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A Fuzzy Goal Programming Approach To Four Dimensional Multi Level Multi Objective Multi Item Fractional Transportation Problem Under Uncertain Environment

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Abstract

A model for four diemensional multi level multi objective multi item fractional transportation problem under uncertain conditions whose parameters in both objectives and constraints are considered as normal uncertain variables has been presented. The proposed model helps handling real life situations more suitably as diverse parameters have been taken into consideration for the first time ever. Applying the properties of expected value and chance constraint models on uncertainty theory, the equivalent deterministic model is obtained for proposed uncertain four diemensional multi level multi objective multi item fractional transportation problem and fuzzy goal programming technique is used to obtain the proposed model's compromise solution. An illustrative example is presented for more clarity about the model proposed.

AMS Subject Classifications: 90C08,90C29,90C32,90C70

Keywords : Multi level transportation problem, Uncertain fractional programming, Uncertain variable, Fuzzy goal programming

1. INTRODUCTION

A linear transportation problem consisting objective in the form of ratio of two different linear functions is known as linear fractional transportation problem. The fractional transportation problem was proposed for the first time by Swarup [32], which was then utilised extensively by many authors like Khurana and Arora [15], Joshi and Gupta [13]. The method to solve linear fractional programming problem was developed by Charnes and Cooper [5]. To solve a non-linear fractional programming problem, a parametric iterative method was proposed by Dinkelbach [9]. When two or more objectives are considered in fractional transportation problem, then it is known as multi objective fractional transportation problem. Multi level programming problems are used to make interactive decisions in hierarchical mangagement levels. It works on the principles of higher level to lower level. The first level decision maker decides the objectives and preferences and asks his inferiors to individually obtained solutions. The solutions provided by the lower level decison makers are compared and the best solution for the general advantage of the organisation is considered. YafengYin [33] obtained a compromise solution of transportation problem involving bilevel optimization and multiple objectives by using the genetic alogorithm base method. Shi and Xia [31] proposed an interactive alogorithm for obtaining solution for decision making problems involving a couple of levels. If the multi level linear fractional transportation problem considered, has two or more objectives, it is known as multi level multi objective fractional linear transportation problem. To solve multi level non-linear multi objective decision making problem, the technique for order perference by similarity to ideal solution was proposed by Baky [3]. When we consider the choices of routes in a solid transportation problem(STP), it becomes a four diemensional transportation problem(4DTP). Considering two or more items in four diemensional multi level multi objective fractional transportation problem(4DMLMOFTP) converts it into four diemensional multi level multi objective multi item fractional transportation problem(4DMLMOMIFTP). A special class of profit fixed charge multi item four diemensional transportation problem considering breakable items was proposed by Halder et al [12].

The data for the parameters such as demand, supply and unit transportation cost are highly imprecise and not accurate in real life suitations. Zadeh [34] introduced fuzzy sets to deal with these impreciseness of data. The method of using fuzzy programming approach for solving MOFTP was proposed by Sadia et al [27]. Later, they have developed a model for fully fuzzy multi objective fractional transportation problem [30]. Khalifa et al [14] employed fractional programming under fuzzy environment for solving solid transportation problem consisting multiple objectives and items. To solve bi - level linear fractional programming problem(BL-LFPP) with fuzzy parameters, a method was proposed by Sakawa et al [28]. Since Zadeh's fuzzy set theory did not hold good for suitations having imprecise data, we use uncertainty theory proposed by Liu [18]. The transportation problem consisting uncertain supply and demands was solved by Guo [11] in his article. Uncertain linear fractional problem and conversion of optimization problem into equivalent crisp problem was proposed by Seyyed Mojtaba [29]. To solve uncertain linear fractional transportation problem, Ali Mahmooderad [2] proposed a model. The multi objective fractional transportation problem under uncertain environment was studied by Revathi et. al. [25]. Solution algorithm for solving a multi level programming problem under uncertain conditions was proposed by Liu and Yao [19]. Gao and Kar [10] proposed a method to solve uncertain solid transportation problem involving product blending. To solve fixed charge multi item STP, few uncertain programming models were proposed by Liu et al [37]. A multi item STP under uncertain environment was studied by Dalman [8]. Cheng, Rao and Chen [7] studied about the multi diemensional Knapsack problem which was based on uncertain measures. The model for uncertain multi objective multi item four dimensional fractional transportation problems was studied by Revathi et. al. [26].

Hiearcherical decision making of a transportation problem is important in dealing with complex cases considering conflicting fractional objectives, multiple items and multiple routes. Till date there has been no studies regarding four diemensional multi level multi objective multi item fractional transportation problem(4DMLMOMIFTP) to handle such real life suitations. For the first time, an effort has been made to create 4DMLMOMIFTP under uncertain environment inspired by the concepts of uncertainty theory and fractional programming. The primary motive of this paper is to present a method to solve uncertain 4DMLMOMIFTP. Utilising expected value and chance constraint models on uncertainty theory, we convert the above said problem into deterministic problem. Since the problem has multiple objectives, we make use of the fuzzy goal programming technique as it yields suitable compromise solution.

We have reviwed some definitions and theorems of uncertainty theory which are used in the model in section 2. Notations are given under section 3. In section 4, the mathematical model of uncertain four dimension multi level multi objective multi-item fractional transportation problem (U4DMLMOMIFTP) is introduced. Equivalent deterministic models by using expected value method and chance constraint method are given in the sections 5 and 6 respectively. In section 7, fuzzy goal programming technique for solving multi level fractional programming is presented. Section 8 contains the procedure for solving the U4DMLMOMIFTP, followed by section 9 consisting of a numerical example and section 10 containing conclusion.

2. PRELIMINARIES

Here, we review some basic definitions and the concepts of uncertainty theory, which will be applied in the subsequent sections.

Definition 2.1. [18,20] Let $\mathcal{L}$ be a σ - algebra of collection of events Λ of a universal set Γ. A set function $\mathcal{M}$ is said to be uncertain measure defined on the σ - algebra where $\mathcal{M}\{\Lambda\}$ indicate the belief degree with which we believe that the event will happens and satisfies the following axioms:

1. Normality Axiom: For the universal set Γ, we have $\mathcal{M}(\Gamma) = 1$.
2. Duality Axiom: For any event Λ, we have $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^C\} = 1$.
3. Subadditivity Axiom: For every countable sequence of events $\Lambda_1, \Lambda_2, \cdots$, we have $\mathcal{M}\{\cup_{i=1}^{\infty}\Lambda_i\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\} = 1$
4. Product Axiom: Let $(\Gamma_i, \mathcal{L}_i, \mathcal{M}_i)$ be uncertainty spaces for $i = 1, 2, 3, \cdots$ The product uncertain measure is an uncertain measure holds $\mathcal{M}\{\Pi_{i=1}^{\infty}\Lambda_i\} \leq \Lambda_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}$ where $\Lambda_i \in \mathcal{L}_i$ for $i = 1, 2, 3, \cdots, \infty$.

Definition 2.2. [18] A function $\xi : (\Gamma, \mathcal{L}, \mathcal{M}) \to \mathcal{R}$ is said to be an uncertain variable such that $\{\xi \in B\} = \{\gamma \in \Gamma / \xi(\gamma) \in B\}$ is an event for any Borel set B of real numbers.

Definition 2.3. [18] An uncertain variable ξ defined on the uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ is said to be non- negative if $\mathcal{M}\{\xi < 0\} = 0$ and positive if $\mathcal{M}\{\xi \leq 0\} = 0$.

Definition 2.4. [18] The uncertainty distribution $\varphi(x)$ of an uncertain variable ξ for any real number x is defined by $\varphi(x) = \mathcal{M}\{\xi \leq x\}$.

Definition 2.5. Let $\varphi(x)$ be the regular uncertainty distribution of an uncertain variable ξ. Then $\varphi^{-1}(\alpha)$ is called inverse uncertainty distribution of ξ and it exists on (0, 1).

Definition 2.6. [18] The uncertain variable ξ_i $(i = 1, 2, \cdots n)$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n}(\xi_n \in B_n)\right\} = \Lambda_{i=1}^{n}\mathcal{M}(\xi_n \in B_n) \tag{2.1}$$

where B_i $(i = 1, 2, \cdots n)$ are called Borel sets of real numbers.

Theorem 2.7. *Let ξ be an uncertain variable with regular uncertain distribution function ψ. Then its α-optimistic value and α-pessimistic values are*

$$\xi_{\sup}(\alpha) = \psi^{-1}(1-\alpha), \;\; \xi_{\inf}(\alpha) = \psi^{-1}(\alpha) \tag{2.2}$$

Theorem 2.8. *[21] The regular uncertainty distributions of independent uncertain variables ξ_i $(i = 1, 2, \cdots n)$ are ϕ_i $(i = 1, 2, \cdots n)$ respectively. If the function $f(x_1, x_2, \cdots, x_n)$ is strictly increasing and strictly decreasing with respect to $x_1, x_2, \cdots, x_m$ and $x_{m+1}, x_{m+2}, \cdots, x_n$ respectively then the uncertain variable $\xi = f(\xi_1, \xi_2, \cdots, \xi_m, \cdots, \xi_n)$ has an inverse uncertainty distribution*

$$\psi^{-1}(\alpha) = f(\phi_1^{-1}(\alpha), \phi_2^{-1}(\alpha), \cdots, \phi_m^{-1}(\alpha), \phi_{m+1}^{-1}(1-\alpha), \phi_{m+2}^{-1}(1-\alpha), \cdots, \phi_n^{-1}(1-\alpha)) \tag{2.3}$$

Definition 2.9. [18] The expected value of uncertain variable ξ is given by

$$E(\xi) = \int_0^\infty \mathcal{M}\{\xi \geq x\}dx - \int_{-\infty}^0 \mathcal{M}\{\xi \leq x\}dx \tag{2.4}$$

This is valid only if at least one of the integral is finite.

Theorem 2.10. *[22] Let ϕ_i $(i = 1, 2, \cdots n)$ be regular uncertainty distributions of independent ξ_i $(i = 1, 2, \cdots n)$ with respectively. If the function $f(x_1, x_2, \cdots, x_n)$ is strictly increasing and strictly decreasing w.r.to $x_1, x_2, \cdots, x_m$ and $x_{m+1}, x_{m+2}, \cdots, x_n$ respectively, then*

$$E(\xi) = \int_0^1 f(\phi_1^{-1}(\alpha), \cdots, \phi_m^{-1}(\alpha), \phi_{m+1}^{-1}(1-\alpha), \cdots, \phi_n^{-1}(1-\alpha))d\alpha \tag{2.5}$$

From the above theorem, we know that

$$E(\xi) = \int_0^1 \phi^{-1}(\alpha)d\alpha \tag{2.6}$$

where ξ is an uncertain variable with regular uncertainty distribution ϕ.

Definition 2.11. [18] A linear uncertain variable ξ is defined as

$$\phi(x) = \begin{cases} 0 & \text{if } x \leq l \\ \dfrac{x-l}{m-l} & \text{if } 1 \leq x \leq m \\ 1 & \text{if } x \geq m \end{cases} \tag{2.7}$$

represented by $L(1, m)$, where 1 and $m \in R$ with $1 < m$.
The inverse distribution function of a linear uncertain variable $L(1, m)$ is given by

$$\phi^{-1}(\alpha) = (1-\alpha)l + \alpha m \tag{2.8}$$

and its expected value is given by

$$E(\xi) = \frac{l+m}{2} \tag{2.9}$$

Definition 2.12. [18] The distribution function of a normal uncertain variable is

$$\phi(x) = \left(1 + \exp^{\left(\frac{\pi(\mu - x)}{\sigma\sqrt{3}}\right)}\right)^{-1}, \ x \geq 0 \tag{2.10}$$

and it is denoted as $N(\mu, \sigma)$; $\mu, \sigma \in R$ with $\sigma > 0$.
The inverse uncertainty distribution and the expected value of $N(\mu, \sigma)$ is defined as follows

$$\phi^{-1}(\alpha) = \mu + \frac{\sigma\sqrt{3}}{\pi} \ln \frac{\alpha}{1 - \alpha} \tag{2.11}$$

$$E(\xi) = \mu \tag{2.12}$$

3. NOMENCLATURE

The following notations have been introduced for constructing the proposed U4DMLMOMIFTP model.

i	–	index for sources
j	–	index for destinations
(tn)	–	index for n^{th} objective function of t^{th} level
k	–	index of conveyance
r	–	index of route of transportation
g	–	index of product
$\tilde{Z}^{(tn)}$	–	n^{th} uncertain objective function at t^{th} level, where n=1,2,. . . N and t=1,2....T.
$\frac{\tilde{C}^{(tn)}_{ijkrg}}{\tilde{D}^{(tn)}_{ijkrg}}$	–	ratio of unit transportation actual cost and standard cost of g^{th} good from i^{th} origin to j^{th} destination by k^{th} transport over r^{th} road per unit distance in n^{th} objective of t^{th} level.
$\frac{\tilde{A}^{(tn)}_{ijkrg}}{\tilde{S}^{(tn)}_{ijkrg}}$	–	ratio of actual transportation time to the standard transportation time.
$\tilde{a}_{ig}$	–	quantity of g^{th} good available at i^{th} origin
$\tilde{b}_{jg}$	–	the demand of the g^{th} good at the j^{th} destination.
$\tilde{e}_k$	–	capacity of a single k^{th} type transport
$d^{(tn)}_{N^+}, d^{(tn)}_{D^+}$	–	positive deviational variable of the n^{th} objective's numerator and denominator at t^{th} level respectively.
$d^{(tn)}_{N^-}, d^{(tn)}_{D^-}$	–	negative deviational variable of the n^{th} objective's numerator and denominator at t^{th} level respectively.
$d^{(t)}_{+}, d^{(t)}_{-}$	–	positive and negative deviational variables for t^{th} level decision vectors.

4. MATHEMATICAL FORMULATION OF U4DMLMOMIFTP

The general model for an uncertain four dimensional multi level multi-objective multi-item fractional transportation problem is presented below.

The general form of U4DMLMOMIFTP is given by

$$
\left\{
\begin{array}{l}
\text{[Level1]} \\
\underset{\bar{X}^{1}}{Min\tilde{Z}^{(1n)}} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\tilde{C}_{ijkrg}^{(1n)}x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\tilde{D}_{ijkrg}^{(1n)}x_{ijkrg}},\ n=1,2,\cdots,N \\
\text{[Level2]} \\
\underset{\bar{X}^{2}}{Min\tilde{Z}^{(2n)}} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\tilde{C}_{ijkrg}^{(2n)}x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\tilde{D}_{ijkrg}^{(2n)}x_{ijkrg}},\ n=1,2,\cdots,N \\
\vdots \\
\text{[Level t]} \\
\underset{\bar{X}^{tn}}{Min\tilde{Z}^{(tn)}} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\tilde{C}_{ijkrg}^{(tn)}x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\tilde{D}_{ijkrg}^{(tn)}x_{ijkrg}},\ n=1,2,\cdots,N \\
\text{subject to} \\
\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}x_{ijkrg} \le \tilde{a}_{ig},\ i=1,2,\cdots,I, g=1,2,\cdots,G \\
\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R}x_{ijkrg} \ge \tilde{b}_{jg},\ j=1,2,\cdots,J, g=1,2,\cdots,G \\
\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{g=1}^{G}x_{ijkrg} \le \tilde{e}_{k},\ k=1,2,\cdots,K \\
x_{ijkrg} \ge 0, \forall i,j,k,r,g
\end{array}
\right.
\tag{4.1}
$$

where g products can be transported from i origins $\tilde{a}_i$ to j destinations $\tilde{b}_j$ by means of $\tilde{e}_k$ conveyances and n objectives are to be minimized at each level t, where $t = 1, 2, \cdots, T$ and $\bar{X}^t = \{X_1^t, X_2^t, \cdots, X_M^t\}$, decision variables under the control of t^{th} level decision maker. Here, $\bar{X} = \bar{X}^1 \cup \bar{X}^2 \cup \bar{X}^3, \cdots, \bar{X}^T$.

The above model (4.1) is created considering all the parameters involved to be known quantities. Contrastingly, in real life situations, these parameters like supply, demand, costs and capacities exists with some uncertainty which make the model as the complex one. Due to the uncertainty that exists in the parameters, it cannot be optimized directly. Instead we will use expected value and chance constraint model on uncertainty theory for the above U4DMLMOMIFTP and obtain the equivalent deterministic model.

5. EXPECTED VALUE MODEL FOR U4DMLMOMIFTP

An equivalent deterministic model for U4DMLMOMIFTP has been presented in this section.

By using the expected value method and its properties, the equivalent deterministic model for U4DMLMOMIFTP is given as follows.

$$
\begin{cases}
[\text{Level1}] \\
\underset{\bar{X}^1}{Min \tilde{Z}^{(1n)}} = E\left(\dfrac{\sum\limits_{i=1}^{I}\sum\limits_{j=1}^{J}\sum\limits_{k=1}^{K}\sum\limits_{r=1}^{R}\sum\limits_{g=1}^{G}\tilde{C}_{ijkrg}^{(1n)}x_{ijkrg}}{\sum\limits_{i=1}^{I}\sum\limits_{j=1}^{J}\sum\limits_{k=1}^{K}\sum\limits_{r=1}^{R}\sum\limits_{g=1}^{G}\tilde{D}_{ijkrg}^{(1n)}x_{ijkrg}}\right), \ n = 1,2,\cdots,N \\
[\text{Level2}] \\
\underset{\bar{X}^2}{Min \tilde{Z}^{(2n)}} = E\left(\dfrac{\sum\limits_{i=1}^{I}\sum\limits_{j=1}^{J}\sum\limits_{k=1}^{K}\sum\limits_{r=1}^{R}\sum\limits_{g=1}^{G}\tilde{C}_{ijkrg}^{(2n)}x_{ijkrg}}{\sum\limits_{i=1}^{I}\sum\limits_{j=1}^{J}\sum\limits_{k=1}^{K}\sum\limits_{r=1}^{R}\sum\limits_{g=1}^{G}\tilde{D}_{ijkrg}^{(2n)}x_{ijkrg}}\right), \ n = 1,2,\cdots,N \\
\vdots \\
[\text{Level t}] \\
\underset{\bar{X}^t}{Min \tilde{Z}^{(tn)}} = E\left(\dfrac{\sum\limits_{i=1}^{I}\sum\limits_{j=1}^{J}\sum\limits_{k=1}^{K}\sum\limits_{r=1}^{R}\sum\limits_{g=1}^{G}\tilde{C}_{ijkrg}^{(tn)}x_{ijkrg}}{\sum\limits_{i=1}^{I}\sum\limits_{j=1}^{J}\sum\limits_{k=1}^{K}\sum\limits_{r=1}^{R}\sum\limits_{g=1}^{G}\tilde{D}_{ijkrg}^{(tn)}x_{ijkrg}}\right), \ n = 1,2,\cdots,N \\
\text{subject to} \\
E\left(\sum\limits_{j=1}^{J}\sum\limits_{k=1}^{K}\sum\limits_{r=1}^{R} x_{ijkrg} - \tilde{a}_{ig}\right) \leq 0, \ i = 1,2,\cdots,I, g = 1,2,\cdots,G \\
E\left(\tilde{b}_{jg} - \sum\limits_{i=1}^{I}\sum\limits_{k=1}^{K}\sum\limits_{r=1}^{R} x_{ijkrg}\right) \leq 0, \ j = 1,2,\cdots,J, g = 1,2,\cdots,G \\
E\left(\sum\limits_{i=1}^{I}\sum\limits_{j=1}^{J}\sum\limits_{r=1}^{R}\sum\limits_{g=1}^{G} x_{ijkrg} - \tilde{e}_k\right) \leq 0, \ k = 1,2,\cdots,K \\
x_{ijkrg} \geq 0, \forall i,j,k,r,g
\end{cases}
\tag{5.1}
$$

By using the properties of expected value method in the above model (5.1), we have

$$
\begin{cases}
\text{[Level1]} \\
\underset{\bar{X}^1}{Min\tilde{Z}^{(1n)}} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G} E(\tilde{C}_{ijkrg}^{(1n)})x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G} E(\tilde{D}_{ijkrg}^{(1n)})x_{ijkrg}},\ n=1,2,\cdots,N \\
\text{[Level2]} \\
\underset{\bar{X}^2}{Min\tilde{Z}^{(2n)}} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G} E(\tilde{C}_{ijkrg}^{(2n)})x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G} E(\tilde{D}_{ijkrg}^{(2n)})x_{ijkrg}},\ n=1,2,\cdots,N \\
\vdots \\
\text{[Level t]} \\
\underset{\bar{X}^t}{Min\tilde{Z}^{(tn)}} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G} E(\tilde{C}_{ijkrg}^{(tn)})x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G} E(\tilde{D}_{ijkrg}^{(tn)})x_{ijkrg}},\ n=1,2,\cdots,N \\
\text{subject to} \\
\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrg} - E(\tilde{a}_{ig}) \le 0,\ i=1,2,\cdots,I, g=1,2,\cdots,G \\
E(\tilde{b}_{jg}) - \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrg} \le 0,\ j=1,2,\cdots,J, g=1,2,\cdots,G \\
\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{g=1}^{G} x_{ijkrg} - E(\tilde{e}_k) \le 0,\ k=1,2,\cdots,K \\
x_{ijkrg} \ge 0, \forall i,j,k,r,g
\end{cases}
\tag{5.2}
$$

6. CHANCE CONSTRAINED METHOD FOR U4DMLMOMIFTP

An equivalent deterministic model for an U4DMLMOMIFTP by using chance constraint method is presented in this section.

Suppose $\tilde{C}^{(tn)}_{ijkrg}, \tilde{D}^{(tn)}_{ijkrg}, (t = 1,2,\cdots,T, \& n = 1,2,\cdots,N), \tilde{a}_{ig}, \tilde{b}_{jg}, \tilde{e}_k$ are independent uncertain variables with regular uncertain distributions $\chi^{(tn)}_{ijkrg}, \phi^{(tn)}_{ijkrg}, \psi_{ip}, \theta_{jp}, \lambda_k (t = 1,2,\cdots,T, \& n = 1,2,\cdots,N)$ respectively. The proposed U4DMLMOMIFTP's equivalent deterministic model using the chance constrained method is given as below.

$$\begin{cases}
\text{[Level1]} \\
\underset{\bar{X}^1}{Min \tilde{Z}^{(1n)}} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}(\chi^{(1n)})^{-1}_{ijkrg}(\alpha_{1n})x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}(\phi^{(1n)})^{-1}_{ijkrg}(\gamma_{1n})x_{ijkrg}}, \; n = 1,2,\cdots,N \\
\text{[Level2]} \\
\underset{\bar{X}^2}{Min \tilde{Z}^{(2n)}} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}(\chi^{(2n)})^{-1}_{ijkrg}(\alpha_{2n})x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}(\phi^{(2n)})^{-1}_{ijkrg}(\gamma_{2n})x_{ijkrg}}, \; n = 1,2,\cdots,N \\
\vdots \\
\text{[Level t]} \\
\underset{\bar{X}^t}{Min \tilde{Z}^{(tn)}} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}(\chi^{(tn)})^{-1}_{ijkrg}(\alpha_{tn})x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}(\phi^{(tn)})^{-1}_{ijkrg}(\gamma_{tn})x_{ijkrg}}, \; n = 1,2,\cdots,N \\
\text{subject to} \\
\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrg} \leq \psi^{-1}_{ig}(1-\alpha_{ig}), \; i = 1,2,\cdots,I, g = 1,2,\cdots,G \\
\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrg} \geq \theta^{-1}_{jg}(\beta_{jg}), \; j = 1,2,\cdots,J, g = 1,2,\cdots,G \\
\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{g=1}^{G} x_{ijkrg} \leq \lambda^{-1}_{k}(1-\beta_k), \; k = 1,2,\cdots,K \\
x_{ijkrp} \geq 0, \forall i,j,k,r,g
\end{cases} \tag{6.1}$$

where $\alpha_{tn}, \gamma_{tn}, \beta_1, \beta_2$ and $\beta_3 (t = 1,2,\cdots,T, \& n = 1,2,\cdots,N)$ are predetermined confidence level and $\alpha_{tn}, \gamma_{tn}, \beta_1, \beta_2$ and $\beta_3 \in (0,1), \forall n, t$.

By applying the properties of chance constraint method, (6.1) becomes as follows.

$$
\begin{cases}
\text{[Level1]} \\
\underset{\bar{X}^1}{Min}\tilde{Z}^{(1n)} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\left(e_{ijkrg}^{(1n)} + \dfrac{\sigma_{ijkrg}}{\pi} * \sqrt{3}\ln\dfrac{\alpha_{1n}}{1-\alpha_{1n}}\right)x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\left(e_{ijkrg}^{1} + \dfrac{\sigma_{ijkrg}^{1}}{\pi} * \sqrt{3}\ln\dfrac{\gamma_{1n}}{1-\gamma_{1n}}\right)x_{ijkrg}}, \ n = 1, 2, \cdots, N \\
\text{[Level2]} \\
\underset{\bar{X}^2}{Min}\tilde{Z}^{(2n)} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\left(e_{ijkrg}^{(2n)} + \dfrac{\sigma_{ijkrg}}{\pi} * \sqrt{3}\ln\dfrac{\alpha_{2n}}{1-\alpha_{2n}}\right)x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\left(e_{ijkrg}^{1} + \dfrac{\sigma_{ijkrg}^{1}}{\pi} * \sqrt{3}\ln\dfrac{\gamma_{2n}}{1-\gamma_{2n}}\right)x_{ijkrg}}, \ n = 1, 2, \cdots, N \\
\vdots \\
\text{[Level t]} \\
\underset{\bar{X}^t}{Min}\tilde{Z}^{(tn)} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\left(e_{ijkrg}^{(tn)} + \dfrac{\sigma_{ijkrg}}{\pi} * \sqrt{3}\ln\dfrac{\alpha_{tn}}{1-\alpha_{tn}}\right)x_{ijkrg}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{g=1}^{G}\left(e_{ijkrg}^{1} + \dfrac{\sigma_{ijkrg}^{1}}{\pi} * \sqrt{3}\ln\dfrac{\gamma_{tn}}{1-\gamma_{tn}}\right)x_{ijkrg}}, \ n = 1, 2, \cdots, N \\
\text{subject to} \\
\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R} \le e_{ig} + \dfrac{\sigma_{ig}\sqrt{3}}{\pi}\ln\dfrac{1-\alpha_{ig}}{\alpha_{ig}}, \ i = 1, 2, \cdots, I, g = 1, 2, \cdots, G \\
\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R} \ge e_{jg} + \dfrac{\sigma_{jg}\sqrt{3}}{\pi}\ln\dfrac{\beta_{jg}}{1-\beta_{jg}}, \ j = 1, 2, \cdots, J, g = 1, 2, \cdots, G \\
\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{g=1}^{G} \le e_{k} + \dfrac{\sigma_{k}\sqrt{3}}{\pi}\ln\dfrac{1-\beta_{k}}{\beta_{k}}, \ k = 1, 2, \cdots, K \\
x_{ijkrp} \ge 0, \forall i, j, k, r, g
\end{cases}
\tag{6.2}
$$

Where $\alpha_{tn}, \gamma_{tn}, \beta_1, \beta_2$ and $\beta_3, \forall t = 1, 2, \cdots, T \ \& n = 1, 2, \cdots, N$ are predetermined confidence level and $\alpha_{tn}, \gamma_{tn}, \beta_1, \beta_2$ and $\beta_3 \in (0, 1), \forall n, t$.

Definition 6.1. A point $x^0 \in X$ is said to be an efficient solution of U4DMLMOMIFTP iff there does not exist another $x \in X$ s.t. $Z_n(x) \le Z_n(x^0)$ and $Z_n(x) < Z_n(x^0)$ for at least one n.

7. FUZZY GOAL PROGRAMMING TECHNIQUE FOR U4DMLMOMIFTP

When more than one goal is present, to obtain the satisfactory solution the goal programming technique was proposed by Charnes Clan Cooper [6]. The goal programming technique was further developed by T.Chang [4], Pal [24] etc. To solve multi objective transportation problem (MOTP), a new fuzzy goal programming technique was introduced by Mohammed [23], which was later used by Zangiabadi [35, 36] to solve MOTP containing linear as well as non-linear membership functions. The main aim of goal programming (GP) is to minimize the distance between Z and aspiration (or) target level $\bar{Z}$. The positive and negative deviational variables are defined as follows.

$$D_n^+ = \max(0, Z_n - \bar{Z}_n)$$
$$D_n^- = \max(0, \bar{Z}_n - Z_n)$$

When the aim is to maximize Z_n, we obtain the optimal solution by minimizing the negative deviational variable. Similarly, when the aim is to minimimize Z_n, we obtain the optimal solution by minimizing the positive deviational variable. When we desire $Z_n = \bar{Z}_n$, we obtain the optimal solution by minimizing $D_n^+ + D_n^-$.

To formulate membership functions the fuzzy goals and their aspiration levels has to be defined first. Firstly, we maximize and minimize the numerator and denominator objective functions individually for each level of decision making. After finding the maximum and minimum values of each objective function, we construct the payoff matrices as follows.

$$\begin{bmatrix} \bar{N}^{(11)}(\bar{X}^1) & \bar{N}^{(12)}(\bar{X}^1) & \cdots & \bar{N}^{(1N)}(\bar{X}^1) \\ \bar{N}^{(21)}(\bar{X}^2) & \bar{N}^{(22)}(\bar{X}^2) & \cdots & \bar{N}^{(2N)}(\bar{X}^2) \\ \vdots & \vdots & \vdots & \vdots \\ \bar{N}^{(T1)}(\bar{X}^T) & \bar{N}^{(T2)}(\bar{X}^T) & \cdots & \bar{N}^{(TN)}(\bar{X}^T) \end{bmatrix} \tag{7.1}$$

$$\begin{bmatrix} \underline{N}^{(11)}(\bar{X}^1) & \underline{N}^{(12)}(\bar{X}^1) & \cdots & \underline{N}^{(1N)}(\bar{X}^1) \\ \underline{N}^{(21)}(\bar{X}^2) & \underline{N}^{(22)}(\bar{X}^2) & \cdots & \underline{N}^{(2N)}(\bar{X}^2) \\ \vdots & \vdots & \vdots & \vdots \\ \underline{N}^{(T1)}(\bar{X}^T) & \underline{N}^{(T2)}(\bar{X}^T) & \cdots & \underline{N}^{(TN)}(\bar{X}^T) \end{bmatrix} \tag{7.2}$$

$$\begin{bmatrix} \bar{D}^{(11)}(\bar{X}^1) & \bar{D}^{(12)}(\bar{X}^1) & \cdots & \bar{D}^{(1N)}(\bar{X}^1) \\ \bar{D}^{(21)}(\bar{X}^2) & \bar{D}^{(22)}(\bar{X}^2) & \cdots & \bar{D}^{(2N)}(\bar{X}^2) \\ \vdots & \vdots & \vdots & \vdots \\ \bar{D}^{(T1)}(\bar{X}^T) & \bar{D}^{(T2)}(\bar{X}^T) & \cdots & \bar{D}^{(TN)}(\bar{X}^T) \end{bmatrix} \tag{7.3}$$

$$\begin{bmatrix} \underline{D}^{(11)}(\bar{X}^1) & \underline{D}^{(12)}(\bar{X}^1) & \cdots & \underline{D}^{(1N)}(\bar{X}^1) \\ \underline{D}^{(21)}(\bar{X}^2) & \underline{D}^{(22)}(\bar{X}^2) & \cdots & \underline{D}^{(2N)}(\bar{X}^2) \\ \vdots & \vdots & \vdots & \vdots \\ \underline{D}^{(T1)}(\bar{X}^T) & \underline{D}^{(T2)}(\bar{X}^T) & \cdots & \underline{D}^{(TN)}(\bar{X}^T) \end{bmatrix} \tag{7.4}$$

Each row's maximum values $\bar{N}^{(tn)}$ and $\bar{D}^{(tn)}$, $\forall n = 1, 2, \cdots, N$ are known as the aspired level or upper tolerance limit for the membership function of t^{th} level numerator and denominator objectives respectively. Likewise, the minimum values of each row $\underline{N}^{(tn)}$ and $\underline{D}^{(tn)}$, $\forall n = 1, 2, \cdots, N$ are lower tolerance limit for the membership function of the t^{th} level numerator and denominator respectively.
The linear membership functions for fuzzy goals are defined as follows.

$$\mu(N^{(tn)}(\bar{X})) = \begin{cases} 1 & \text{if } N^{(tn)}(\bar{X}) \leq \underline{N}^{(tn)} \\ \dfrac{\bar{N}^{(tn)} - N^{(tn)}(\bar{X})}{\bar{N}^{(tn)} - \underline{N}^{(tn)}} & \text{if } \underline{N}^{(tn)} \leq N^{(tn)}(\bar{X}) \leq \bar{N}^{(tn)}, \\ & \quad \forall t = 1, 2, \cdots, T \ \&n = 1, 2, \cdots, N \\ 0 & \text{if } \underline{N}^{(tn)}(\bar{X}) \geq \bar{N}^{(tn)} \end{cases} \tag{7.5}$$

$$\mu(D^{(tn)}(\bar{X})) = \begin{cases} 0 & \text{if } D^{(tn)}(\bar{X}) \leq \underline{D}^{(tn)} \\ \dfrac{D^{(tn)}(\bar{X}) - \underline{D}^{(tn)}}{\bar{D}^{(tn)} - \underline{D}^{(tn)}} & \text{if } \underline{D}^{(tn)} \leq D^{(tn)}(\bar{X}) \leq \bar{D}^{(tn)}, \\ & \quad \forall t = 1, 2, \cdots, T \ \&n = 1, 2, \cdots, N \\ 1 & \text{if } \underline{D}^{(tn)}(\bar{X}) \geq \bar{D}^{(tn)} \end{cases} \tag{7.6}$$

Comparably, the decision vector X^t's membership function as follows, where $(t = 1, 2, \cdots, T)$.

$$\mu(X^t) = \begin{cases} 1 & \text{if } X^t \leq X^t \\ \dfrac{\bar{X} - X^t}{\bar{X} - \underline{X}} & \text{if } \underline{X} \leq X^t \leq \bar{X}, \forall t = 1, 2, \cdots, T \ \&n = 1, 2, \cdots, N \\ 0 & \text{if } X^t \geq \bar{X} \end{cases} \tag{7.7}$$

where $\bar{X}^t$ and $\underline{X}^t$ are represents the values of corresponding decision vectors at each level which yield the maximum and minimum values of the numerator part of the objective functions $(\bar{N}^t(\bar{X})$ and $\underline{N}^t(\underline{X}), \forall t = 1, 2, \cdots, T-1)$ at every level respectively is given by

$$\left.\begin{aligned} \bar{X}^t &= \underset{\bar{X}^t \in X}{Max} \ \{\bar{N}^{(tn)}(\bar{X}),\, \forall n = 1, 2, \cdots, N\} \\ \underline{X}^t &= \underset{\bar{X}^t \in X}{Min} \ \{\underline{N}^{(tn)}(\bar{X}),\, \forall n = 1, 2, \cdots, N\} \end{aligned}\right\} \tag{7.8}$$

Since, the objective functions generally conflict each other, the completely satisfactory optimal solution is very rarely obtained; the highest degree of membership value for each fuzzy goal can be 1. So, we need to minimize the regret of each decision maker at all levels and every decision maker should try to maximize the membership function by reducing the distance between membership value and unity and minimize the positive deviational value. In this process all objective functions are simultaneously optimized. The model U4DMLMOMIFTP (5.2) can be written as

$$\begin{cases} \min \delta = \sum_{t=1}^{T} \sum_{n=1}^{N} d_{N^+}^{(tn)} + \sum_{t=1}^{T} \sum_{n=1}^{N} d_{D^+}^{(tn)} + \sum_{t=1}^{T} d_+^t \\ \text{subject to} \\ \mu(N^{(tn)} + d_{N^-}^{(tn)} - d_{N^+}^{(tn)} = 1, \forall t = 1, 2, \cdots, T \ \&n = 1, 2, \cdots, N \\ \mu(D^{(tn)} + d_{N^-}^{(tn)} - d_{N^+}^{(tn)} = 1, \forall t = 1, 2, \cdots, T \ \&n = 1, 2, \cdots, N \\ \mu(X^t) + d_-^{(t)} - d_+^{(t)} = 1, \forall t = 1, 2, \cdots, T-1 \\ \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{r=1}^{R} x_{ijkrg} - E(\tilde{a}_{ig}) \leq 0, \ i = 1, 2, \cdots, I, g = 1, 2, \cdots, G \\ E(\tilde{b}_{jg}) - \sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{r=1}^{R} x_{ijkrg} \leq 0, \ j = 1, 2, \cdots, J, g = 1, 2, \cdots, G \\ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{r=1}^{R} \sum_{g=1}^{G} x_{ijkrg} - E(\tilde{e}_k) \leq 0, \ k = 1, 2, \cdots, K \\ x_{ijkrg} \geq 0, \forall i, j, k, r, g \end{cases} \tag{7.9}$$

Therefore, we can note that only the sum of over deviational variables has to be minimized to reach the aspiration level. When the aspired level is reached, the negative deviational value is zero. When the achievement level is zero, negative deviational value becomes unity.

The GP model formulation of U4DMLMOMIFTP(7.9) becomes as follows:

$$\begin{cases} \min \delta = \sum_{t=1}^{T}\sum_{n=1}^{N} d_{N^+}^{(tn)} + \sum_{t=1}^{T}\sum_{n=1}^{N} d_{D^+}^{(tn)} + \sum_{t=1}^{T} d_+^t \\ \text{subject to} \\ \underline{N}^{(tn)} - N^{(tn)}(\bar{X}) + d_{N^-}^{(tn)}(\bar{N}^{(tn)} - \underline{N}^{(tn)}) = 1, \forall t = 1,2,\cdots,T \ \& n = 1,2,\cdots,N \\ -\bar{D}^{(tn)} + D^{(tn)}(\bar{X}) + d_{N^-}^{(tn)}(\bar{D}^{(tn)} - \underline{D}^{(tn)}) = 1, \forall t = 1,2,\cdots,T \ \& n = 1,2,\cdots,N \\ \underline{X} - X^t + d_-^{(t)}(\bar{X} - \underline{X}) = 1, \forall t = 1,2,\cdots,T-1 \\ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrg} - E(\tilde{a}_{ig}) \leq 0,\ i = 1,2,\cdots,I, g = 1,2,\cdots,G \\ E(\tilde{b}_{jg}) - \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrg} \leq 0,\ j = 1,2,\cdots,J, g = 1,2,\cdots,G \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{g=1}^{G} x_{ijkrg} - E(\tilde{e}_k) \leq 0,\ k = 1,2,\cdots,K \\ x_{ijkrg} \geq 0, \forall i,j,k,r,g \end{cases} \tag{7.10}$$

8. SOLUTION PROCEDURE FOR U4DMLMOMIFTP

Step 1: Formulate the decision making model for uncertain four dimensional multi level multi objective multi item fractional transportation problem as in (4.1).

Step 2: Obtain an equivalent deterministic model for U4DMLMOMIFTP by using expected value model and chance constraint model on uncertainty theory as in (5.2) and (6.2).

Step 3: Under the given constraints, for all objectives, calculate the individual max $\bar{N}^{(tn)}$ and $\bar{D}^{(tn)}$ and min ($\underline{N}^{(tn)}$ and $\underline{D}^{(tn)}$) values of numerator and denominator for all levels respectively.

Step 4: For all levels and all objectives, set the fuzzy goals and aspiration levels $\bar{N}^{(tn)}, \underline{N}^{(tn)}$ (or) $\bar{D}^{(tn)}, \underline{D}^{(tn)}$ for each and every numerator and denominator parts.

Step 5: Compute the highest and lowest value of numerator part of all objectives respectively as defined in (7.8).

Step 6: Set corresponding values of decision variables as aspiration levels for membership functions of decision vector $X^{(t)}, \forall t = 1,2,\cdots,T-1$.

Step 7: Find the membership functions of numerators $\mu(N^{(tn)})$, denominators $\mu(D^{(tn)})$ and decision variables $\mu(X^{(t)})$.

Step 8: For the proposed U4DMLMOMIFTP, formulate the fuzzy goal programming model as in equation (7.10).

Step 9: Using generalized reduced gradient technique (LINGO-18.0 Suite Solver) , solve the fuzzy goal programming model to have the compromise solution of proposed U4DMLMOMIFTP.

9. NUMERICAL EXAMPLE

Problem A numerical example is considered to illustrate the performance of the proposed U4DMLMOMIFTP model. In the problem we have taken, we have considered 2 levels, 2 origins, 2 destinations, 2 possible routes, 2 modes of transports and 2 types of products. In this problem, we aim to minimize the ratio of the actual and standard transportation cost and the ratio of the actual and standard time at each level. The data presented below is based on normal variable $N(e, \mu)$.
The table 9.1 contains the data for the availabilites of the origins.

Table 9.1: Availabilities in origins

i	g	$\tilde{a}_{ig}$
1	1	(100, 9)
	2	(260, 7)
2	1	(125, 5)
	2	(350, 10)

The data for the demands in various destinations are presented in table 9.2.

Table 9.2: Demands in destinations

j	g	$\tilde{b}_{jg}$
1	1	(50, 8)
	2	(25, 9)
2	1	(100, 10)
	2	(200, 20)

Table 9.3 contains data for the capacities of transports.

Table 9.4 contains the data for ratio between the actual and standard transportation cost , ratio between the actual and standard transportation time of level 1 and level 2.

Table 9.3: Capacity of Conveyances

k	r	$\tilde{e}_{kr}$
1	1	(180, 10)
	2	(150, 15)
2	1	(190, 20)
	2	(280, 30)

Table 9.4: Ratios of actual and standard unit transportation cost, actual and standard transportation time

i	j	k	r	Level 1 Ratio of actual unit transportation cost and standard unit transportation cost		Level 1 Ratio of actual transportation and standard transportation time		Level 2 Ratio of actual unit transportation cost and standard unit transportation cost		Level 2 Ratio of actual transportation and standard transportation time	
				$\frac{\tilde{C}^{11}_{ijkr1}}{\tilde{D}^{11}_{ijkr1}}$	$\frac{\tilde{C}^{11}_{ijkr2}}{\tilde{D}^{11}_{ijkr2}}$	$\frac{\tilde{A}^{12}_{ijkr1}}{\tilde{S}^{12}_{ijkr1}}$	$\frac{\tilde{A}^{12}_{ijkr2}}{\tilde{S}^{12}_{ijkr2}}$	$\frac{\tilde{C}^{21}_{ijkr1}}{\tilde{D}^{21}_{ijkr1}}$	$\frac{\tilde{C}^{21}_{ijkr2}}{\tilde{D}^{21}_{ijkr2}}$	$\frac{\tilde{A}^{22}_{ijkr1}}{\tilde{S}^{22}_{ijkr1}}$	$\frac{\tilde{A}^{22}_{ijkr2}}{\tilde{S}^{22}_{ijkr2}}$
1	1	1	1	$\frac{(18,0.5)}{(28,1)}$	$\frac{(22,1)}{(22,0.5)}$	$\frac{(23,1)}{(23,2)}$	$\frac{(22,0.5)}{(22,1)}$	$\frac{(28,2)}{(18,0.5)}$	$\frac{(12,7)}{(27,5)}$	$\frac{(29,2)}{(21,3)}$	$\frac{(29,2)}{(16,7)}$
			2	$\frac{(18,0.5)}{(28,1)}$	$\frac{(28,0.5)}{(28,1)}$	$\frac{(23,2)}{(23,3)}$	$\frac{(22,4)}{(23,5)}$	$\frac{(28,4)}{(18,5)}$	$\frac{(12,7)}{(24,3)}$	$\frac{(29,3)}{(36,4)}$	$\frac{(19,1)}{(26,3)}$
		2	1	$\frac{(19,1)}{(19,2)}$	$\frac{(29,3)}{(29,4)}$	$\frac{(41,0.5)}{(41,0.1)}$	$\frac{(32,4)}{(32,3)}$	$\frac{(12,7)}{(19,2)}$	$\frac{(21,0.5)}{(24,1)}$	$\frac{(19,0.5)}{(21,0.8)}$	$\frac{(19,3)}{(36,8)}$
			2	$\frac{(19,3)}{(19,4)}$	$\frac{(19,1)}{(19,0.5)}$	$\frac{(32,1)}{(22,0.5)}$	$\frac{(32,1)}{(32,2)}$	$\frac{(18,2)}{(19,1)}$	$\frac{(18,4)}{(19,2)}$	$\frac{(39,3)}{(36,2)}$	$\frac{(49,0.2)}{(16,4)}$
	2	1	1	$\frac{(17,4)}{(7,4)}$	$\frac{(29,3)}{(29,2)}$	$\frac{(43,5)}{(43,3)}$	$\frac{(32,1)}{(32,0.5)}$	$\frac{(18,2)}{(18,3)}$	$\frac{(21,4)}{(37,7)}$	$\frac{(39,0.5)}{(22,3)}$	$\frac{(59,2)}{(38,1)}$
			2	$\frac{(29,2)}{(29,1)}$	$\frac{(29,3)}{(29,2)}$	$\frac{(32,1)}{(42,7)}$	$\frac{(32,2)}{(32,3)}$	$\frac{(16,4)}{(37,5)}$	$\frac{(27,2)}{(37,8)}$	$\frac{(49,7)}{(22,3)}$	$\frac{(49,3)}{(22,5)}$
		2	1	$\frac{(19,2)}{(19,0.5)}$	$\frac{(24,2)}{(20,3)}$	$\frac{(29,3)}{(49,8)}$	$\frac{(24,1)}{(24,0.5)}$	$\frac{(16,2)}{(18,1)}$	$\frac{(21,0.5)}{(21,1)}$	$\frac{(49,3)}{(42,2)}$	$\frac{(21,1)}{(31,0.5)}$
			2	$\frac{(29,3)}{(39,7)}$	$\frac{(24,5)}{(24,3)}$	$\frac{(29,3)}{(19,2)}$	$\frac{(29,2)}{(19,4)}$	$\frac{(16,2)}{(28,4)}$	$\frac{(21,3)}{(23,2)}$	$\frac{(16,3)}{(41,5)}$	$\frac{(21,3)}{(41,5)}$
2	1	1	1	$\frac{(29,2)}{(29,3)}$	$\frac{(28,0.5)}{(38,1)}$	$\frac{(49,3)}{(39,2)}$	$\frac{(25,2)}{(25,10)}$	$\frac{(21,3)}{(18,5)}$	$\frac{(21,4)}{(28,7)}$	$\frac{(21,4)}{(22,6)}$	$\frac{(21,3)}{(41,2)}$
			2	$\frac{(29,0.7)}{(29,0.5)}$	$\frac{(29,3)}{(29,2)}$	$\frac{(25,2)}{(25,1)}$	$\frac{(25,1)}{(25,2)}$	$\frac{(18,2)}{(18,1)}$	$\frac{(18,0.5)}{(18,1)}$	$\frac{(36,2)}{(41,3)}$	$\frac{(36,2)}{(21,3)}$
		2	1	$\frac{(21,1)}{(21,2)}$	$\frac{(24,0.5)}{(24,2)}$	$\frac{(49,0.5)}{(59,2)}$	$\frac{(28,3)}{(38,4)}$	$\frac{(18,4)}{(18,5)}$	$\frac{(22,3)}{(25,7)}$	$\frac{(26,2)}{(17,3)}$	$\frac{(41,5)}{(23,2)}$
			2	$\frac{(24,3)}{(24,2)}$	$\frac{(24,0.5)}{(24,0.8)}$	$\frac{(28,3)}{(38,2)}$	$\frac{(28,2)}{(28,3)}$	$\frac{(28,3)}{(25,2)}$	$\frac{(22,4)}{(25,7)}$	$\frac{(36,2)}{(17,5)}$	$\frac{(36,3)}{(17,2)}$
	2	1	1	$\frac{(28,3)}{(28,2)}$	$\frac{(25,0.8)}{(25,0.7)}$	$\frac{(28,1)}{(28,2)}$	$\frac{(25,1)}{(25,1)}$	$\frac{(28,2)}{(18,3)}$	$\frac{(22,0.4)}{(10,1)}$	$\frac{(36,0.5)}{(37,2)}$	$\frac{(22,4)}{(43,0.1)}$
			2	$\frac{(28,4)}{(28,3)}$	$\frac{(23,1)}{(33,5)}$	$\frac{(28,2)}{(28,2)}$	$\frac{(28,3)}{(28,4)}$	$\frac{(28,0.5)}{(28,1)}$	$\frac{(22,7)}{(19,0.6)}$	$\frac{(48,3)}{(43,2)}$	$\frac{(22,1)}{(43,2)}$
		2	1	$\frac{(28,1)}{(28,2)}$	$\frac{(25,0.5)}{(25,0.3)}$	$\frac{(8,2)}{(38,0.5)}$	$\frac{(25,2)}{(25,1)}$	$\frac{(22,3)}{(38,0.1)}$	$\frac{(22,2)}{(38,1)}$	$\frac{(22,5)}{(17,3)}$	$\frac{(22,3)}{(73,0.5)}$
			2	$\frac{(28,1)}{(28,3)}$	$\frac{(28,0.5)}{(28,0.7)}$	$\frac{(25,0.5)}{(25,2)}$	$\frac{(25,0.7)}{(25,0.9)}$	$\frac{(23,1)}{(38,0.8)}$	$\frac{(23,2)}{(18,5)}$	$\frac{(31,4)}{(43,2)}$	$\frac{(41,5)}{(73,2)}$

For the formulation of the problem and getting the compromise solution, we may follow the following steps:

Step 1: The decision making model is formulated for U4DMLMOMIFTP for above data as of (4.1).

Step 2: We convert the above U4DMLMOMIFTP model into deterministic model by making use of expected value method on uncertainity theory as (5.2).

Step 3: Calculate the individual max $(\bar{N}^{(tn)}, \bar{D}^{(tn)})$ and min $(\underline{N}^{(tn)}, \underline{D}^{(tn)})$ for all levels under the given constraints. The optimal values of each numerator and denominator have been presented in table (9.5).

Table 9.5: Minimum and maximum values of the numerator and denominator of all objectives

Levels	Objectives		Max	Min
	$Z^{(11)}$	Numerator	$\bar{N}^{(11)} = 23020$	$\underline{N}^{(11)} = 7875$
		Denominator	$\bar{D}^{(11)} = 25900$	$\underline{D}^{(11)} = 6415$
	$Z^{(12)}$	Numerator	$\bar{N}^{(12)} = 27565$	$\underline{N}^{(12)} = 7410$
1		Denominator	$\bar{D}^{(12)} = 26465$	$\underline{D}^{(12)} = 7600$
2	$Z^{(21)}$	Numerator	$\bar{N}^{(21)} = 19760$	$\underline{N}^{(21)} = 7000$
		Denominator	$\bar{D}^{(21)} = 27615$	$\underline{D}^{(21)} = 5310$
	$Z^{(22)}$	Numerator	$\bar{N}^{(22)} = 37855$	$\underline{N}^{(22)} = 7325$
		Denominator	$\bar{D}^{(22)} = 43485$	$\underline{D}^{(22)} = 7900$

Step 4: The aspiration levels and fuzzy goals has been set for each numerator and denominator.

Step 5: The objectives' numerator parts' highest and lowest values are 27565 and 7410 and the corresponding highest and lowest value for the decision variables are 110 and 0.

Step 6: Respective values of decision variables are set a aspiration levels for each decision vectors' membership functions as shown in (7.8).

Step 7: Membership functions of numerators, denominators and decision vectors of all levels are found as of (7.7).

Step 8: Formulate the fuzzy goal programming model for U4DMLMOMIFTP as given below

$$\begin{cases} \min \delta = \sum_{t=1}^{2}\sum_{n=1}^{2} d_{N^+}^{(tn)} + \sum_{t=1}^{2}\sum_{n=1}^{2} d_{D^+}^{(tn)} + \sum_{t=1}^{2} d_+^{(t)} \\ 7875 + 15145 d_{N^-}^{(11)} - \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{r=1}^{2}\sum_{g=1}^{2} C_{ijkrg}^{(11)} x_{ijkrg} \geq 0 \\ 7410 + 20155 d_{N^-}^{(12)} - \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{r=1}^{2}\sum_{g=1}^{2} A_{ijkrg}^{(12)} x_{ijkrg} \geq 0 \\ 7000 + 12760 d_{N^-}^{(21)} - \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{r=1}^{2}\sum_{g=1}^{2} C_{ijkrg}^{(21)} x_{ijkrg} \geq 0 \\ 7325 + 30530 d_{N^-}^{(22)} - \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{r=1}^{2}\sum_{g=1}^{2} A_{ijkrg}^{(22)} x_{ijkrg} \geq 0 \\ -25900 + 19485 d_{D^-}^{(11)} + \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{r=1}^{2}\sum_{g=1}^{2} D_{ijkrg}^{(11)} x_{ijkrg} \geq 0 \\ -29465 + 21865 d_{D^-}^{(12)} + \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{r=1}^{2}\sum_{g=1}^{2} D_{ijkrg}^{(12)} x_{ijkrg} \geq 0 \\ -27615 + 22305 d_{D^-}^{(21)} + \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{r=1}^{2}\sum_{g=1}^{2} S_{ijkrg}^{(21)} x_{ijkrg} \geq 0 \\ -43485 + 35585 d_{D^-}^{(22)} + \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{r=1}^{2}\sum_{g=1}^{2} S_{ijkrg}^{(22)} x_{ijkrg} \geq 0 \\ -1 * (x_{11111} + x_{11121} + x_{11211} + x_{11221} + x_{12111} + x_{12121} + x_{12211} + x_{12221} + x_{21111} \\ + x_{21121} + x_{21211} + x_{21221} + x_{22111} + x_{22121} + x_{22211} + x_{22221}) - d_-^1 * 110 \geq 0 \\ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrg} - E(\tilde{a}_{ig}) \leq 0,\ i = 1,2,\cdots,I, g = 1,2,\cdots,G \\ E(\tilde{b}_{jg}) - \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrg} \leq 0,\ j = 1,2,\cdots,J, g = 1,2,\cdots,G \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{g=1}^{G} x_{ijkrg} - E(\tilde{e}_k) \leq 0,\ k = 1,2,\cdots,K \\ x_{ijkrg} \geq 0, \forall i,j,k,r,g \end{cases} \tag{9.1}$$

as said in equation(7.10)

Step 9: The problem obtained in step 8 has been solved using the reduced gradient technique to obtain the compromise solution of the proposed U4DMLMOMIFTP problem.

The compromise solution for the proposed U4DMLMOMIFTP is
$\delta = 0$, $x_{11111} = 50$, $x_{12111} = 50$, $x_{21112} = 25$, $x_{22111} = 50$, $x_{22222} = 200$,
$d_{N+}^{(11)} = d_{N+}^{(12)} = d_{N+}^{(21)} = d_{N+}^{(22)} = d_{D+}^{(11)} = d_{D+}^{(12)} = d_{D+}^{(21)} = d_{D+}^{(22)} = 0$ with the corresponding objective values $Z^{(11)} = 0.9742, Z^{(12)} = 1, Z^{(21)} = 1.2607, Z^{(22)} = 0.709$.
Repeat the above said steps from 1 to 9 for chance constraint method for the proposed U4DMLMOMIFTP to obtain the compromise solution of the model. The solution has been obtained by considering predetermined confidence level as
$\alpha_{11} = \alpha_{12} = \alpha_{21} = \alpha_{22} = \gamma_{11} = \gamma_{12} = \gamma_{21} = \gamma_{22} = \beta_1 = \beta_2 = \beta_3 = 0.9$.
The compromise solution has been given in the table 9.6.

Table 9.6: Compromise solution of the objectives from chance constraint method

Variable Values	Objective values
$x_{11111} = 59.69, x_{12121} = 9.99,$ $x_{12221} = 19.4, x_{21112} = 25.46,$ $x_{21122} = 10.448, x_{22111} = 82.72,$ $x_{22222} = 224.24$	$Min\, Z^{(11)} = 0.915,$ $Min\, Z^{(12)} = 0.933,$ $Min\, Z^{(21)} = 1.1467,$ $Min\, Z^{(22)} = 0.72,$

The decision makers can obtain the optimal solutions flexibly, as per their desired conditions, by using the chance constrained method. Considering diverse set of values for various parameters in the proposed model, will benefit the decision making under uncertain environment.

10. CONCLUSIONS AND FUTURE RESEARCH WORK

In this work, a four dimensional multi-level multi objective multi item fractional transportation problem under uncertain environment has been investigated. Four diemensional multi level multi objective multi item fractional transportation problem (4DMLMOMIFTP) to handle real life suitations has been studied for the first time ever in this paper. We have solved the transportation problem taken using fractional programming instead of linear programming as the ratio to be optimized in fractional programming often describes the efficiency of the system considered. The problem has been converted into equivalent deterministic problem using expected value method and chance constrained method. The compromise solution has then been obtained for hierarchical decision making system suitably to avoid dead locks in situations involving conflicting objectives, using goal programming method. An illustrative numerical example's solution using above method has been presented to showcase the validity of the method presented. The above work can be extended further in future by including time constraint considering vehicle speed in the above problem.

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