$$F_A^* = \frac{\chi B[(\alpha_1 S + \alpha_2 R)(\delta\gamma + \theta T) + \alpha_2 \delta \eta_2 Q]}{TQSP(\chi + \mu)\mu}$$

where P, Q, R, S, T are defined as above.

3. STABILITY ANALYSIS

Here, using the linearization method and matrix analysis the local and global stability at E_0 and E^* are to be studied in this section.

3.1 Local Stability

Theorem-1: (Stability of E_0) If $R_0 < 1$ then the model free equilibrium point E_0 is locally asymptotically stable. If $R_0 = 1$ then it is locally stable. If $R_0 > 1$ then it is unstable.

Proof: Jacobian Matrix evaluated at the equilibrium point $E_0 = \left(\frac{B}{\mu}, 0, 0, 0, 0, 0\right)$ is

$$J = \begin{bmatrix} -\mu & 0 & -A & -V & -C & 0 \\ 0 & -P & A & V & C & 0 \\ 0 & \alpha_1 & -Q & R & 0 & 0 \\ 0 & \alpha_2 & 0 & -S & 0 & 0 \\ 0 & 0 & \gamma & \eta_2 & -T & 0 \\ 0 & 0 & \theta & 0 & \delta & -\mu \end{bmatrix}$$

where A, V, P, Q, R, S, T are defined as above.

The eigen value of the characteristic equation are

$\lambda_{1,2} = -\mu < 0$ and $\lambda_3, \lambda_4, \lambda_5, \lambda_6$ satisfy the equation,

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

where,

$$a_1 = T + S + Q + P$$
$$a_2 = PQ + PS + PT + QS + QT + ST + (A\alpha_1 - B\alpha_2)$$

Here, $a_2 > 0$ as it can be seen that

$$PQ + PS + PT + QS + QT + ST > -(A\alpha_1 + B\alpha_2)$$

which is obvious.

$$a_3 = PQS + PQT + PST + QST - (AR\alpha_2 + AS\alpha_1 + AT\alpha_1 + VQ\alpha_2 + VT\alpha_2 + C\alpha_1\gamma + C\alpha_2\eta_2)$$

we can write it as.

$$a_3 = k_1 + k_2 + k_3$$

where,

$$k_1 = PQS + PQT - (A\alpha_1(S + T) + B\alpha_2(Q + T))$$
$$k_2 = PST - (C\alpha_1\gamma + C\alpha_2\eta_2)$$
$$k_3 = QST - AR\alpha_2$$

$$\begin{aligned} a_4 &= TQSP - CART\alpha_2 + A\alpha_1 TS + BQT\alpha_2 + CQ\alpha_2\eta_2 + CR\alpha_2\gamma + CS\alpha_1\gamma \\ &= TQSP(1 - R_0) \end{aligned}$$

If $R_0 < 1$ then $a_4 > 0$

$\therefore a_1, a_2, a_3, a_4 > 0$ and the eigenvalues $\lambda_3, \lambda_4, \lambda_5, \lambda_6$ have negative real parts.

Thus, all the eigenvalues have negative real part and hence E_0 is locally asymptotically stable.

If $R_0 = 1$ then $a_1, a_2, a_3 > 0,\ a_4 = 0$ thus E_0 is locally stable.

If $R_0 > 1$ then $a_4 < 0$.

$\therefore E_0$ is unstable.

Theorem-2: (stability of E^*) If $R_0 < 1$ and $U > \mu$ then the modelling existence equilibrium E^* is locally asymptotically stable.

Proof: Jacobian matrix evaluated at the endemic equilibrium $E^* = (S^*, E_S^*, M_S^*, M_U^*, T_A^*, F_A^*)$ gives

$$J(E^*) = \begin{bmatrix} -U-\mu & 0 & -X & -Y & -Z & 0 \\ U-\mu & -P & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & -Q & R & 0 & 0 \\ 0 & \alpha_2 & 0 & -S & 0 & 0 \\ 0 & 0 & \gamma & \eta_2 & -T & 0 \\ 0 & 0 & \theta & 0 & \delta & -\mu \end{bmatrix}$$

where,

$U = \dfrac{(\beta M_S^* + \beta\beta_1 M_U^* + \beta\beta_2 T_A^*)}{N}, X = \dfrac{\beta S^*}{N}, Y = \dfrac{\beta\beta_1 S^*}{N}, Z = \dfrac{\beta\beta_2 S^*}{N}$ and P, Q, R, S, T are defined as above.

The eigen values of the characteristic equation are $\lambda_1 = -\mu < 0$ and $\lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$ satisfy the equation

$$\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0$$

where,

$$a_1 = T + S + Q + P + U + \mu$$
$$a_2 = PQ + PS + PT + PU + QS + QT + QU + ST + SU + TU + \mu(P + Q + S + T)$$
$$a_3 = PQS + PQT + PQU + PST + PSU + PTU + QST + QSU + QTU + STU$$
$$+ \mu(PQ + PS + PT + QS + QT + ST) + (U - \mu)(A\alpha_1 + B\alpha_2)$$

But here $U > \mu \Rightarrow U - \mu > 0$

$\therefore a_3 > 0$

$$a_4 = PQST + (U + \mu)(PQS + PQT + PST + QST)$$
$$+ (U - \mu)(AR\alpha_2 + AS\alpha_1 + AT\alpha_1 + BQ\alpha_2 + BT\alpha_2 + C\alpha_1\gamma + C\alpha_2\eta_2)$$

Here also, $U > \mu \Rightarrow a_4 > 0$.

$$
\begin{aligned}
a_5 &= TQSP(U+\mu)+(U-\mu)(ART\alpha_2+AST\alpha_1+BQT\alpha_2+CQ\alpha_2\eta_2+CR\alpha_2\gamma+CS\alpha_1\gamma)\\
&= TQSP(U+\mu)+(U-\mu)TQSP(R_0)\\
&= TQSP[(U-\mu)R_0+(U+\mu)]\\
&= TQSP[\mu(1-R_0)+U(1+R_0)]
\end{aligned}
$$

So, $a_5 > 0$ if $R_0 < 1$.

Hence, $a_1, a_2, a_3, a_4, a_5 > 0$ if $U > \mu$ and $R_0 < 1$.

Thus, by Routh-Hurwitz criteria,

$\lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$ have negative real parts.

Therefore, all eigen values have negative real part which implies that E^* is locally asymptotically stable.

3.2 Global Stability

Theorem- 3: (stability of E_0): If $\min\{\beta B, \beta\beta_1 B\} \le \mu N(\mu+\alpha_3)\, and\, \beta\beta_2 B \le \mu^2 N$ then E_0 is globally asymptotically stable.

Proof: Consider the Lyapunov function:

$$
\begin{aligned}
L(t) &= E_S(t)+M_S(t)+M_U(t)+T_A(t)+F_A(t)\\
&= \lambda S-\mu(E_S+T_A+F_A)-(\mu+\alpha_3)(M_S+M_U)\\
&= (\beta S-(\mu+\alpha_3))M_S+(\beta\beta_1 S-(\mu+\alpha_3))M_U+(\beta\beta_2 S-\mu)T_A-\mu(E_S+F_A)
\end{aligned}
$$

Since, $E_0 \in \Lambda$ then $S \le \dfrac{B}{\mu}$ and we have,

$$L'(t) \le 0 \text{ if } \min\left\{\frac{\beta B}{\mu N}, \frac{\beta\beta_1 B}{\mu N}\right\} \le \mu+\alpha_3 \ and \ \frac{\beta\beta_2 B}{\mu N} \le \mu .$$

Since, μ is non-negative, it follows that

$$L'(t) \le 0 \text{ if } \min\{\beta B, \beta\beta_1 B\} \le \mu N(\mu+\alpha_3)\, and\, \beta\beta_2 B \le \mu^2 N$$

with $L'(t) = 0$ if and only if $E_S = M_S = M_U = T_A = F_A = 0$.

Hence, the only solution of system (1) in Ω on which $L'(t) = 0$ is E_0.

Therefore, by LaSalle’s Invariance Principle, every solution of the system (1), with initial conditions in Ω, as approaches E_0, as $t \to \infty$. Hence, E_0 is globally asymptotically stable.

Theorem- 4: (stability at E^*): The modelling existence equilibrium E^* is globally stable.

Proof: Consider the Lyapunov function:

$$L(t) = \frac{1}{2}\left[(S(t)-S^*)+(E_S(t)-E_S^*)+(M_S(t)-M_S^*)+(M_U(t)-M_U^*)+(T_A(t)-T_A^*)+(F_A(t)-F_A^*)\right]^2$$

$$\therefore L'(t) = \left[(S-S^*)+(E_S-E_S^*)+(M_S-M_S^*)+(M_U-M_U^*)+(T_A-T_A^*)+(F_A-F_A^*)\right]$$
$$\left[S' + E_S' + M_S' + M_U' + T_A' + F_A'\right]$$
$$= \left[(S-S^*)+(E_S-E_S^*)+(M_S-M_S^*)+(M_U-M_U^*)+(T_A-T_A^*)+(F_A-F_A^*)\right]$$
$$\left[\mu S^* + \mu E_S^* + \mu M_S^* + \mu M_U^* + \mu T_A^* + \mu F_A^* - \mu S - \mu E_S - \mu M_S - \mu M_U - \mu T_A - \mu F_A\right]$$
$$= -\mu\left[(S-S^*)+(E_S-E_S^*)+(M_S-M_S^*)+(M_U-M_U^*)+(T_A-T_A^*)+(F_A-F_A^*)\right]^2 \le 0$$

Hence, E^* is globally stable.

4. OPTIMAL CONTROL MODEL

In this section, a control function has been taken into consideration to get the maximum number of successful artist. The objective function along with the optimal control variable for the stage artist is given by

$$J(u_1,\Omega) = \int_0^T (A_1S^2 + A_2E_S{}^2 + A_3M_S{}^2 + A_4M_U{}^2 + A_5T_A{}^2 + A_6F_A{}^2 + w_1^2u_1{}^2)dt \qquad (3)$$

where Ω denotes the set of all compartmental variables, $A_1, A_2, A_3, A_4, A_5, A_6$ denotes non-negative weight constants for the compartments $S, E_S, M_S, M_U, T_A, F_A$ respectively and w_1 is the weight constant for the control variable u_1.

The optimal control condition is standardized by the weight w_1 which is a constant parameter for the professional training (u_1).

Now, we will calculate the value of control variable u_1 from $t=0$ to $t=T$ such that

$$J(u_1(t)) = \min\{J(u_1{}^*,\Omega) / u_1 \in \phi\}$$

where ϕ is smooth function on the interval $[0,1]$.

Making use of Fleming and Rishel (2012) results, the optimal control denoted by u_i^* is obtained by collecting all the integrands of the objective function (3) using the lower bound a_1 and upper bound b_1 of the control variable u_1 respectively.

In order to minimize the cost function in (3) we construct a Lagrangian function consisting of state equation and adjoint variables $A_V = (\lambda_S, \lambda_{E_S}, \lambda_{M_S}, \lambda_{M_U}, \lambda_{T_A}, \lambda_{F_A})$ using Pontrygin's principle which is as follows:

$$\begin{aligned}
&L(\Omega, A_V) \\
&= A_1 S^2 + A_2 E_S{}^2 + A_3 M_S{}^2 + A_4 M_U{}^2 + A_5 T_A{}^2 + A_6 F_A{}^2 + w_1^2 u_1^2 \\
&\quad + \lambda_S \left[B - \chi S - \mu S\right] \\
&\quad + \lambda_{E_S} \left[\chi S - \alpha_1 E_S - \alpha_2 E_S - \mu E_S\right] \\
&\quad + \lambda_{M_S} [\alpha_1 E_S + (\eta_1 + u_1) M_U - \theta M_S - \gamma M_S - (\mu + \alpha_3) M_S] \\
&\quad + \lambda_{M_U} \left[\alpha_2 E_S - (\eta_1 + u_1) M_U - \eta_2 M_U - (\mu + \alpha_3) M_U\right] \\
&\quad + \lambda_{T_A} \left[\eta_2 M_U + \gamma M_S - \delta T_A - \mu T_A\right] \\
&\quad + \lambda_{F_A} \left[\delta T_A + \theta M_S - \mu F_A\right]
\end{aligned} \tag{4}$$

Now, the partial derivative of the Lagrangian function with respect to each variable of the compartment gives us the adjoint equation such that

$$\begin{aligned}
\dot{\lambda}_S &= -\frac{\partial L}{\partial S} \\
&= -2A_1 S + \chi(\lambda_S - \lambda_{E_S}) + \mu \lambda_S \\
\dot{\lambda}_{E_S} &= -\frac{\partial L}{\partial E_S} \\
&= -2A_2 E_S + \alpha_1(\lambda_{E_S} - \lambda_{M_S}) + \alpha_2(\lambda_{E_S} - \lambda_{M_U}) + \mu \lambda_{E_S} \\
\dot{\lambda}_{M_S} &= -\frac{\partial L}{\partial M_S} \\
&= -2A_3 M_S + \theta(\lambda_{M_S} - \lambda_{F_A}) + \gamma(\lambda_{M_S} - \lambda_{T_A}) + (\mu + \alpha_3)\lambda_{M_S} \\
\dot{\lambda}_{M_U} &= -\frac{\partial L}{\partial M_U} \\
&= -2A_4 M_U + (\eta_1 + u_1)(\lambda_{M_U} - \lambda_{M_S}) + \eta_2(\lambda_{M_U} - \lambda_{T_A}) + (\mu + \alpha_3)\lambda_{M_U} \\
\dot{\lambda}_{T_A} &= -\frac{\partial L}{\partial T_A} \\
&= -2A_5 T_A + \delta(\lambda_{T_A} - \lambda_{F_A}) + \mu \lambda_{T_A} \\
\dot{\lambda}_{F_A} &= -\frac{\partial L}{\partial F_A} \\
&= -2A_6 F_A + \mu \lambda_{F_A}
\end{aligned}$$

The necessary conditions for Lagrangian function L to be optimal for control are

$$\frac{\partial L}{\partial u_1} = 2w_1u_1 + M_U\lambda_{M_S} - M_U\lambda_{M_U} = 0 \tag{5}$$

Solving equation (5) we get,

$$u_1 = \frac{M_U(\lambda_{M_U} - \lambda_{M_S})}{2w_1}$$

Thus, the required optimal control condition is obtained as

$$u_1^* = \max\left(a_1, \min\left(b_1, \frac{M_U\left(\lambda_{M_U} - \lambda_{M_S}\right)}{2w_1u_1}\right)\right)$$

where a_1 = lower bound and b_1 = upper bound

5. NUMERICAL SIMULATION

In this section, we will study the numerical results of all compartments. Also, the effect of control on each compartment is analysed here.

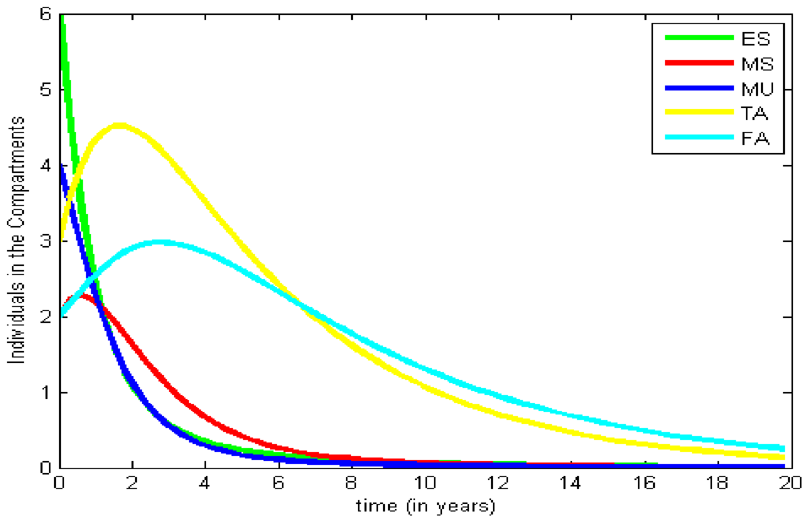

Figure 2: Motion of an individual in each compartment

It can be seen from the figure 2 that a small screen artist survives longer than a film artist as new and new enters into the system and successful modellers always lose their glamour after certain time. Exposed stage artist decreases because professional training costs huge amount and that makes modeller unsuccessful.

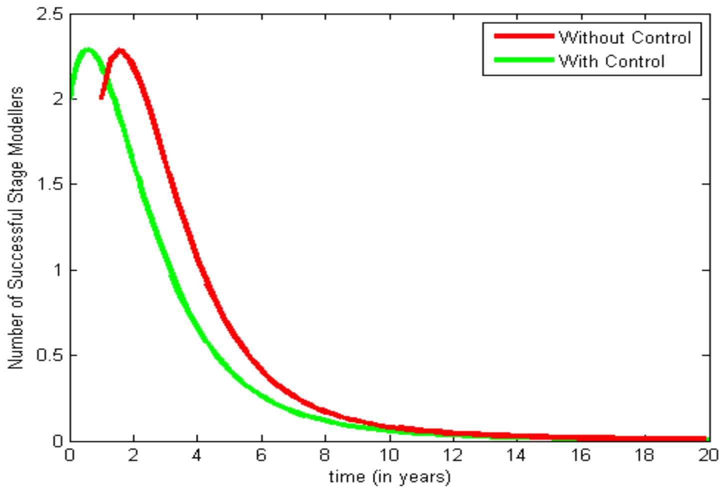

Figure 3: Effect of Control on Successful Stage Modellers

Figure 3 shows that the control in terms of professional training (u_1) should be taken from the beginning itself from when artist enters into the modelling world. This will help in making success faster than the one taking training after a year or more.

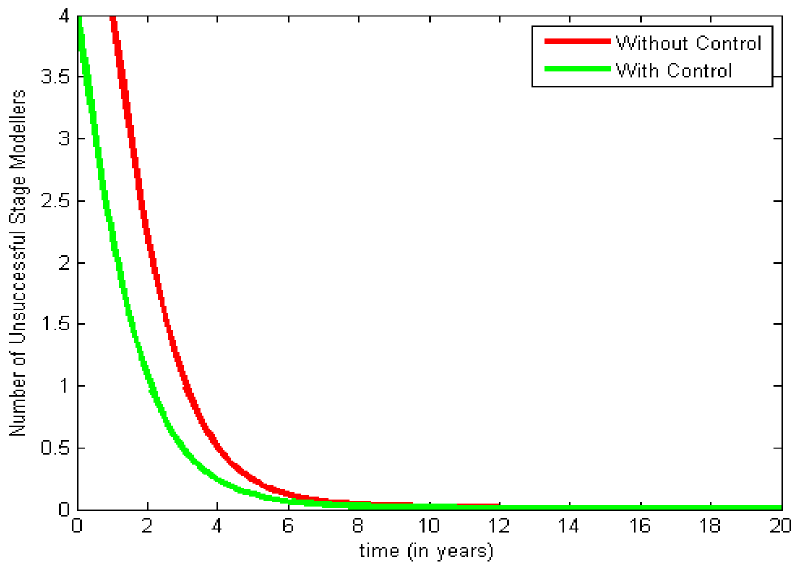

Figure 4: Effect of Control on Unsuccessful Stage Modellers

It can be seen from the above figure 4 that a professional training should be taken by an unsuccessful modellers so as to make their dream of becoming film artist to come true. The decreasing trend in a graph shows that some of the modellers may either give up and if they are truly interested they will continue by moving further to be T.V. screen players, drama artist and many small stage programmes.

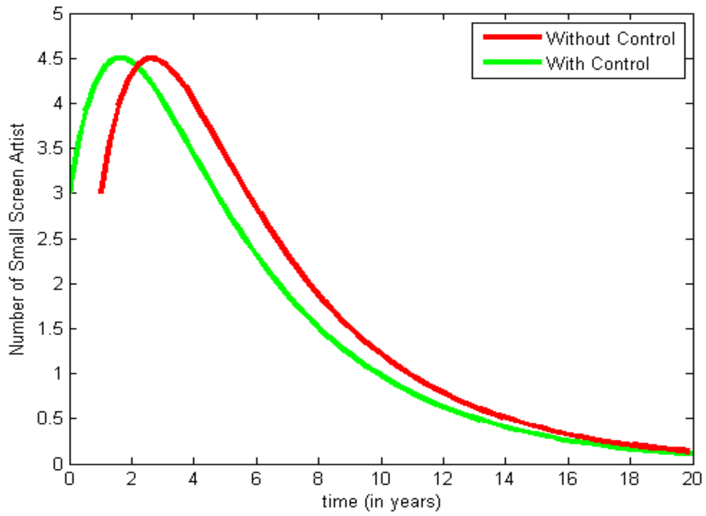

Figure 5: Effect of Control on Small Screen Artist

Figure 5 shows that the control plays a vital role in making film artist from a small screen artist in less duration as compared to without control.

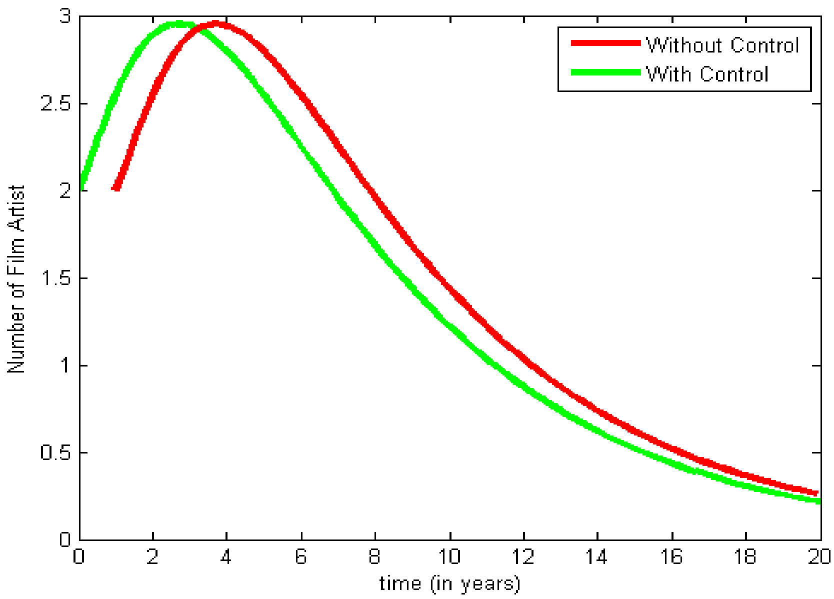

Figure 6: Effect of Control on Film Artist

Figure 6 shows that a successful modellers or a small screen artist becomes film artist in 3 years if they take training from the initial phase, whereas if they don't it takes about 4.5 years.

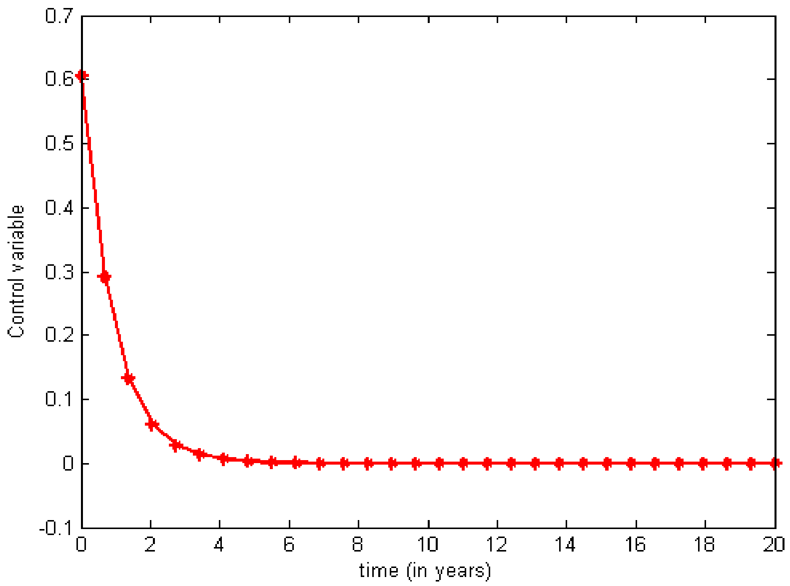

Figure 7: Time verses Control Variable

It can be seen from the figure 7 that the maximum control in terms of professional training is required in the initial period to become successful, but as the time passes it becomes stable which means that the modellers becomes habituated and continue with their daily routine training which is fruitful for them.

6. CONCLUSION

In this paper, a mathematical model for the dynamics of an artist to be a film artist is formulated. The control in terms of professional training is provided to an unsuccessful modeller through which they can become successful and become a film artist or by participating in the small stage programmes such as T.V. screen players, drama etc. Every individual has to work hard and has to give maximum effort in building their bright future and for their dreams to come true.

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