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Thermoelastic Strain Field Due to a Cylindrical Inclusion in an Elastic Half-Space

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Abstract

Present work deals with the investigation of thermoelastic strain field outside and inside a cylindrical inclusion embedded in an elastic half-space. The study is carried out in the context of steady-state uncoupled linear theory of thermoelasticity. The closed form analytical solution of the problem is obtained by using thermoelastic displacement potential functions. The cylindrical inclusion as well as the surrounding material is characterized by the same elastic constants. The thermoelastic deformation field is generated due to differences in the coefficients of linear thermal expansion between a cylindrical inclusion and the surrounding material. The surface of the half-space is considered to be free of tractions. The variation of thermoelastic strain components with depth is also shown graphically.

Keywords: Strain field, Elastic half-space, Potential functions, Cylindrical inclusion, Uncoupled thermoelasticity

INTRODUCTION

Thermoelasticity is the extension of the theory of elasticity to include thermal effects. The theory of thermoelasticity is concerned with the interaction between thermal field and elastic bodies. The detailed studies on the topics of thermoelasticity have been documented in the several classical texts such as Parkus (1976), Nowinski (1978), Truesdell (1984), Nowacki (1986), Boley and Weiner (1997), Boresi *et al.* (2011) etc. The study of thermoelasticity has begun with Duhamel (1837) and Neumann (1885), who postulated the equations of linear thermoelasticity for isotropic bodies. Goodier (1937) solved the static problem of uncoupled thermoelasticity by employing the method of superposition using displacement potential functions. In the recent years, increased attention has been drawn to three-dimensional problems especially to those involving cylindrical geometry. Such problems find applications in the rapidly developing field of nuclear technology. Barber (1983) determined the closed form expressions for the distortion of a traction free cylinder subjected to arbitrary non-uniform axisymmetric heat flux using potential function approach. For solid rods having space dependent energy generation, Arpaci (1984) calculated the steady axially symmetric three-dimensional thermoelastic stresses in terms of the Goodier and the Love-Galerkin or the Boussinesq- Papkovich potentials.

Krichevets *et al.* (1993) proposed a method for determining the thermally stressed state of an elastic half-space containing a defect in the form of a cylindrical inclusion. The displacements and stresses were obtained in the cylindrical coordinates using Hankel transform technique. The problem of axial displacement of a rigid circular disk-shaped inclusion embedded at the interface between two bonded dissimilar isotropic elastic solids was examined by Selvadurai (2000). Li *et al.* (2005) studied the problem of a 2D plane strain circular inclusion in a finite representative volume element. The exact and closed form solution of the elastic fields was determined due to a circular inclusion embedded in a two-dimensional isotropic finite circular domain that was subjected to prescribed displacement boundary conditions. Xiao *et al.* (2009) considered the problem of interaction between a circular elastic inclusion and an interfacial crack under a heat dipole. The closed form solutions of the temperature and stress fields were established by using the analytical extension technique, the generalized Liouville theorem, and the Muskhelishvili boundary value theory.

Kedar *et al.* (2012) gave the explicit expressions for the thermal stresses in a semi-infinite solid circular cylinder subjected to an arbitrary initial heat supply on the lower surface. The results were obtained in a series form in terms of Bessel's functions. Itou (2014) derived basic equations for the thermoelastic plane stress conditions and thermoelastic plane strain conditions. Two problems have been solved using thermoelastic displacement potential functions- (i) axisymmetric thermal stresses for a hollow thin disk, (ii) thermal stresses for an infinite thin plate with a circular hole under uniform heat flow. Using thermoelastic displacement potential functions, the displacement field due to a cylindrical inclusion in a thermoelastic half-space was obtained by Singh and Renu (2017). Also the stress field for the same problem was determined by Muwal and Singh (2019).

In the present paper, the plane strain thermoelastic deformation problem of an elastic half-space due to a cylindrical inclusion is studied in the context of steady-state uncoupled linear theory of thermoelasticity. Firstly the strain field is obtained for an infinite region using thermoelastic displacement potential functions. Then following the procedure opted by Davies (2003), the strain field in the semi-infinite region is derived using the corresponding field for an infinite region. The closed form expressions for this field are derived for both exterior and interior points of a cylindrical inclusion. The surface of the half-space is considered to be free of tractions.

THEORY

It is well known that temperature changes in an unrestrained elastic solid generally produce changes in strains and stresses. Thus a general strain field emanates from both mechanical and thermal effects. In the linear theory of thermoelasticity, the total strain can be written as the sum of mechanical and thermal strains (Goodier 1937, Nowinski 1978 and Sadd 2005):

$$e_{ij} = e_{ij}^{(M)} + e_{ij}^{(T)} \tag{1}$$

in which the thermal strain takes the form $e_{ij}^{(T)} = \alpha T \delta_{ij}$ for an isotropic material. Here α is the coefficient of linear thermal expansion, T is the temperature change/difference, δ_{ij} is the Kronecker delta and suffices *i*, *j* range from 1 to 3. Let e_{ij} and τ_{ij} are the components of strain and stress tensor respectively, then the usual stress-strain relations (generalized Hooke's law) including the thermal effects takes the following form:

$$\tau_{ij} = 2\mu e_{ij} + \frac{2\mu\nu}{1-2\nu}\delta_{ij} e_{kk} - \frac{2\mu(1+\nu)}{1-2\nu}\alpha\delta_{ij} T \tag{2}$$

or the above relation can be written equivalently as

$$\tau_{ij} = 2\mu e_{ij} + \lambda\delta_{ij} e_{kk} - \beta\,\delta_{ij} T \tag{3}$$

with

$$\beta = 2\mu\left(\frac{1+\nu}{1-2\nu}\right)\alpha = (3\lambda + 2\mu)\,\alpha\,,$$

$$\lambda = \frac{2\mu\nu}{1-2\nu}\ \text{, the Lame's constant}$$

and μ and ν are the shear modulus and Poisson's ratio respectively.

Substituting the relations (2) together with total strain-displacement relations in the equilibrium equations (with no body forces), the Navier's equation can be written as

$$\nabla^2\mathbf{u} + \frac{1}{1-2\nu}\nabla(\nabla.\mathbf{u}) = \frac{2(1+\nu)}{1-2\nu}\alpha\,\nabla T \tag{4}$$

where $\mathbf{u}$ is the displacement vector.

The uncoupled heat conduction equation for the steady state temperature field (T) with Q as heat supply and λ_0 as the thermal conductivity can be written as

$$\nabla^2 T = -\frac{Q}{\lambda_0} \tag{5}$$

The solution of inhomogeneous equation (4) can be expressed as

$$\mathbf{u} = \mathbf{u}^c + \mathbf{u}^p \tag{6}$$

where $\mathbf{u}^c$ is the complementary function satisfying the corresponding homogeneous equation of (4) and $\mathbf{u}^p$ represents the particular solution of the displacement field generated by the temperature field (T).

According to Goodier's method (Timoshenko and Goodier, 1951), the displacement $\mathbf{u}^{(\infty)}(\mathbf{r})$ for an infinite solid is given by

$$\mathbf{u}^{(\infty)} = \nabla\phi, \tag{7}$$

where the potential function ϕ satisfies the following partial differential equation

$$\nabla^2\phi = \left(\frac{1+\nu}{1-\nu}\right)\alpha\, T(\mathbf{r}) \tag{8}$$

Then the function ϕ is obtained as

$$\phi(\mathbf{r}) = \frac{-1}{4\pi}\left(\frac{1+\nu}{1-\nu}\right)\alpha \int \frac{T(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\, d^3(\mathbf{r}') \tag{9}$$

where $|\mathbf{r}-\mathbf{r}'| = |(x,y,z)-(\xi,\eta,\varsigma)| = [(x-\xi)^2+(y-\eta)^2+(z-\varsigma)^2]^{1/2}$ is the distance between the points (x,y,z) and (ξ,η,ς).

Let u_x , u_y and u_z denotes the components of displacement vector **u** along *x*, y and *z* axes respectively. Then the displacement and strain components in Cartesian coordinates $(x,\ y,\ z)$ are expressed in terms of potential function ϕ as (Barber, 2002 and Sadd, 2005):

$$u_x = \frac{\partial\phi}{\partial x} \quad ; \quad u_y = \frac{\partial\phi}{\partial y} \quad ; \quad u_z = \frac{\partial\phi}{\partial z} \tag{10}$$

$$\begin{aligned} &e_{xx} = \frac{\partial u_x}{\partial x} \quad ; \quad e_{yy} = \frac{\partial u_y}{\partial y} \quad ; \quad e_{zz} = \frac{\partial u_z}{\partial z} \quad ; \\ &e_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right) \quad ; \quad e_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right) \quad ; \quad e_{zx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right) \end{aligned} \tag{11}$$

FORMULATION OF THE PROBLEM

We consider the plane strain problem of a cylindrical inclusion in the upper half-space ($x \ge 0$) having different coefficient of linear thermal expansion to that of the half-space but both has same elastic constants (as Min-zhong and Ke-fu, 1991). Due to this difference in the coefficients of linear thermal expansion between a cylindrical inclusion and its surrounding material, say η_0, the thermoelastic deformation field is generated. The axis of the cylinder is taken parallel to the surface of half-space and the center of axis is located on the line $x = h$ and $y = 0$. The radius of the cylinder is '*a*', where $h \ge a$ as shown in Fig. 1. The surface $x = 0$ is taken as the traction free surface, i.e. at $x = 0$, $\tau_{xx} = \tau_{yx} = 0$. Then according to Min-zhong and Ke-fu (1991), the thermoelastic potential function ϕ satisfies the following Poisson's equations, when the temperature of the semi-infinite region increases by T_0 is:

$$\nabla^2\phi = \frac{1+\nu}{1-\nu}\,\alpha T = \frac{1+\nu}{1-\nu}\,\eta_0 T_0 \quad \text{for} \quad R_1 \le a \tag{12}$$

and

$$\nabla^2\phi = 0 \quad \text{for} \quad R_1 > a \tag{13}$$

where $R_1^2 = (x-h)^2 + y^2$ is the distance of the point (x, y) from $(h, 0)$.

The function ϕ for this problem is taken as (Min-zhong and Ke-fu, 1991):

$$\phi = \frac{1}{4}KR_1^2 \quad \text{for} \quad R_1 \le a \tag{14}$$

and

$$\phi = \frac{1}{2}Ka^2\left[\ln\frac{R_1}{a} + \frac{1}{2}\right] \quad \text{for} \quad R_1 > a \tag{15}$$

where $K = \dfrac{1+\nu}{1-\nu}\eta_0 T_0$ (16)

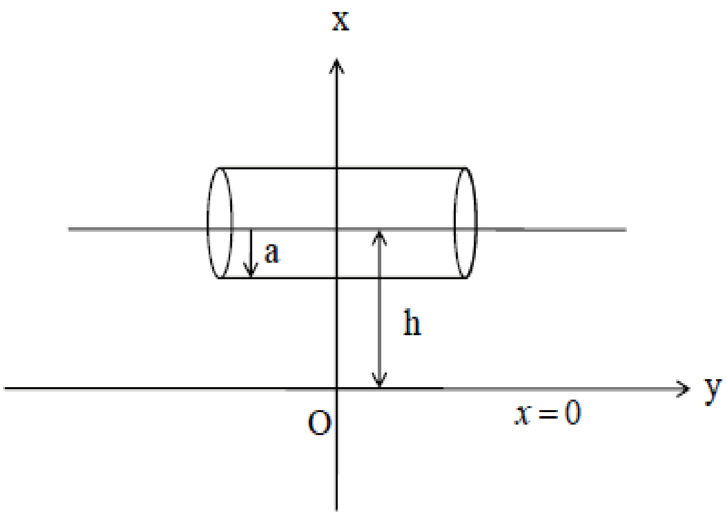

Figure 1: A cylindrical inclusion in a half-space

The displacement field in an infinite region and that at the image point for exterior points $(R_1 > a)$ of the cylindrical inclusion is obtained from $\mathbf{u}^{(\infty)} = \nabla\phi$ on using equation (15),

$$\mathbf{u}^{(\infty)} = \frac{1}{2}Ka^2\frac{(x-h,\ y)}{R_1^2} \tag{17}$$

$$\overline{\mathbf{u}}^{(\infty)} = \frac{1}{2}Ka^2\frac{(-x-h,\ y)}{R_2^2} \tag{18}$$

where $(-h, 0)$ is the image of point $(h, 0)$ and $R_2^2 = (x+h)^2 + y^2$ is the distance of the point (x, y) from $(-h, 0)$.

GOVERNING EQUATIONS FOR STRAIN FIELD

The strain components for the plane strain problem in the *xy* - plane are reduced in the following form:

$$e_{xx} = \frac{\partial u_x}{\partial x} \quad ; \quad e_{yy} = \frac{\partial u_y}{\partial y} \quad ; \quad e_{yx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$
$$e_{zz} = 0 \quad ; \quad e_{yz} = 0 \quad ; \quad e_{zx} = 0 \tag{19}$$

Now for the plane strain problem in *xy* - plane, the non-zero components of strain within the semi-infinite region $x \geq 0$ with the traction free surface at $x = 0$ are reduced in terms of strain components for an infinite region, in the form as given below (using Davies, 2003):

$$e_{xx} = e_{xx}^{(\infty)} - (1-4\nu)\,\overline{e}_{xx}^{(\infty)} + 2x\,\frac{\partial}{\partial x}\overline{e}_{xx}^{(\infty)}$$
$$e_{yy} = e_{yy}^{(\infty)} + (3-4\nu)\overline{e}_{yy}^{(\infty)} + 2x\,\frac{\partial}{\partial x}\overline{e}_{yy}^{(\infty)} \tag{20}$$
$$e_{yx} = e_{yx}^{(\infty)} - \overline{e}_{yx}^{(\infty)} - 2x\,\frac{\partial}{\partial x}\overline{e}_{yx}^{(\infty)}$$

Also cubical dilatation ϑ is given as

$$\vartheta = \vartheta^{(\infty)} - 4(1-2\nu)\overline{e}_{xx}^{(\infty)} \tag{21}$$

STRAIN FIELD FOR EXTERIOR POINTS

On using equations (10) - (11) and (17) - (18), the strain fields in the infinite region and those at the image point for exterior points $(R_1 > a)$ of the cylindrical inclusion (where $\nabla^2\phi = 0$) are obtained as under:

$$e_{xx}^{(\infty)} = \frac{1}{2}Ka^2\left[\frac{1}{R_1^2} - \frac{2(x-h)^2}{R_1^4}\right] \tag{22}$$

$$e_{yy}^{(\infty)} = \frac{1}{2}Ka^2\left[\frac{1}{R_1^2} - \frac{2y^2}{R_1^4}\right] \tag{23}$$

$$e_{yx}^{(\infty)} = \frac{1}{2}Ka^2\,\frac{(-2)(x-h)\,y}{R_1^4} \tag{24}$$

$$\overline{e}_{xx}^{(\infty)} = \frac{1}{2}Ka^2\left[\frac{1}{R_2^2} - \frac{2(x+h)^2}{R_2^4}\right] \tag{25}$$

$$\bar{e}_{yy}^{(\infty)} = \frac{1}{2} K a^2 \left[\frac{1}{R_2^2} - \frac{2y^2}{R_2^4} \right] \tag{26}$$

$$\bar{e}_{yx}^{(\infty)} = \frac{1}{2} K a^2 \frac{2(x+h)y}{R_2^4} \tag{27}$$

Also cubical dilatation,

$$\vartheta^{(\infty)} = 0 \tag{28}$$

Substituting equations (22) - (27) into (20), the non-zero components of strain for exterior points $(R_1 > a)$ of the cylindrical inclusion in the thermoelastic half-space are expressed as

$$e_{xx} = \frac{1}{2} K a^2 \left[\frac{1}{R_1^2} - \frac{2(x-h)^2}{R_1^4} - \frac{1-4\nu}{R_2^2} - \frac{12x(x+h)}{R_2^4} + \frac{2(1-4\nu)(x+h)^2}{R_2^4} + \frac{16x(x+h)^3}{R_2^6} \right] \tag{29}$$

$$e_{yy} = \frac{1}{2} K a^2 \left[\frac{1}{R_1^2} + \frac{3-4\nu}{R_2^2} - \frac{4x(x+h)}{R_2^4} - 2y^2 \left\{ \frac{1}{R_1^4} + \frac{3-4\nu}{R_2^4} - \frac{8x(x+h)}{R_2^6} \right\} \right] \tag{30}$$

$$e_{yx} = -K a^2 y \left[\frac{x-h}{R_1^4} + \frac{3x+h}{R_2^4} - \frac{8x(x+h)^2}{R_2^6} \right] \tag{31}$$

Further on substituting equations (25) and (28) into equation (21), the cubical dilatation is expressed as

$$\vartheta = e_{xx} + e_{yy} = 4\,(1-2\nu)\,\frac{1}{2} K a^2 \left[\frac{1}{R_2^2} - \frac{2y^2}{R_2^4} \right] \tag{32}$$

RELATIONS BETWEEN STRAIN FIELDS OF THE EXTERIOR POINTS AND THE INTERIOR POINTS OF A CYLINDRICAL INCLUSION

For the interior points $(R_1 \le a)$ of the cylindrical inclusion,

$$\mathbf{u}_{\text{int.}} = \mathbf{u}_{ext.} + \frac{1}{2} K a^2 \mathbf{R}_1 \left(\frac{1}{a^2} - \frac{1}{R_1^2} \right) \tag{33}$$

Equation (33) is in a similar form for a cylindrical inclusion as in Mindlin and Cheng (1950) for the interior points $(R_1 \le a)$ of a spherical inclusion.

Using the equation (33), the relations between the non-zero components of strain for the exterior points $(R_1 > a)$ and the interior points $(R_1 \le a)$ of the cylindrical inclusion are given below:

$$e_{xx}^{\text{int.}} = e_{xx}^{ext.} + \frac{1}{2}Ka^2\left(\frac{1}{a^2} - \frac{1}{R_1^2} + \frac{2\,(x-h)^2}{R_1^4}\right)$$
$$e_{yy}^{\text{int.}} = e_{yy}^{ext.} + \frac{1}{2}Ka^2\left(\frac{1}{a^2} - \frac{1}{R_1^2} + \frac{2\,y^2}{R_1^4}\right) \tag{34}$$
$$e_{yx}^{\text{int.}} = e_{yx}^{ext.} + Ka^2\,\frac{(x-h)\,y}{R_1^4}$$

Also cubical dilatation is

$$\vartheta_{\text{int.}} = \vartheta_{ext.} + K \tag{35}$$

STRAIN FIELD FOR INTERIOR POINTS

Substituting equations (29) - (31) into (34), the non-zero strain components for the interior points $(R_1 \le a)$ of the cylindrical inclusion in the thermoelastic half-space are expressed as

$$e_{xx} = \frac{1}{2}Ka^2\left[\frac{1}{a^2} - \frac{1-4\nu}{R_2^2} + \frac{2(1-4\nu)\,(x+h)^2}{R_2^4} - \frac{12x(x+h)}{R_2^4} + \frac{16x(x+h)^3}{R_2^6}\right] \tag{36}$$

$$e_{yy} = \frac{1}{2}Ka^2\left[\frac{1}{a^2} + \frac{3-4\nu}{R_2^2} - \frac{4x(x+h)}{R_2^4} - 2y^2\left\{\frac{3-4\nu}{R_2^4} - \frac{8x\,(x+h)}{R_2^6}\right\}\right] \tag{37}$$

$$e_{yx} = -Ka^2 y\left[\frac{3x+h}{R_2^4} - \frac{8x\,(x+h)^2}{R_2^6}\right] \tag{38}$$

Further on substituting equation (32) into equation (35), the cubical dilatation is expressed as

$$\vartheta = \frac{1}{2}Ka^2\left[\frac{2}{a^2} + \frac{4(1-2\nu)}{R_2^2} - \frac{8\,(1-2\nu)\,y^2}{R_2^4}\right] \tag{39}$$

NUMERICAL RESULTS AND DISCUSSIONS

In this section, the graphical representations of the strain components at the point $(0, 0)$ and at the point $(h, 0)$ of a cylindrical inclusion in the thermoelastic half-space are obtained using MATLAB software programming. The numerical computations are carried out for the value of Poisson's ratio ν = 0.25. Figures 2 and 3 depict the variation of the strain component e_{xx} and e_{yy} respectively, at the point (0, 0) for exterior points of a cylindrical inclusion in the thermoelastic half-space. The strain component e_{xy} at the point (0, 0) for exterior points and at the point $(h, 0)$ for interior points takes the value zero. From figure 2, it is observed that the strain component e_{xx} is negative for all values of distance h/a. The absolute value of this strain decreases gradually from its peak value as the distance h/a increases and then approaches to zero for infinitely large values of distance h/a. From figure 3, it is noticed that the strain component e_{yy} start with its maximum value and then it decreases rapidly with increasing distance h/a and finally tends to zero at infinity. Figures 4 and 5 illustrate the variation of strain components e_{xx} and e_{yy} respectively, at the point $(h, 0)$ for interior points of a cylindrical inclusion. It can be seen from figure 4 that the strain component e_{xx} decreases gradually as the values of distance h/a increases and it approaches a finite value 0.625 for infinitely large values of distance h/a. From figure 5, it is observed that the strain component e_{yy} takes only a finite constant value 0.625 for all the values of distance h/a.

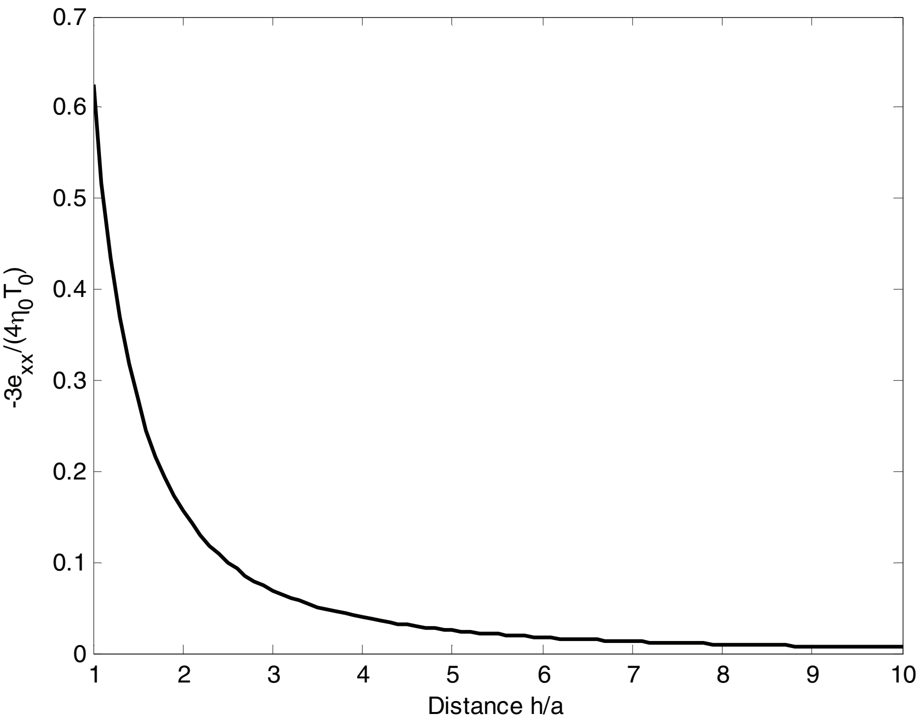

Figure 2 Variation of strain component e_{xx} at the point (0, 0) with distance h/a for exterior points of the cylindrical inclusion

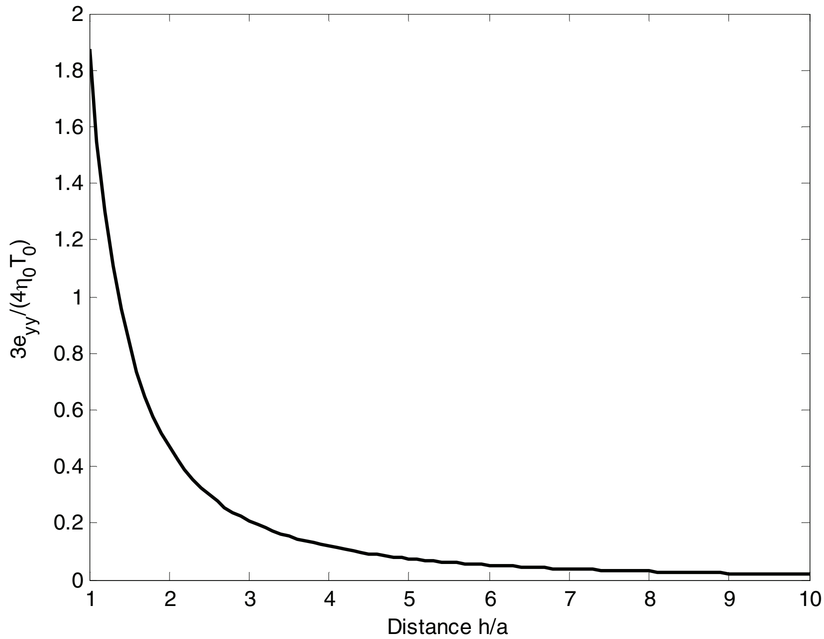

Figure 3 Variation of strain component e_{yy} at the point (0, 0) with distance h/a for exterior points of the cylindrical inclusion

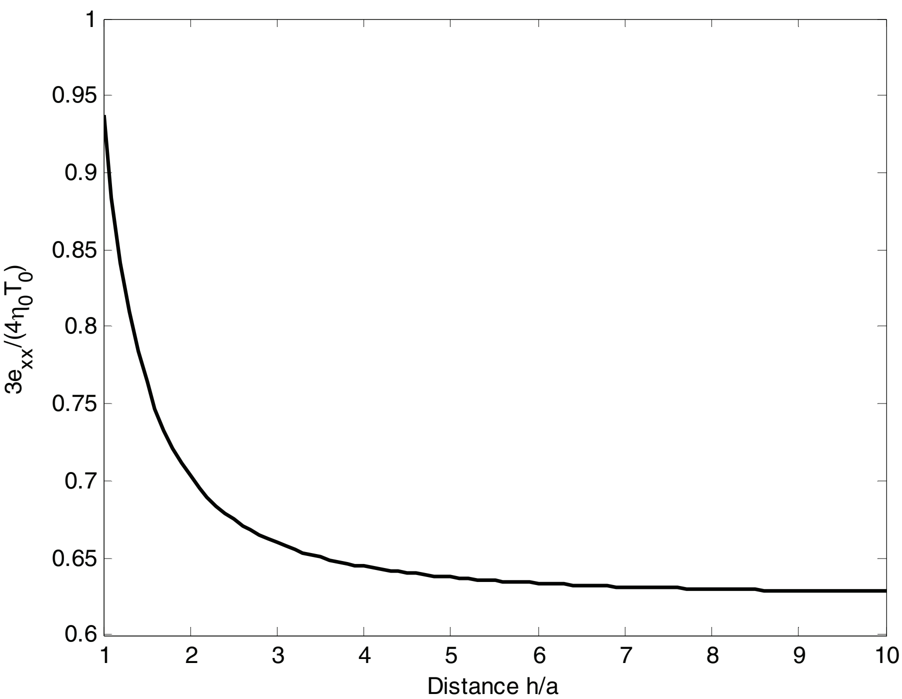

Figure 4 Variation of strain component e_{xx} at the point $(h, 0)$ with distance h/a for interior points of the cylindrical inclusion

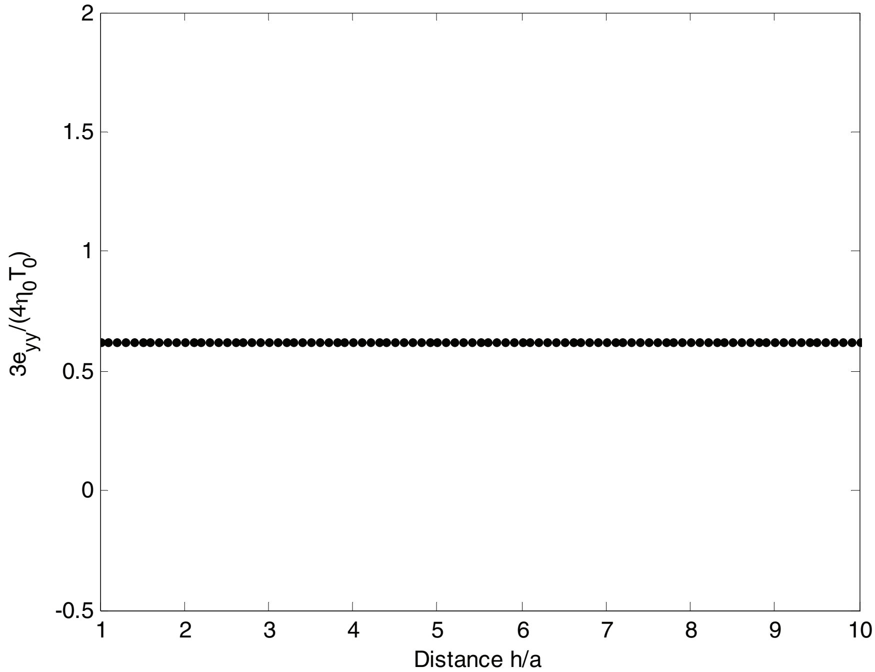

Figure 5 Variation of strain component e_{yy} at the point $(h, 0)$ with distance h/a for interior points of the cylindrical inclusion

CONCLUSIONS

The strains and stresses induced by the thermal mismatch of dissimilar media have been an important topic since the thermal stresses due to temperature differences become the main criterion to cause failure in modern electron devices. In this paper, the plane strain thermoelastic deformation problem of an elastic half-space due to a cylindrical inclusion has been studied by using thermoelastic displacement potential functions. A complete solution of thermoelastic strain fields for outside and inside a cylindrical inclusion in a half-space has been derived. The computed numerical results are also depicted graphically to study the strain fields induced in the considered medium.

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