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Modelling the Dynamics of Marital Interactions with Optimal Effort Plan

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Abstract

Marital relationship is one unique institution every adult wishes to experience. Due to lack of understanding of its underlying dynamics even the perfect fit couple gets frustrated about how to maintain their relationship beyond the feelings. This study uses the second law of thermodynamics and the Eisner-Strotz model by means of the Euler equation in calculus of variation to resolve the satisfaction effort problem of the dynamics. Lyapunov instability theorem was used to determine that marriage interaction is unstable. The result analyzed in phase portrait yield a saddle with the stable branch implying that daily efforts to keep relationships stable and alive should be admissible. A conceptual model hinged on sinusoidal graphs was constructed based on empirical and available data is adopted in predicting the various transition phases the interaction dynamics go through.

Keywords: Marriage, optimal effort, Eisner-Strotz model, Lyapunov stability, Satisfaction.

1. INTRODUCTION

Marriage is one of the pillars that hold the world. Different people in different parts of the world have their own views about it. However, it is an undeniable fact that marriage remains one of the world's most powerful institutions that ever existed. Beyond the wedding night, it has always been the desire of every couple to live happily forever [1, 2, 3].

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However, the question that remains unanswered has always been how much couples should invest in the relationship to keep it strong and everlasting. Marriage or elationships usually truncate or terminate with separation or divorce for a variety of reasons. Hence, the need to apply mathematical models to investigate the optimal investment pathway by which couples could invest to achieve the overall satisfaction they so desire [4, 5].

Mathematical models usually explains the dynamics of natural phenomena and these help understand conplex natural happenings in a more simplied way for better solutions to these problems [6, 7, 8, 9, 10, 11, 12]. Marriage is the foundation of all human societies. This clearly suggests that the quality of individual marriages and the families formed through them go a long way to influence every facet of society and the world at large. According to studies, the honeymoon stage that is; the first two years of the marriage was a major deciding period if the relationship was going to be stable or not [13].

Initial feelings may play a part in bringing couples together in marriage, but the feelings alone may not be sufficient to sustain the relationship for a life time as couples desire. This is because marital interaction seemingly follows the second law of thermodynamics and as such things would collapse if couples did nothing about it [14, 15].

Without any effort policy, the relationship will decay exponentially. In order to remedy the decay in the system namely marriage, the feeling of the couples was considered as a state variable and the intervention policy as a control variable which set the platform for which an optimal control model could be used to analyze the interaction [15].

1.1. Marriage Interaction Models

The main model employed in this paper is hinged on four core claims and three assumption

1.2. Decrease In Partners' Happiness After Marriage

Marriage is accompanied with higher levels of happiness as compared to singleness. According to Rey, the average happiness recounted after marriage decreases [15]. For many couples their satisfaction peaks in the honeymoon stage around the time of marriage and decreases over time [4, 13].

1.3. Ongoing Deterioration Process Results in Disruption

In a survey where men and women were interrogated, the main reason they gave for their divorce was the fact that they 'gradually grew further apart, lost a sense of closeness while staying together and emotionally detached until their loneliness was no longer bearable' [15]. This was largely due to unmet emotional needs, boredom with the relationship and high conflict which rank among the top of the list that contribute to marital stress.

1.4. Instability of Romantic Relationships

A primary way to assess the health of a marital relationship is to know how stable and satisfied couples are [3]. Many couples believe that marriage is the main factor for happiness and building a happy life [15]. Many times couples begin on a good note with the notion that their deep euphoric feelings for each other and their confidence will sail them through [4, 16].

These newly married couples who initially had a happy marriage see their relationship crumble right before their very eyes. They find themselves in an unending series of conflict. They begin to think to themselves that their marriage was a mistake in the first place [3].

1.5. Relationships Work If You Work It

There is a general belief that nothing rises by itself. Pertaining marriage, there is a universal assertion that you cannot get more out of it than you invest. In a broader sense, the 'work' has more to do with decisions and choices couples make [5].

Many couples find it difficult choosing to love their mate particularly in the face of pain or hurt. And this simply points out that choosing to love in relationships can be and is hard work [5] . Couples who invest in building the roots of their marriage through their deliberate choices and decisions enjoy the fruits they bear [17].

2. MARRIAGE ASSUMPTIONS

2.1. Second Law of Thermodynamics for Marital Relationships

The second law of thermodynamics says that energy spontaneously disperses from being localized to becoming spread out, if it is not hindered. Simply put everything in nature will decay unless some form of work or energy is applied to the system to prevent that decay. There exist something like second law of thermodynamics for marital relationships that things fall apart unless energy is supplied to keep the relationship alive and well [14].

2.2. Optimal Investment Path

In reality, marriage can be seen as a business arrangement[17]. Like any business, the business owner looks at how much returns he can get from the efforts and investment he pumps into it [17]. Assume the level of overall satisfaction in a marital relationship varies directly as the rate of effort put into it. Then it implies, the higher the effort the higher the level of affection.

The essence of optimal control in modelling is to determine the best control measure in combating a phenomena [18, 11, 19, 20, 21, 22].

2.3. Parity in Couple's Characteristics

Literature shows that couples often look out for partners who share mutual traits [23]. It has also been indicated that couples who have comparable sentiments are happy in their relationships [24]. Even though total similarities in personality may not guarantee overall satisfaction, similarities among couples are very crucial and cannot be overlooked [25]. In accordance to the specifications of the model, the assumption is made that couples share the comparable traits. According to Rey, this assumption stands as a rule rather than an exception [15].

3. METHODS

x	Happy feeling of couples
u	Effort variable
r	Rate at which feeling fade
s	Effort efficiency
m	Overall satisfaction
n	Effort term or policy
π	Intertemporal satisfaction
c	Cost of executing effort term or policy
ρ	Discount factor
$\prod$	Optimal effort policy

Table 1: Parameters and interpretations

Table 1 shows paramters used in the marriage modelling and their interpretations.

a	4.1
b	0.7
α	2.4
β	3.1
ρ	0.08

Table 2: Numerical values of constant

Table 2 shows the various assumed numericcal values of constants used in the dynamics of the marriage modelling.

3.1. The Decay Model

At $t = 0$ the shared feeling $x(0) = x_0$ is presumed to be very large [14, 13, 15]. If energy is supplied to the system, it could be kept from falling apart. This is written as:

$$\frac{dx}{dt} = -rx(t) + su(t) \tag{1}$$

3.2. The Satisfaction-Effort Model

The level of overall satisfaction in a marital relationship varies directly as the rate of effort put into it. Taking the proportionality constant $k = 1$ then
$m(t) \propto \frac{d}{dt}n(t)$
$m(t) = k\frac{d}{dt}n(t)$

$$m(t) = \frac{d}{dt}n(t) \tag{2}$$

4. METHOD OF SOLUTION

4.1. The Eisner-Strotz Model

Equation (2) is solved using the Eisner-Strotz model . The magnitude of the cost is directly proportional to rate at which couples increase the level of their effort in sustaining their relationship. If couples could choose an ideal effort$n(t)$ that will sustain and give them the satisfaction they need, then is the optimal effort $n'(t)$ couples could invest to achieve the overall satisfaction they desire. The objective of this effort model is to choose an effort pathway that maximizes the total present level of their satisfaction over time after all factors have been taken into account. Maximize:

$$\prod[n] = \int_0^{\infty} [\pi(n(t)) - c(n'(t))]e^{-pt}dt \tag{3}$$

subject to $n(0) = n_0$

4.2. The Quadratic Inter-temporal Satisfaction Cost

Assuming the choice for$\pi(n(t))$ and $c(n'(t))$ in (3) are both quadratic because every marital interaction has the basic nature of a quadratic curve (rise,stationary and fall) given as:

$$\pi = \alpha n - \beta n^2 \qquad \alpha, \beta > 0 \tag{4}$$

and

$$c = an'^2 + bn' \qquad a, b > 0 \tag{5}$$

then (3) becomes

$$\prod[n] = \int_0^{\infty} [\alpha n - \beta n^2 - an'^2 + bn']e^{-pt} dt \tag{6}$$

4.3. The Euler Equation

The problem arising in 6 is solved with the Euler equation in calculus of variation given as:

$F_n = \frac{d}{dt} F_{n'} \forall t \in [0,t]$

$$\frac{d}{dt} F_{n'} = \frac{\partial F_{y'}}{\partial t} + \frac{\partial F_{y'}}{\partial n} \frac{dy}{dt} + \frac{\partial F_{y'}}{\partial n'} \frac{dy'}{dt} - F_n + F_{tn} + F_{nn'} n'(t) + F_{n'n'} n''(t) = 0 \tag{7}$$

Let

$$F = \pi(n) - c(n') = (\alpha n - \beta n^2 - an'^2 - bn')e^{-pt} \tag{8}$$

4.4. The Eisner-Strotz Model In Terms of Laplace Transforms

Consider the Eisner-Strotz model given in (3) without the constrain as;

$\prod[n] = \int_0^{\infty} [\pi(n(t)) - c(n'(t))]e^{-pt} dt$

Where e^{-pt} is the kernel of the transformation and is invariant to the Laplace transform $\mathscr{L}[\pi(n(t)) - c(n'(t))]$ converges if the limit on the RHS of $\prod[n]$ give as

$= lim_{b\to\infty} \int_0^b [\pi(n(t)) - c(n'(t))]e^{-pt} dt$ exist and is finite.

$$\mathscr{L}[\pi(n(t)) - c(n'(t))] = lim_{b\to\infty} \int_0^b [\pi(n(t)) - c(n'(t))]e^{-pt} dt \tag{9}$$

Special Cases:

Case I: $n(t) = t^a$

The case where ideal effort $n(t)$ is a polynomial

It follows that;

$$n'(t) = at^{a-1} \text{ and } \pi(n(t)) = \pi(t^a)$$
$$c(n'(t)) = a(at^{a-1})$$

If $c(at^{a-1})$ is a linear functional then, $c(at^{a-1}) = ac(e^{at})$

(9) the becomes

$= \int_0^{\infty} [\pi(e^{at}) - ac(ae^{at})]$

$= \mathscr{L}[\pi(e^{at}) - ac(ae^{at})]$

$= \mathscr{L}[\pi(e^{at})] - a\mathscr{L}[c(e^{t^{a-1}})]$

Case II: $n(t) = e^{at}$

The case where ideal effort is $n(t)$ is exponential

It follows that,

$$n'(t) = ae^{at} \text{ and } \pi(n(t)) = \pi(e^{at})$$
$$c(n'(t) = c(ae^{at})$$

If $c(at^{a-1})$ is a linear functional then, $c(at^{a-1}) = ac(e^{at})$ then 9 the becomes
$= \int_0^{\infty}[\pi(e^{at}) - ac(ae^{at})]e^{-pt}dt$
$= \mathscr{L}[\pi(e^{at}) - ac(ae^{at})]$
$= \mathscr{L}[\pi(e^{at})] - a\mathscr{L}[c(e^{at})]$
Case III: The case where ideal effort is trigonometric
a) sine function: $n(t) = sinat$

$$n'(t) = acosat \text{ and } \pi(n(t)) = \pi(sinat)$$
$$c(n'(t)) = c(acosat)$$

If $c(acosat)$ is a linear function then, $c(acosat) = ac(cosat)$ 9 the becomes
$= \int_0^{\infty}[\pi(sinat) - ac(cosat)]e^{-pt}dt$
$= \mathscr{L}[\pi(sinat) - ac(cosat)]$
$= \mathscr{L}[\pi(e^{at})] - a\mathscr{L}[c(\cos at)]$
b) *cosine* function :$n(t) = \cos at$

$$n'(t) = sinat \text{ and } \pi(n(t)) = \pi(\cos at). \text{ then}$$
$$c(n'(t)) = c(-a\sin at)$$

If $c(-a\sin at)$ is a linear functional then,

$$c(-a\sin at) = \begin{cases} -ac(\sin at), & odd \\ ac(\sin at), & even \end{cases}$$

Let $b = -a$ then, $c(n'(t)) = c(b\sin at)$, then (9) becomes
$\int[\pi(\sin at) - bc(\sin at)]e^{-pt}dt$
$= \mathscr{L}[\pi(\cos at) - bc(\sin at)]$
$= \mathscr{L}[\pi(e^{at})] - b\mathscr{L}[c(\sin at)]$
Among all these 3 cases it is both the polynomial and sinusoidal that are realistic to be an effort plan. Naturally in any relationship or marriage interaction. The effort plan may increase, be stationary or decrease.

5. CONCEPTUAL MODEL

This model gives a new outlook of marriage interaction. It is built on three main facts based on various research findings and empirical data from different fields and disciplines about marital interactions. Mathematical models are real representation of a natural phenomenon [26, 27, 28, 29].

5.1. Balance between Positivity and Negativity in Marriage

The one phenomenon which is a clear predictor if a particular marital relationship would eventually end in divorce or otherwise is the triumph of negative over positive affect". The balance between positivity and negativity in interactions is crucial in predicting the state of any marriage at any given point [14, 25].

5.2. Seasons in Marriage

There are four seasons in marriage and this alternate sequence of spring (positivity) and winter (negativity) as well as summer (positivity) and fall (negativity) [16]. It is worthy of notice that marriage and all its associated interactions are always in a state of transition from one season to another chronologically or emotionally or both [25].

5.3. Stability in Marriage

The honeymooning period which lasts for an average period of two years, is a major phase which decides if a marital relationship will be stable or not [13]. Research shows that half of all the divorce happens in the first seven years of marriage with more than half of divorces occurring in a forty-year span [14]. It follows that marital interaction is not linear. We intuitively pattern this phenomenon after the sinusoidal function which is represented generally by:

$$y(t) = A\cos(Bt - C) + D \tag{10}$$

Where; A represents amplitude, The period is given by $T = 2\pi/B$, C represents the horizontal shift, D represents the vertical shift from midline

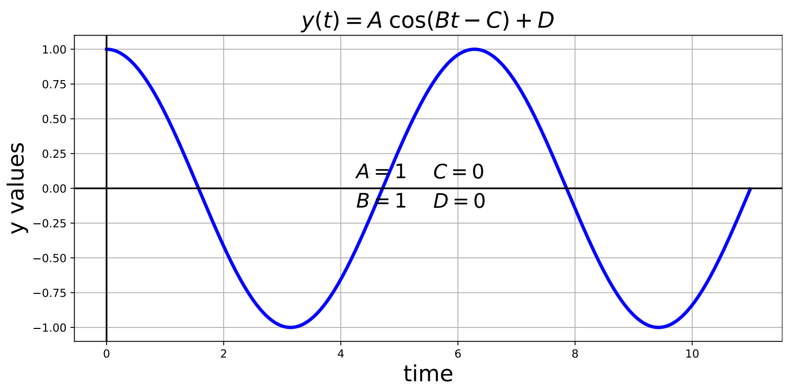

Figure 1: A General Cosine Graph With The Horizontal Shift Being Zero

From equation 10 in any marital relationships and in Figure 1, 'A' represents peak of positivity (affection and satisfaction) and or peak of negativity (misery). 'T, the period' represents the extent to which couples may go thru various seasons in marriage. 'C' characterizes couples who though legally married are not living as couples yet. An example is betrothal marriage. 'D' characterizes couples who get to interact martially before they are legally put together as married couples. An example is cohabitation. Also, let's assume:

1. That every marital relationship starts off at a time,$t = 0$, where couples have great affection for each other at a degree of satisfaction or misery if otherwise which peaks at a rise or otherwise a fall that ranges between $[1, -1]$. This implies that $|A| = 1$. Since the relationship starts at $t = 0$, it implies that $C = 0$ and $D = 0$.
2. That every marriage is balanced between positivity and negativity as the love that exists between couples will rise and fall from time to time.
3. That both partners are but one unit plied along the path carved by over time.

Special Case
Let the period $T = 2\pi/B = \frac{\pi}{7}$ define the extent to which couples may go through three various seasons in marriage. And let $C = D = 0$ and $|A| = 1$. Equation 10 which becomes;

$$y = \cos(\frac{\pi t}{7}) \tag{11}$$

models marital relationships in particular by assumption 1 where affection between couples is at its peak at $t = 0$.

6. ANALYSIS AND DISCUSSION

6.1. Analysis of Decay Model

If no energy (effort), $u(t)$, is put into the system, implying $u(t) = 0$, then $x(t)$, fades at a constant rate of r. That is:

$$\frac{dx}{dt} = rx \rightarrow \frac{dx}{x} = -rdt \tag{12}$$

$$\int \frac{dx}{x} = -\int rdt$$
$$\ln|x| = -rt + c$$
$$x(t) = x_0 e^{-rt}$$

The analysis of the decay Model shows that, the level of satisfaction $x(t)$ of couples over time decays exponentially and asymptotically decreases that is $x(t) \to 0,\ t \to \infty$ as shown in Figure 2. At time, $t = 0$, when couples get engaged to start their marital journey, affection for each other is high enough [4, 15]. That is, $x(0) = x_0$ but over the course of time there is a decline in happiness after marriage as also supported by claims and assumptions

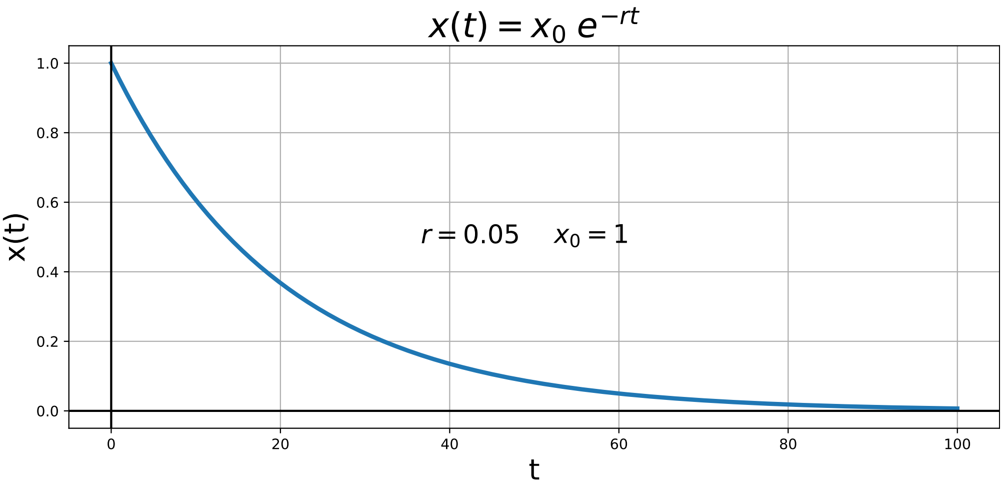

Figure 2: Graph of a typical marriage relationship with no effort input by couples.

6.2. Analysis of Satisfaction-Effort Model

6.2.1 Intertemporal Satisfaction

The choice of the effort every couple decides to put in their marriage yields a corresponding rate of satisfaction at that point in time which influences the possibilities of satisfaction they could enjoy available for other points in time. Referring to 4, the choice of the rate of satisfaction selected for this model informs us that its turning point is bounded above for $\alpha, \beta > 0$. Recalling, 4 was given as;

$$\pi = \alpha n - \beta n^2$$

Which implies

$$\frac{d\pi}{dn} = \alpha - 2\beta n = 0 \implies n = \frac{\alpha}{2\beta} \tag{13}$$

Putting 13 into 4 gives;

$$\pi(n = \tfrac{\alpha}{2\beta}) = \alpha(\tfrac{\alpha}{2\beta}) - \beta(\tfrac{\alpha}{2\beta})^2 = \tfrac{\alpha^2}{4\beta}$$

Equation (13) further confirms that there is a ceiling to the rate of satisfaction couples can enjoy in corresponding to their effort. That is, (13) reveals that the peak achievable inter-temporal satisfaction occurs when effort invested is $n = \alpha/2\beta$. In real life satisfaction beyond $\pi = \alpha^2/4\beta$. is usually termed unmet fantasies. As couples increase their effort the trajectory of satisfaction grows until it reaches this point where it cannot be increased anymore even if they continuously invested more effort. Thus satisfaction begins to wane as supported by claim. Figure 3 below shows a graph of 13.

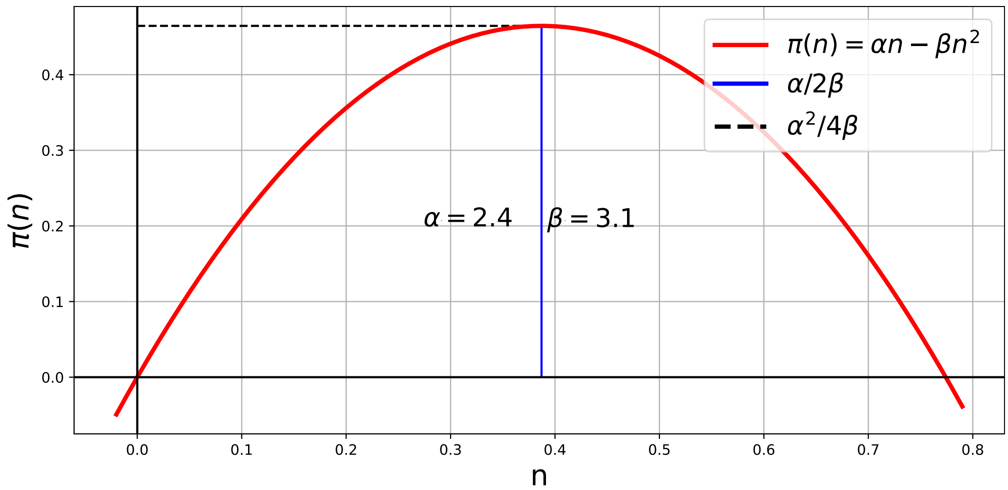

Figure 3: Graph of rate of Intertemporal Satisfaction

6.3. The Cost Associated with Increase in Effort

Couples begin their marital journey on the wings of emotional love, and they desiring that their marriages will be entirely different from everyone else's put in their best possible effort which is $n' > 0$. This explains why for 5 as shown in Figure 4 below we consider the curve only in the first quadrant. Every couple anticipates living happily ever after. And truly so, no one gets married with the intention of making their spouse miserable or becoming unhappy themselves [4]. The graph of 4 shows that as effort increases, its corresponding cost also increases quadractically. This implies that, with increased effort comes sacrifice. However, at a point couples may feel that how much they sacrifice by investing effort into the relationship is higher as compared to the rate of satisfaction they get out of it. They find it unbearable because of the stress it brings them. Thus over time, there is a higher tendency for them to rather grow further apart as supported by claims.

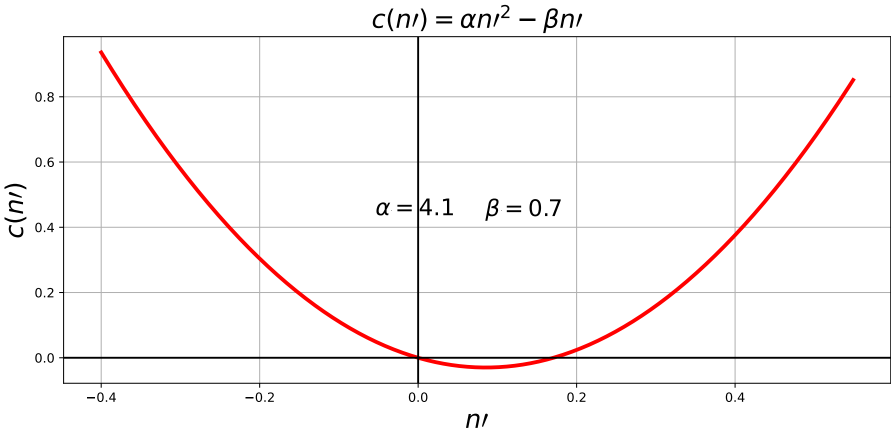

Figure 4: Graph of cost that comes with increasing effort

7. MODELING A SYSTEM OF DIFFERENTIAL EQUATION

Applying the explicit form of Euler equations in 7 to 6 the resulting partial derivatives with respect to $n, nn, n', nn', n'n', tn'$ are given below

$$F_n = (\alpha - 2\beta n)e^{-pt}, \, F_{nn} = -2\beta e^{-pt}$$

$$F_{n'} = -(2\alpha n' + b)e^{-pt}, F_{nn'=0} \tag{14}$$

$$F_{n'n'} = 2ae^{-pt}, F_{tn} = p(2\alpha n' + b)e^{-pt}$$

Substituting 14 into (7) results;

$$\begin{gathered} -(\alpha - 2\beta n)e^{-pt} - p(2\alpha n' + b)e^{-pt} + 0.n' + [2ae^{-pt}].n'' = 0 \\ \to \alpha - 2\beta n - p(2\alpha n' + b) + 2an'' = 0 \\ \alpha - pb - 2\beta n - 2\alpha pn' + 2an'' = 0 \end{gathered}$$

$$n'' - pn' - \frac{\beta}{a}n = \frac{pb - \alpha}{2a} \tag{15}$$

From 2 $m(t) = n'(t)$ it implies,

$$m'(t) = n''(t) \tag{16}$$

Putting (2) into (16) and (15) which gives;

$$m'(t) = pm(t) + \frac{\beta}{a}n(t) + \frac{bp - \alpha}{2a} \tag{17}$$

(2) and (17) give rise to two differential equations

$$m' = pm + \frac{\beta}{a}n + \frac{bp - \alpha}{2a} \quad (18)$$

$$n' = m \quad (19)$$

$$At \quad equilibrium, m' = 0 \quad and \quad n' = 0 \quad (20)$$

Putting (20) into (18) and (19) gives;

$$m = 0 \quad (21)$$

and

$$n = \frac{\alpha - bp}{2\beta} \quad (22)$$

The equilibrium point of the model is thus given as;

$$(0, \frac{\alpha - bp}{2\beta}) \quad (23)$$

Let $m' = f(m,n)$ and $n' = g(m,n)$
Then, the Jacobian Matrix of (f,g) is given as

$$J_{(f,g)} = \begin{bmatrix} \rho & \frac{\beta}{a} \\ 1 & 0 \end{bmatrix}$$

$$|J - \lambda I| = \begin{vmatrix} \rho - \lambda & \frac{\beta}{a} \\ 1 & 0 - \lambda \end{vmatrix} = 0$$

$$\lambda^2 - \rho\lambda - \frac{\beta}{a} = 0 \quad (24)$$

For $\rho, \beta > 0$ for $\lambda_{1,2} \in \lambda$ it implies that:
$\lambda_1 = \frac{1}{2}(\rho + \sqrt{\rho^2 + \frac{4\beta}{a}}) > 0$ and $\lambda_2 = \frac{1}{2}(\rho - \sqrt{\rho^2 + \frac{4\beta}{a}}) < 0$
Since $\lambda_1 > 0$ and $\lambda_2 < 0$ and $det(J) = -\frac{\beta}{a} < 0$. It implies that the qualitative dynamic behavior of the system $\mu = (f,g)$ is a saddle one and thus unstable.

8. OPTIMAL EFFORT APPROACHES PARTICULAR INTEGRAL

In mathematical modelling, optimal control is usually employed to determine the best control measure in achieving certain goals or objectives [30, 31, 20, 32] Since $\lambda_{1,2} \in \lambda$ is saddle it implies that the complementary solution of 15 is given by:

$$n_c = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \quad (25)$$

And the particular integral given by

$$n_p = \frac{\alpha - \rho b}{2\beta} \tag{26}$$

The general solution is thus given as:

$$n_t = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \frac{(\alpha - \rho b)}{2\beta} \tag{27}$$

With $n(0) = n_0$ and $t = 0$ it implies that (27) is

$$n_0 = c_1 + c_2 + \frac{(\alpha - \rho)}{2\beta} \tag{28}$$

As $t \to \infty, c_1 e^{\lambda_1 t} \to \pm\infty$ in 27 is not admissible and thus since $e^{\lambda_1 t} \neq 0$ implies $c_1 = 0$. Thus putting $c_1 = 0$ into (28) yields.

$$n(t) = (n_0 - \frac{a - \rho b}{2\beta})e^{\lambda_2 t} + \frac{\alpha - \rho b}{2\beta} \tag{29}$$

With $\lambda < 0$, it implies that

$$n(t) \to \frac{\alpha - \rho b}{2\beta} \tag{30}$$

In essence $\frac{(\alpha - \rho b)}{2\beta}$ appears to be the optimal effort couples should put into their marriage. This is seen as $n = \frac{\alpha}{2\beta}$ in 13 and $n(t) \to \frac{\alpha - \rho b}{2\beta}$ in 30 are compared. It seems the initial effort n_0 increases and decays exponentially and converges asymptotically towards n over the course of time as shown in Figure 5. Moreover, Figure 6 shows the rate of satisfaction and cost comparatively.

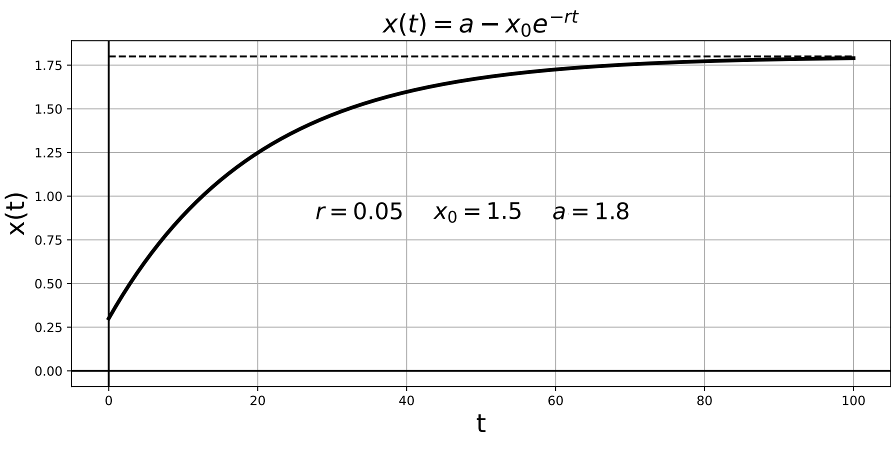

Figure 5: Graph of optimal effort required to make the relationship work

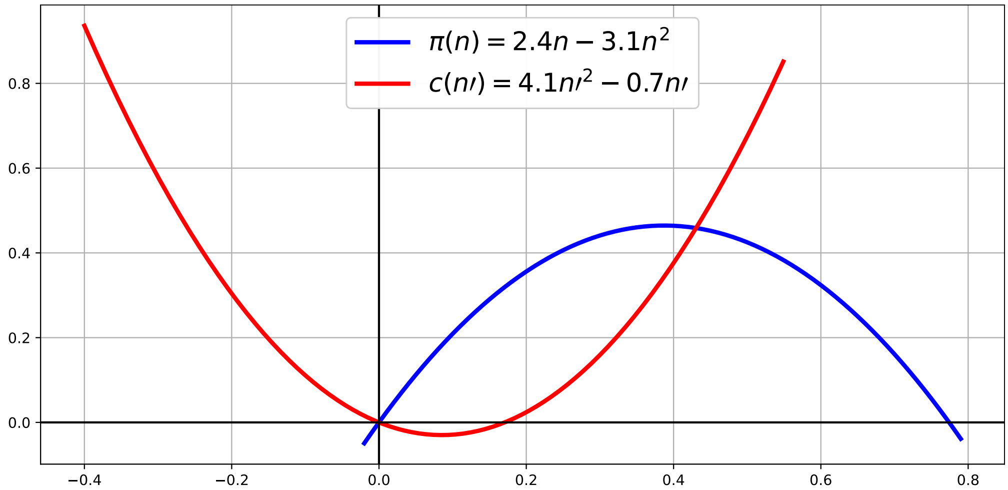

Figure 6: Graph of rate of satisfaction and cost when considered together

8.1. Lyapunov Instability Theorem

Let a function $V(\mu)$ be continuously differentiable in a neighborhood of the origin, where $V(\mu)$ is the Lyapunov function for an autonomous system $\mu = (m,n)$ [33, 34, 35]. Suppose that in a neighbourhood U of the zero solution $\mu = 0$ there is a continuously differentiable function $V(\mu)$ such that:

1. $V(0) = 0$

$$V(0) = \begin{bmatrix} \rho & \frac{\beta}{a} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

2. $\frac{dv}{dt} > 0.$

   Let $V(\mu) = V(m,n) = m^2 - \frac{\beta}{a}n^2$
   $\frac{dm}{dt} = \rho m \frac{\beta}{a} n$
   $\frac{dn}{dt} = m$
   Then $\frac{dV}{dt} = \frac{\partial V}{\partial m}\frac{dm}{dt} + \frac{\partial V}{\partial n}\frac{dn}{dt}$
   $= [2m.(\rho m + \frac{\beta}{a}n)] - [\frac{2\beta n}{a}.m]$
   $= 2\rho m^2 + \frac{2\beta mn}{a} - \frac{2\beta mn}{a} = 2\rho m^2$ Since $\rho > 0$ and $m^2 > 0$

3. If in the neighborhood U there are points at which $V(\mu)$>0 , then the zero solution $\mu = 0$ is unstable $\forall a, \beta, \rho > 0$.

8.2. The Separation Curve

Taking $m = 0$ and substituting in 17 we get
$0 = \rho m(t) + \frac{\beta}{a}n(t) + \frac{b\rho-\alpha}{2a}$
$\frac{\beta}{a}n(t) = \frac{\alpha-b\rho}{2a} - pm(t)$

$$n(t) = \frac{\alpha - b\rho}{2\beta} - \frac{a\rho}{\beta}m(t) \tag{31}$$

Which is the separation line with negative slope of $-\frac{a\rho}{\beta}$ and cutting the n-axis with coordinate $E(0, \frac{\alpha-b\rho}{2\beta})$ as show in figure 7.

8.3. Phase Portrait Diagram

The dynamical outline of Figure 7 is a saddle one. Along the stable branches, perturbations will expand as time passes thus making the system unstable [36]. The curves labeled $\prod_s$ and $\prod_\mu$ break up into stable and unstable branches respectively. Equilibrium occurs at E . Trajectories sitting in $R1$ and $R2$ require more effort thus makes it intolerable. All other streamlines will result ultimately in either excess exertion or even lowering of effort which results in failure to achieve satisfaction as couples may desire. The only way to reach the target level of effort at E is to get onto $\prod_s^+$ For any assumed initial point of satisfaction along m in between 0 and Z there is a corresponding level of effort, which places a couple on the stable branch, moving towards E as shown in Figure 7.

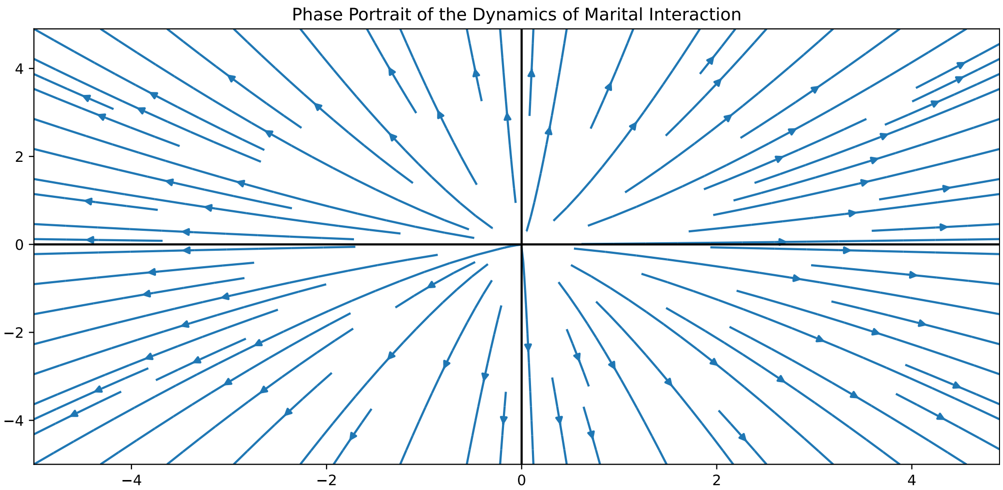

Figure 7: Phase Portrait of the Dynamics of Marital Interaction

9. ANALYSIS AND DISCUSSION OF CONCEPTUAL MODEL

The conceptual model gives general predictive information about what couples could expect in the course along their marital journey. From the Figure 8 below, in phases F, I,

and K couples could expect to have a level satisfaction as compared to phases G and H where they may be faced with misery. According to [37] couples who are stable use negativity and misery they face as an opportunity for bonding and deepening intimacy whereas unstable relationships disrupt.
Figure 8 and 9 predicts the specific time points couples could expect these changes and transition.

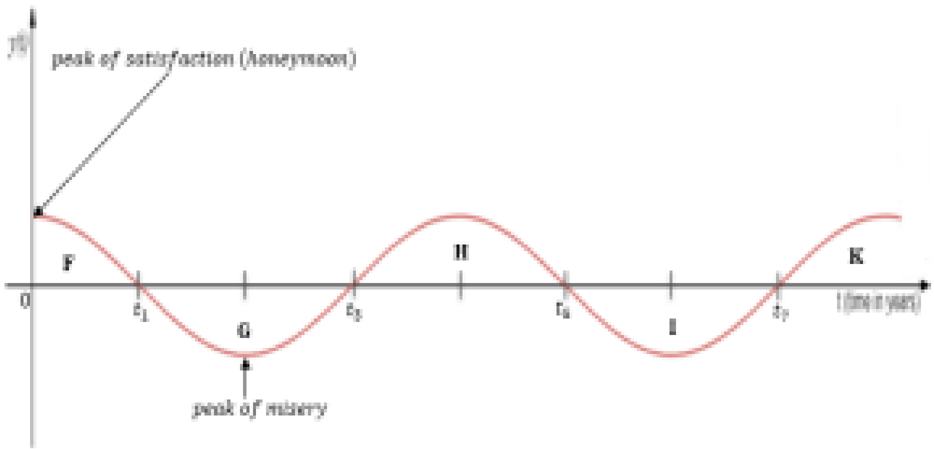

Figure 8: Graph of the overview of the marital pathway and likely pattern

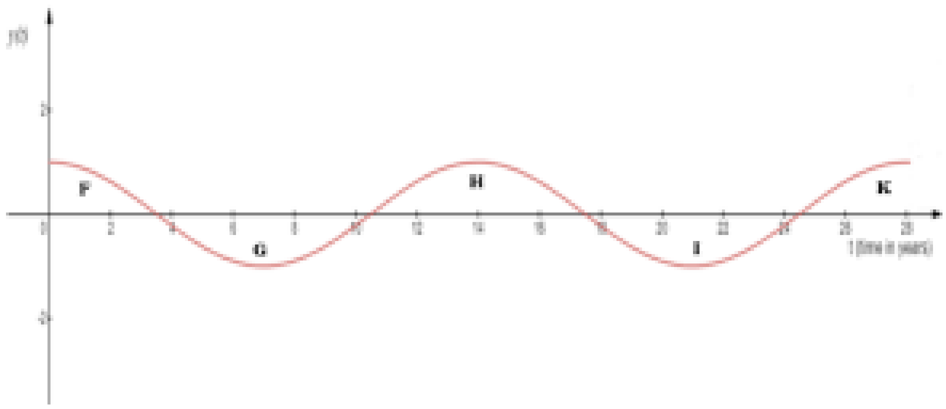

Figure 9: Graph of marital pathway with specific time points.

From assumption three of the conceptual model, both partners are considered as a unit (particle) plied along the path carved by . Assume the particle plied along the path carved by moves at a constant velocity, then it could go through all the phases provided the velocity of the unit (particle) is greater the resistance opposed with at the points of inflection and transition through phases G and I from phases F and K. Likewise, if the velocity is less than then, the resistance opposed with at the points of inflection and transition there is a higher probability the particle will assume a pendulum motion as it would be trapped in the trough phases G or I as shown below in Figure 10.

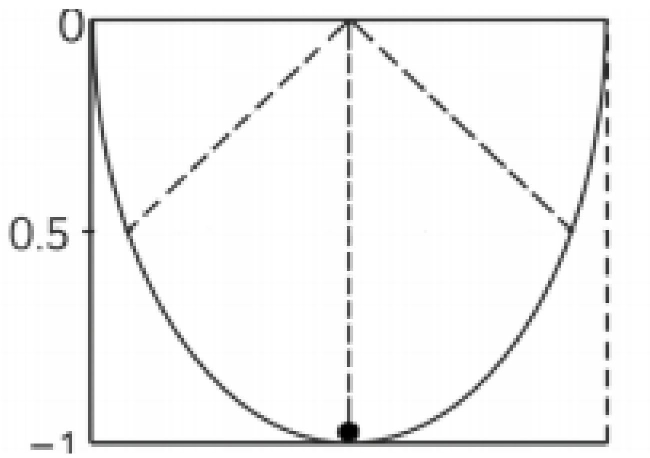

Figure 10: A section of the phase of misery.

As a couple (like a unit particle) plied along the graph pathway get trapped in the trough.

10. CONCLUSION

A degree of decay of satisfaction which obeys the second law of thermodynamics sets in at the commencement of the interaction and continues in the course of time. Also, because couples desire to stay married as long as forever, daily efforts to keep it stable and alive should be admissible. The phase portrait dynamic analysis of the optimal effort that would yield desired overall satisfaction reveals that marital interactions are generally unstable. On the contrary, couples whose effort places them on the stable branch leading to the equilibrium could achieve the satisfaction they desire. Paradoxically, intense effort even on the stable branch leads to a decline in satisfaction. The overall satisfaction depends of the intertemporal decisions and its resultant experiences (intertemporal satisfaction). With the big picture of how couples would want their relationship to turn out eventually, if they endeavor to make every day count through their best possible consistent little efforts they can keep the interaction along the stable trajectory. Secondly, the theoretical model of the sinusoidal graph used to predict what couples could expect (either satisfaction or misery) at what time of their marriage shows that marital interactions are generally in transition from a phase of satisfaction to that of misery. The analysis could advice couples on how to maintain their long term relationship. Lasting relationships are possible only if the effort is admissible and the optimal effort is continuously watched over to stay on the target dynamics. In view of this, couples should invest a little to just enough effort they can keep up with consistently over time. Little efforts like consistent acceptance of good influence and keeping the slightest hint of contempt far off is encouraged. Couples should always use the high moments (period of inter temporal satisfaction) in their relationship to strengthen what incites conflicts and pain in order to be able to go through the down moments without becoming destabilized.

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Data Availability Statement
The data supporting this Marriage model analysis is from are taken from published articles and are cited in this paper. Some of the parameter values are assumed. These published articles are also cited at relevant places within the text as references.

Conflict of interest
Authors declare that there are no conflict of interest regarding the publication of this paper.

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