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A Frequency-Domain Analysis and Design of a Ball and Plate System Controller

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Abstract

In this article, we have developed a state feedback controller to regulate the position of the ball on the plate. First, we determine the linear model of the ball and plate system based on retarded functional differential equations (RFDE), and secondly, according to the geometric approach and considering the feedback delay in the closed-loop, we integrated a controller based on a state feedback of the system. Finally, we tested the effectiveness of our methodology through simulations.

Keywords: Frequency-Domain, Ball-plate system, State-feedback control, Delay systems.

Introduction

In this article, we are interested in the balancing problem in the ball-plate system. In particular, we focus on the determination of the set of gain values of the state feedback control law allowing to regulate the position of the ball. This result can be used subsequently to meet the other requirements of the specification.

The ball and plate structure is an extension of the ball and beam system. Because of its simple configuration and easy-to-implement functions, it has become a popular target device for controller realization. For the purpose of tracking control, an approximate input and output linearization method based on imprecise models is introduced [2]. By using the touch screen as a position sensor with a conventional PID controller, actual experiments can be carried out [1]. In addition, a ball and plate system for teaching was developed [4,5]. Many researchers have established an online learning fuzzy control algorithm based on this teaching equipment [6,7]. By using a CCD camera instead of a touch screen device to obtain the position of the ball, a fuzzy control scheme is also proposed [8,9, 10]. In [12] a large mechanical ball and plate system was built to execute a state feedback control using a camera to obtain the position of the ball.

In this article, we also use a state feedback controller, however, our difference from previous studies is the use of geometric considerations (see [3,11]) taking into account feedback delay. The rest of this article is structured as follows. In Section 2, we introduced the mathematical model of the ball and plate system. Then, in Section 3, we reveal the main results of this paper, including the formulation of the controller to ensure system stability. In Section 4, we verified the results obtained in the previous sections through simulation. Finally, we make a conclusion in Section 5.

Mathematical Model of Ball and Plate System

In this section we will determine the state space model of the ball and plate system. For this, using the Lagrangian equation (1) we will determine the equation that describes the dynamics of the system and which will be linearized afterwards.

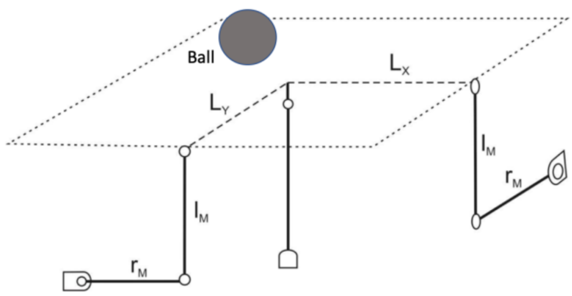

Fig.1. Schematic diagram of the ball-plate system

According to Lagrange's method, the equation that describes the dynamics of the ball and plate system is described by :

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \tag{1}$$

With

$$L = E_{k,T} + E_{k,R} - E_p \tag{2}$$

$E_{k,T}$, $E_{k,R}$ and E_p represent respectively the kinetic energy of translation, the kinetic energy of rotation and the potential energy of the ball.

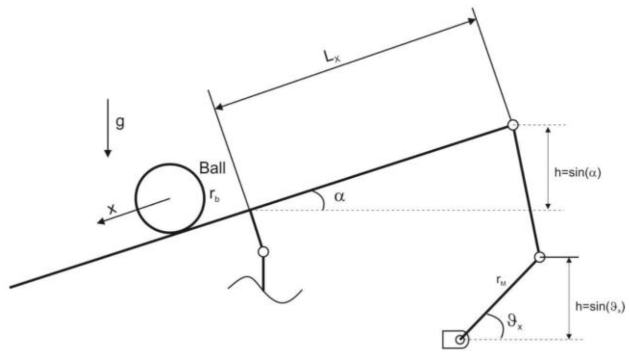

Fig.2. Lateral view of the ball-plate system

Based on the different parameters of the system (see Figure 1 and Figure 2), we have :

$$E_{k,T} = \frac{1}{2}\, m_b v_b^2 \;=\; \frac{1}{2}\, m_b(\dot{x}_b^2 + \dot{y}_b^2) \tag{3}$$

$$E_{k,R} = \frac{1}{2}\, J_b \omega_b^2 \;=\; \frac{1}{2}\, J_b \frac{(\dot{x}_b^2+\dot{y}_b^2)}{r_b^2} \tag{4}$$

$$E_p \;=\; -m_b g x_b \sin(\alpha) - m_b g y_b \sin(\beta) \tag{5}$$

After calculation, we thus have the differential equation which describes the dynamics of the system along the x-axis :

$$\ddot{x}_b = \frac{m_b g r_b^2}{m_b r_b^2 + J_b}\, \sin(\alpha) \tag{6}$$

The equation linking angle α and angle ϑ_x is as follows (see Figure 2) :

$$\sin(\vartheta_x)\, r_M = sin(\alpha) L_X = h \tag{7}$$

So from (6) and (7) we have :

$$\ddot{x}_b = \frac{m_b g r_b^2\, r_M}{(m_b r_b^2 + J_b)\, L_X}\, \sin(\vartheta_x) \tag{8}$$

Similarly, proceeding in the same way, the differential equation that describes the dynamics of the system along the y-axis is as follows :

$$\ddot{y}_b = \frac{m_b g r_b^2\, r_M}{(m_b r_b^2 + J_b)\, L_Y}\, \sin(\vartheta_y) \tag{9}$$

Considering small variation of ϑ_x and ϑ_y we have :

$$\sin(\vartheta_x) \approx \vartheta_x \,, \quad \sin(\vartheta_y) \approx \vartheta_y$$

Equations (13) and (13) become thus :

$$\ddot{x}_b = G_x \, \vartheta_x \tag{10}$$

$$\ddot{y}_b = G_y \, \vartheta_y \tag{11}$$

with

$$G_x = \frac{m_b g r_b^2 \, r_M}{(m_b r_b^2 + J_b) \, L_X} \quad \text{and} \quad G_y = \frac{m_b g r_b^2 \, r_M}{(m_b r_b^2 + J_b) \, L_Y}$$

Remarks: Considering the similarity of the x-axis and y-axis models, we will start the controller synthesis based on the x-axis model and infer the results of the y-axis at the end.

Therefore, considering the feedback delay τ_x, The state-space model along the x-axis can be defined by :

$$\begin{cases} \dot{x}(t) \;=\; A\,x(t) \;+\; B\,u(t-\tau_x) \\ y(t) \;=\; C\,x(t) \end{cases} \tag{12}$$

Where

$$x(t) = \; [x_b(t) \quad \dot{x}_b(t)]^{\,T}, u(t) = \; \vartheta_x(t)$$

and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ G_x \end{bmatrix}, \qquad C = [1 \quad 0]$$

Ball And Plate Controller : Geometric Considerations

Based on a geometric approach, we will develop in this section a state feedback control law acting on the packet loss rate and allowing the stabilization of the system (12).

On the basis of a geometrical analysis, we will develop a state feedback controller acting on the angles ϑ_x and ϑ_y and providing stabilization of the closed-loop system (12).

A. State Feedback Controller

The state feedback controller is defined by the equation :

$$u(t) \;=\; -K_x \, x(t) \tag{13}$$

Where $K_x = [k_{x1} \quad k_{x2}]$ represents the state feedback gain, $x(t)$ is the internal state of the system $x(t) = [x_b(t) \quad \dot{x}_b(t)]^T$, i.e. the position and velocity of the ball, and $u(t)$ is the control signal.

Using the Laplace transform, equations (17) and (18) are transformed into :

$$\begin{cases} s\,x(s) = A\,x(s) + B\,u(s)e^{-\tau_x s} \\ y(s) = C\,x(s) \\ u(s) = -K_x\ x(s) \end{cases} \tag{14}$$

Consequently, the characteristic equation of the system defined by (12) is expressed as:

$$H(s, k_{x1}, k_{x2}, \tau_x) = \det(sI_2 - (A - BK_x\,e^{-\tau_x s}))=0 \tag{15}$$

which can be written as :

$$H(s, k_{x1}, k_{x2}, \tau_x) = Q(s) + P(s)e^{-\tau_x s} \tag{16}$$

with

$$Q(s) = s^2\ ,\ P(s) = G_x(sk_{x2} + k_{x1}) \tag{17}$$

To study the stability of the system, we will first consider a zero delay, equation (16) becomes :

$$H(s, k_{x1}, k_{x2}, 0) = s^2 + G_x(sk_{x2} + k_{x1}) = 0 \tag{18}$$

Using Routh's criterion, we have found the conditions :

$$k_{x1} > 0,\ \ k_{x2} > 0 \tag{19}$$

B. Analysis in the (k1, k2) plane

Let's now consider the real case, corresponding to a non-zero feedback delay.

In order to determine the conditions that guarantee the stability of the system (12), we will first consider the case of a system at the limit of stability, which corresponds to the existence of a pure imaginary root of equation (16), so we have :

$$H(j\omega, k_{x1}, k_{x2}, \tau_x^*) = 0 \tag{20}$$

This is equivalent to :

$$-\omega^2 + G_x(j\omega k_{x2} + k_{x1})e^{-j\omega\,\tau_x^*} = 0$$

By separating the real and imaginary parts, after simplification, we have the following two equations:

$$k_{x1} = \frac{\omega^2 \cos(\omega\tau_x^*)}{G_x} \tag{21}$$

$$k_{x2} = \frac{\omega\, sin(\omega\tau_x^*)}{G_x} \tag{22}$$

Equations (21) and (22) define the set of values of k_{x1} and k_{x2} as a function of ω, thus, by varying ω, we find the crossing curves [3]. Thus, we have found a second condition on the stability of the system (12) in the plane (k_{x1}, k_{x2}). To illustrate these results, we plotted the crossing curves for different values of the delay τ_x (see Figure 3, Figure 4 and Figure 5).

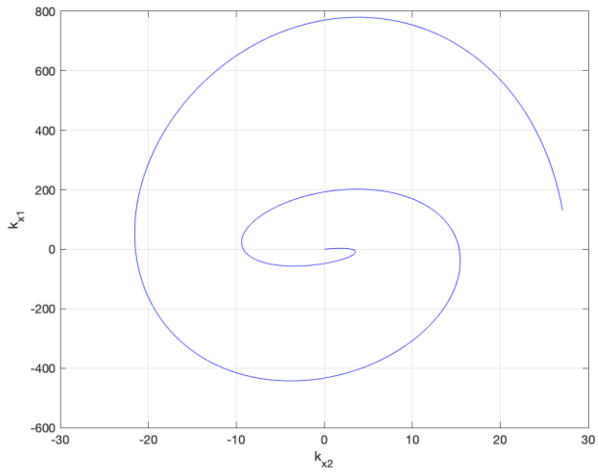

Fig.3. Crossing curves in the (k_{x1}, k_{x2}) space with $\tau_x = 0.4$

We are now going to determine the direction of the crossing [3], which is defined by the sign of $R_2 I_1 - R_1 I_2$, with :

$$R_1 + jI_1 = -\frac{1}{s}\left.\frac{\partial H(s, k_{x1}, k_{x2}, \tau_x^*)}{\partial k_{x2}}\right|_{s=j\omega}$$

$$R_2 + jI_2 = -\frac{1}{s}\left.\frac{\partial H(s, k_{x1}, k_{x2}, \tau_x^*)}{\partial k_{x1}}\right|_{s=j\omega}$$