

Oblong Mean Prime Labeling of Some Cycle Graphs

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Abstract

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The graph for which every edge (uv) , the labels assigned to u and v are oblong numbers and for each vertex of degree at least 2, the $g.c.d$ of the labels of the incident edges is 1. Here we characterize some cycle related graphs for oblong mean prime labeling.

Keywords: Graph labeling, oblong numbers, prime graphs, prime labeling, cycle graph.

1. INTRODUCTION

All graphs in this paper are connected, simple, finite and undirected. The symbol $V(G)$ and $E(G)$ denotes the vertex set and edge set of a graph G . The graph whose cardinality of the vertex set is called the order of G , denoted by p and the cardinality of the edge set is called the size of the graph G , denoted by q . A graph with p vertices and q edges is called a (p,q) - graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1], [2] and [3]. Some basic concepts are taken from Frank Harary [1]. In [4], we introduced the concept of oblong mean prime labeling and proved the result for some path related graphs. In this paper we investigated the oblong mean prime labeling of some cycle graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common divisor of a vertex of degree greater than or equal to 2, is the g c d of the labels of the incident edges.

Definition: 1.2 An oblong number is the product of a number with its successor, algebraically it has the form $n(n+1)$. The oblong numbers are 2, 6, 12, 20, -----.

2. MAIN RESULTS

Definition 2.1 Let G be a graph with p vertices and q edges . Define a bijection

$f : V(G) \rightarrow \{2,6,12,20, \dots, p(p+1)\}$ by $f(v_i) = i(i+1)$, for every i from 1 to p and define a 1-1 mapping $f_{ompl}^* : E(G) \rightarrow$ set of natural numbers N by $f_{ompl}^*(uv) = \frac{f(u)+f(v)}{2}$. The induced function f_{ompl}^* is said to be an oblong mean prime labeling, if the g c d of each vertex of degree at least 2, is one.

Definition 2.2 A graph which admits oblong mean prime labeling is called an oblong mean prime graph.

Theorem: 2.1 The cycle C_n admits oblong mean prime labeling, when $n+1 \equiv 0 \pmod{4}$ and $n \equiv 0 \pmod{4}$.

Proof : Let $G = C_n$ and let v_1, v_2, \dots, v_n are the vertices of G .

Here $|V(G)| = n$ and $|E(G)| = n$.

Define a function $f : V \rightarrow \{2,6,12, \dots, n(n+1)\}$ by

$$f(v_i) = i(i+1) , i = 1, 2, \dots, n.$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{ompl}^* is defined as follows

$$f_{ompl}^*(v_i v_{i+1}) = (i+1)^2, \quad i = 1, 2, \dots, n-1$$

$$f_{ompl}^*(v_1 v_n) = \frac{n^2+n+2}{2}$$

Clearly f_{ompl}^* is an injection.

$$\begin{aligned} \text{g c d of } (v_{i+1}) &= \text{g c d of } \{ f_{ompl}^*(v_i v_{i+1}), f_{ompl}^*(v_{i+1} v_{i+2}) \} \\ &= \text{g c d of } \{ (i+1)^2, (i+2)^2 \} \\ &= \text{g c d of } \{ (i+1), (i+2) \} = 1, \quad i = 1, 2, \dots, n-2 \end{aligned}$$

$$\begin{aligned} \text{g c d of } (v_1) &= \text{g c d of } \{ f_{ompl}^*(v_1 v_2), f_{ompl}^*(v_1 v_n) \} \\ &= \text{g c d of } \left\{ 4, \frac{n^2+n+2}{2} \right\} = 1 \end{aligned}$$

$$\begin{aligned} \text{g c d of } (v_n) &= \text{g c d of } \{ f_{ompl}^*(v_1 v_n), f_{ompl}^*(v_{n-1} v_n) \} \\ &= \text{g c d of } \left\{ n^2, \frac{n^2+n+2}{2} \right\} = 1 \end{aligned}$$

So, g c d of each vertex of degree greater than one is 1.

Hence C_n admits, oblong mean prime labeling.

Theorem 2.2 The corona of cycle C_n admits oblong mean prime labeling.

Proof : Let $G = C_n \odot K_1$ and let v_1, v_2, \dots, v_{2n} are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 2n$.

Define a function $f : V \rightarrow \{2, 6, 12, \dots, 2n(2n + 1)\}$ by

$$f(v_i) = i(i+1), \quad i = 1, 2, \dots, 2n.$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{ompl}^* is defined as follows

$$f_{ompl}^*(v_i v_{i+1}) = (i+1)^2, \quad i = 1, 2, \dots, n+1$$

$$f_{ompl}^*(v_2 v_{n+1}) = \frac{n^2 + 3n + 8}{2}.$$

$$f_{ompl}^*(v_{2+i} v_{2n-i+1}) = \frac{(i+2)(i+3) + (2n-i+1)(2n-i+2)}{2}, \quad i = 1, 2, 3, \dots, n-2$$

Clearly f_{ompl}^* is an injection.

$$\text{g c d of } (v_{i+1}) = 1, \quad i = 1, 2, \dots, n.$$

So, g c d of each vertex of degree greater than one is 1.

Hence $C_n \odot K_1$, admits oblong mean prime labeling.

Theorem 2.3 The graph $C_n \odot K_2$ admits oblong mean prime labeling, when n is even.

Proof :Let $G = C_n \odot K_2$ and let v_1, v_2, \dots, v_{3n} are the vertices of G.

Here $|V(G)| = 3n$ and $|E(G)| = 4n$.

Define a function $f : V \rightarrow \{2, 6, 12, \dots, 3n(3n + 1)\}$ by

$$f(v_i) = i(i+1) , i = 1, 2, \dots, 3n.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ompl}^* is defined as follows

$$\begin{aligned} f_{ompl}^*(v_{3i-2} v_{3i-1}) &= 9i^2 - 6i + 1, & i = 1, 2, \dots, n. \\ f_{ompl}^*(v_{3i-2} v_{3i}) &= 9i^2 - 3i + 1, & i = 1, 2, \dots, n. \\ f_{ompl}^*(v_{3i-1} v_{3i}) &= 9i^2 & i = 1, 2, \dots, n. \\ f_{ompl}^*(v_{3i-2} v_{3i+1}) &= 9i^2 + 2, & i = 1, 2, \dots, n-1. \\ f_{ompl}^*(v_1 v_{3n-2}) &= \frac{9n^2 - 9n + 4}{2}. \end{aligned}$$

Clearly f_{ompl}^* is an injection.

$$\begin{aligned} \text{g c d of } (v_{3i-2}) &= \text{g c d of } \{ f_{ompl}^*(v_{3i-2} v_{3i-1}), f_{ompl}^*(v_{3i-2} v_{3i}) \} \\ &= \text{g c d of } \{ (3i-1)^2, (3i-1)^2 + 3i \} = \text{g c d of } \{ (3i-1)^2, 3i \} \\ &= 1, & i = 1, 2, \dots, n \end{aligned}$$

$$\begin{aligned} \text{g c d of } (v_{3i-1}) &= \text{g c d of } \{ f_{ompl}^*(v_{3i-2} v_{3i-1}), f_{ompl}^*(v_{3i-1} v_{3i}) \} \\ &= \text{g c d of } \{ (3i-1)^2, (3i)^2 \} = 1, & i = 1, 2, \dots, n \end{aligned}$$

$$\text{g c d of } (v_{3i}) = \text{g c d of } \{ f_{ompl}^*(v_{3i-2} v_{3i}), f_{ompl}^*(v_{3i-1} v_{3i}) \}$$

$$= \text{g c d of } \{ 9i^2-3i+1,(3i)^2 \},$$

$$= \text{g c d of } \{ 3i-1, 9i^2-3i+1 \} = 1, \quad i = 1,2,\dots,n$$

So, g c d of each vertex of degree greater than one is 1.

Hence $C_n \odot K_2$, admits oblong mean prime labeling.

Theorem: 2.4 The cycle $C_n(P_m)$ admits oblong mean prime labeling, when $n+1 \equiv 0(\text{mod } 4)$ and $n \equiv 0(\text{mod } 4)$.

Proof :Let $G = C_n(P_m)$ and let $v_1, v_2, \dots, v_{n+m-1}$ are the vertices of G .

Here $|V(G)| = n+m-1$ and $|E(G)| = n+m-1$.

Define a function $f : V \rightarrow \{ 2,6,12,\dots,(n+m-1)(n+m) \}$ by

$$f(v_i) = i(i+1) , i = 1,2,\dots,n+m-1.$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{ompl}^* is defined as follows

$$f_{ompl}^*(v_i v_{i+1}) = (i+1)^2, \quad i = 1,2,\dots,n+m-2$$

$$f_{ompl}^*(v_1 v_n) = \frac{n^2+n+2}{2}$$

Clearly f_{ompl}^* is an injection.

$$\text{g c d of } (v_{i+1}) = 1, \quad i = 1,2,\dots,n+m-3$$

$$\text{g c d of } (v_1) = \text{g c d of } \{ f_{ompl}^*(v_1 v_2), f_{ompl}^*(v_1 v_n) \}$$

$$= \text{g c d of } \{ 4, \frac{n^2+n+2}{2} \} = 1$$

So, g c d of each vertex of degree greater than one is 1.

Hence $C_n(P_m)$ admits, oblong mean prime labeling.

Theorem 2.5 The graph $C_n \odot 2K_1$ admits oblong mean prime labeling.

Proof :Let $G = C_n \odot 2K_1$ and let v_1, v_2, \dots, v_{3n} are the vertices of G .

Here $|V(G)| = 3n$ and $|E(G)| = 3n$.

Define a function $f : V \rightarrow \{2, 6, 12, \dots, 3n(3n + 1)\}$ by

$$f(v_i) = i(i+1), \quad i = 1, 2, \dots, 3n.$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{ompl}^* is defined as follows

$$f_{ompl}^*(v_{3i-2} v_{3i-1}) = (3i-1)^2, \quad i = 1, 2, \dots, n.$$

$$f_{ompl}^*(v_{3i-1} v_{3i}) = (3i)^2, \quad i = 1, 2, \dots, n.$$

$$f_{ompl}^*(v_{3i-1} v_{3i+2}) = 9i^2 + 6i + 3, \quad i = 1, 2, \dots, n-1.$$

$$f_{ompl}^*(v_2 v_{3n-1}) = \frac{9n^2 - 3n + 6}{2}.$$

Clearly f_{ompl}^* is an injection.

$$\begin{aligned} \text{g c d of } (v_{3i-1}) &= \text{g c d of } \{ f_{ompl}^*(v_{3i-2} v_{3i-1}), f_{ompl}^*(v_{3i-1} v_{3i}) \} \\ &= \text{g c d of } \{ (3i-1)^2, (3i)^2 \} = 1, \quad i = 1, 2, \dots, n \end{aligned}$$

So, g c d of each vertex of degree greater than one is 1.

Hence $C_n \odot 2K_1$, admits oblong mean prime labeling.

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