Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 18, Number 2 (2023), pp. 181-188

https://dx.doi.org/10.37622/AFM/18.2.2023.181-188

Fuzzy α – cut Evaluation of Priority Bi- Tandem Queue Network

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Abstract

This paper presents an effective fuzzy solution of complex priority bi-serial queue network. This study uses Zadeh's **α** -cut approach to transform fuzzy variables into crisp values in order to identify the membership solution of the system's performance measures. To establish the effectiveness of the proposed model Triangular Fuzzy numbers and their arithmetic operations are taken in consideration. The effectiveness of performance indicators for the purposed model are demonstrated numerically.

Keywords: Bi-tandem, fuzzy, α – cut, Priority, Queue Performance

INTRODUCTION

In literature, most of the work was devoted to fuzzy logics because fuzzy set theory determines uncertainty. Basically, in standard queuing model there is assumption of Poisson arrival process and exponential service time, but in reality, arrival rate is not probabilistic, it is uncertain. Zadeh [1] was the first who solved the problems on vague data by the use of problems regarding decision-making. The investigators like Li and Lee [2], Kao [3], Chen [4,5], Singh & Pardeep [6], Robert and Ritha [7], Seema et al [8], Sharma Sameer [9], Meenu Mittal [10-12], Ritha and Menon [13], W. Ritha, Josephine S. [14], Saini A, Gupta D and Tripathi A. K. [15,16] derive the membership function value of the system by α – cut fuzzy approach. Both Kao [3] and Chen [4] suggest parametric linear programming strategy for the fuzzy membership solutions. Selvakumaria K., Revathi S. [17] introduced new ranking technique to transform the triangular and trapezoidal fuzzy numbers into precise numbers with pre-emption priority of unequal service rates.

In this paper, fuzzy logics are used to define the membership functions of bi-tandem queue network with priority. Here, fuzzy analysis of the study Saini A, Gupta D and Tripathi A. K. [18] will be done. For this, Sameer Sharma [9] methodology will be used.

Definitions

Fuzzy Set

If the result of the membership function for a function $\tilde{G}$ defined on the universal set X is either $\mu_{\tilde{G}}(\text{x}) = 1; x \in X \; or \; \mu_{\tilde{G}}(\text{x}) = 0; x \notin X$, where x is modal value of $\tilde{G}$ the function is said to be fuzzy.

α – cut approach

A fuzzy set $\tilde{K}: X \to [0,1]$, for any $\alpha \in [0,1]$, the $\alpha - cut \;\; \alpha_K = \{x \in X, \mu_{\tilde{K}}(\text{X}) \geq \alpha\}$ is a crisp set.

Strong $\alpha - cuts$: $\alpha +_K = \{x \in X, \mu_{\tilde{K}}(\text{X}) > \alpha\}$ whenever α *lies between* 0 & 1

Weak $\alpha - cuts$: $\alpha_K = \{x \in X, \mu_{\tilde{K}}(\text{X}) \geq \alpha\}$ *whenever* α *lies between* 0 & 1

As all members of a fuzzy set must be greater than or equal to α, it is important to view fuzzy sets as crisp sets.

Fuzzy Triangular Number

A number $\tilde{K} = (k_1, a, k_2)$ is a fuzzy triangular number and membership function $\mu_{\tilde{K}}(x) \; of \; \tilde{K}$ is defined as

$$\mu_{\tilde{K}}(\text{X}) = \begin{cases} \dfrac{x - k_1}{a - k_1}, & k_1 \leq x \leq a \\ \dfrac{k_2 - x}{k_2 - a}, & a \leq x \leq k_2 \\ 0, otherwise & \end{cases}$$

Notations

$\widetilde{m_{ij}}$	fuzzy low and high priority arriving customers, i = 1,2 & j = L, H
$\widetilde{\lambda_{ij}}$	fuzzy Priority input rate, i = 1,2 & j = L, H
$\widetilde{\lambda_i}$	fuzzy general arrivals, i = 1,2
$\widetilde{\mu_{ij}}$	fuzzy cost of service for low and high priority visitors, i = 1,2 & j = L, H
$\widetilde{\mu_i}$	fuzzy service rate, i = 3,4,5
$\tilde{L}$	fuzzy queue length of the system
$\widetilde{\alpha_{ij}}$	fuzzy possibilities from i'th server to j'th server

Model's Description

The proposed model consists three servers C_1, C_2, C_3. The server C_1& C_2 contains two biserial subsystems C_{11}, C_{12} & C_{21}, C_{22} respectively. The server C_3 commonly connected in series to both of the server C_1 & C_2 for completion of phase type service either of the server C_1 or C_2. The exponential service time with service rates $\widetilde{\mu_{1L}}$, $\widetilde{\mu_{1H}}, \widetilde{\mu_{2L}}, \widetilde{\mu_{2H}}, \widetilde{\mu_3}, \widetilde{\mu_4}$ is assumed at $C_{ij};\ i,j = 1,2$ and $\widetilde{\mu_5}$ at C_3. Two type of Customers arrive in the system with arrival rates $\widetilde{\lambda_{ij}}$ $where\ i = 1,2$ & $j = L, H$ at C_{11} & C_{12}. After the completion of service at C_{11}, Low or high priority customer move to the server C_{12} with possibilities $\widetilde{\alpha_{12}}$ or C_3 with $\widetilde{\alpha_{15}}$. Those who arrive at C_{12} move to C_{11} or C_3 with moving possibilities $\widetilde{\alpha_{21}}$ or $\widetilde{\alpha_{25}}$ respectively. Customers arrive with arrivals $\widetilde{\lambda_j}$ (j =1,2) at C_{2j} will go to C_{22} or C_3 with possibility $\widetilde{\alpha_{34}}+\widetilde{\alpha_{35}}$ =1 or go to the server C_{21} or C_3 with possible condition $\widetilde{\alpha_{43}} + \widetilde{\alpha_{45}} = 1$ respectively.

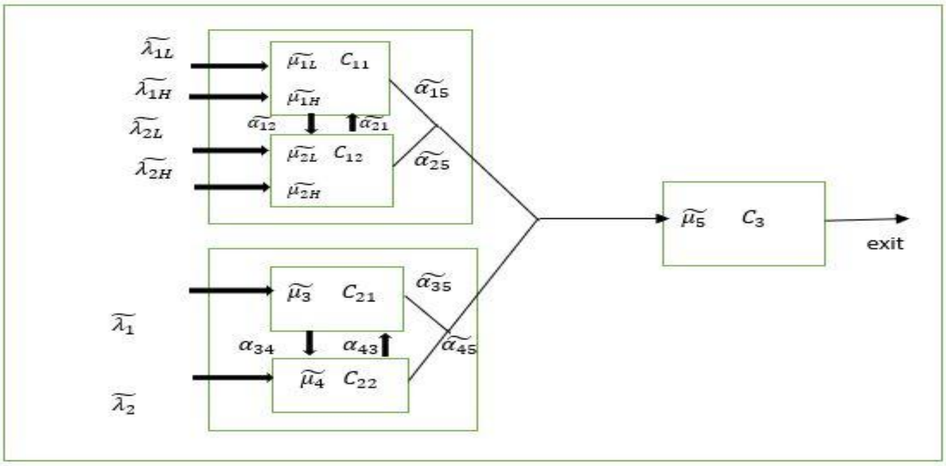

Figure 1. Bi-tandem Priority Queue Model

As from the results obtained by Saini A, Gupta D and Tripathi A.K. [18], the Queue characteristics of the system in stochastic environment are as

$$\gamma_1 = \frac{\lambda_{1H}+\lambda_{2H}\alpha_{21}}{\mu_{1H}(1-\alpha_{12}\alpha_{21})} < 1$$

$$\gamma_2 = \frac{\lambda_{2H}+\lambda_{1H}\alpha_{12}}{\mu_{2H}(1-\alpha_{12}\alpha_{21})} < 1$$

$$\gamma_3 = \frac{\lambda_1+\lambda_2\alpha_{43}}{\mu_3(1-\alpha_{34}\alpha_{43})} < 1$$

$$\gamma_4 = \frac{\lambda_2+\lambda_1\alpha_{34}}{\mu_4(1-\alpha_{34}\alpha_{43})} < 1$$

$$\gamma_5 = \left(\frac{\alpha_{35}(\lambda_1+\lambda_2\alpha_{43})+\alpha_{45}(\lambda_2+\lambda_1\alpha_{34})}{\mu_5(1-\alpha_{34}\alpha_{43})} + \frac{\alpha_{15}\,[(\lambda_{1H}+\lambda_{2H}\alpha_{21})+(\lambda_{1L}+\lambda_{2L}\alpha_{21})]+\alpha_{25}\,[(\lambda_{2H}+\lambda_{1H}\alpha_{12})+(\lambda_{2L}+\lambda_{1L}\alpha_{12})]}{\mu_5(1-\alpha_{12}\alpha_{21})}\right) < 1$$

$$\gamma_6 = \frac{\mu_{1L}(\lambda_{1H}+\lambda_{2H}\alpha_{21})+\mu_{1H}(\lambda_{1L}+\lambda_{2L}\alpha_{21})}{\mu_{1L}\mu_{1H}(1-\alpha_{12}\alpha_{21})} < 1$$

$$\gamma_7 = \frac{\mu_{2L}(\lambda_{2H}+\lambda_{1H}\alpha_{12})+\mu_{2H}(\lambda_{2L}+\lambda_{1L}\alpha_{12})}{\mu_{2L}\mu_{2H}(1-\alpha_{12}\alpha_{21})} < 1$$

a) Expected Length of Queue

$$L = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7$$

Where $L_7 = \frac{\gamma_7}{(1-\gamma_7)}$, $L_1 = \frac{\gamma_1}{(1-\gamma_1)}$, $L_6 = \frac{\gamma_6}{(1-\gamma_6)}$, $L_2 = \frac{\gamma_2}{(1-\gamma_2)}$,

$L_3 = \frac{\gamma_3}{(1-\gamma_3)}$, $L_4 = \frac{\gamma_4}{(1-\gamma_4)}$, $L_5 = \frac{\gamma_5}{(1-\gamma_5)}$

b) Average waiting time

$E = \frac{L}{\lambda}$, $\lambda = \lambda_{1L} + \lambda_{1H} + \lambda_{2L} + \lambda_{2H} + \lambda_1 + \lambda_2$

Solution Methodology

Let $\widetilde{\lambda_{ij}} = (\lambda_{ij}^1, \lambda_{ij}^2, \lambda_{ij}^3), \widetilde{\mu_{ij}} = (\mu_{ij}^1, \mu_{ij}^2, \mu_{ij}^3); \lambda_{ij}^1 < \lambda_{ij}^2 < \lambda_{ij}^3$ & $\mu_{ij}^1 < \mu_{ij}^2 < \mu_{ij}^3$ be triangular fuzzy numbers. The membership function of $\widetilde{\lambda_{ij}}$ & $\widetilde{\mu_{ij}}$ are defined as

$$\mu_{\widetilde{\lambda_{ij}}} = \begin{cases} 0, & \lambda_{ij} < \lambda_{ij}^1 \\ \dfrac{\lambda_{ij} - \lambda_{ij}^1}{\lambda_{ij}^2 - \lambda_{ij}^1}, & \lambda_{ij}^1 \le \lambda_{ij} < \lambda_{ij}^2 \\ \dfrac{\lambda_{ij}^3 - \lambda_{ij}}{\lambda_{ij}^3 - \lambda_{ij}^2}, & \lambda_{ij}^2 \le \lambda_{ij} < \lambda_{ij}^3 \\ 0, & \lambda_{ij} \ge \lambda_{ij}^3 \end{cases}$$

$$\mu_{\widetilde{\mu_{ij}}} = \begin{cases} 0, & \mu_{ij} < \mu_{ij}^1 \\ \dfrac{\mu_{ij} - \mu_{ij}^1}{\mu_{ij}^2 - \mu_{ij}^1}, & \mu_{ij}^1 \le \mu_{ij} < \mu_{ij}^2 \\ \dfrac{\mu_{ij}^3 - \mu_{ij}}{\mu_{ij}^3 - \mu_{ij}^2}, & \mu_{ij}^2 \le \mu_{ij} < \mu_{ij}^3 \\ 0, & \mu_{ij} \ge \mu_{ij}^3 \end{cases}$$

Membership function of $\tilde{L} = (s_1, s_2, s_3)$ is

$$\mu_{\tilde{L}} = \begin{cases} 0, & L < s_1 \\ \dfrac{L - s_1}{s_2 - s_1}, & s_1 \le L < s_2 \\ \dfrac{s_3 - L}{s_3 - s_2}, & s_2 \le L < s_3 \\ 0, & L \ge s_3 \end{cases}$$

The performance of the system characterized by membership function and maintain the fuzziness of system's input data.

Using α – cut methodology as used by Sharma Sameer et al 2015, the following utilization factors are obtained in fuzzy

$$\widetilde{\gamma_1}=\left(\frac{\lambda_{1H}^1+\lambda_{2H}^1\alpha_{21}^1}{\mu_{1H}^3(1-\alpha_{12}^3\alpha_{21}^3)},\frac{\lambda_{1H}^2+\lambda_{2H}^2\alpha_{21}^2}{\mu_{1H}^2(1-\alpha_{12}^2\alpha_{21}^2)},\frac{\lambda_{1H}^3+\lambda_{2H}^3\alpha_{21}^3}{\mu_{1H}^1(1-\alpha_{12}^1\alpha_{21}^1)}\right)$$

$$\widetilde{\gamma_2}=\left(\frac{\lambda_{2H}^1+\lambda_{1H}^1\alpha_{12}^1}{\mu_{2H}^3(1-\alpha_{12}^3\alpha_{21}^3)},\frac{\lambda_{2H}^2+\lambda_{1H}^2\alpha_{12}^2}{\mu_{2H}^2(1-\alpha_{12}^2\alpha_{21}^2)},\frac{\lambda_{2H}^3+\lambda_{1H}^3\alpha_{12}^3}{\mu_{2H}^1(1-\alpha_{12}^1\alpha_{21}^1)}\right)$$

$$\widetilde{\gamma_3}=\left(\frac{\lambda_1^1+\lambda_2^1\alpha_{43}^1}{\mu_3^3(1-\alpha_{34}^3\alpha_{43}^3)},\frac{\lambda_1^2+\lambda_2^2\alpha_{43}^2}{\mu_3^2(1-\alpha_{34}^2\alpha_{43}^2)},\frac{\lambda_1^3+\lambda_2^3\alpha_{43}^3}{\mu_3^1(1-\alpha_{34}^1\alpha_{43}^1)}\right)$$

$$\widetilde{\gamma_4}=\left(\frac{\lambda_2^1+\lambda_1^1\alpha_{34}^1}{\mu_4^3(1-\alpha_{34}^3\alpha_{43}^3)},\frac{\lambda_2^2+\lambda_1^2\alpha_{34}^2}{\mu_4^2(1-\alpha_{34}^2\alpha_{43}^2)},\frac{\lambda_2^3+\lambda_1^3\alpha_{34}^3}{\mu_4^1(1-\alpha_{34}^1\alpha_{43}^1)}\right)$$

$$\widetilde{\gamma_5}=\left(\begin{array}{c}\frac{\alpha_{35}^1(\lambda_1^1+\lambda_2^1\alpha_{43}^1)+\alpha_{45}^1(\lambda_2^1+\lambda_1^1\alpha_{34}^1)}{\mu_5^3(1-\alpha_{34}^3\alpha_{43}^3)}+\frac{\alpha_{15}^1[(\lambda_{1H}^1+\lambda_{2H}^1\alpha_{21}^1)+(\lambda_{1L}^1+\lambda_{2L}^1\alpha_{21}^1)]+\alpha_{25}^1[(\lambda_{2H}^1+\lambda_{1H}^1\alpha_{12}^1)+(\lambda_{2L}^1+\lambda_{1L}^1\alpha_{12}^1)]}{\mu_5^3(1-\alpha_{12}^3\alpha_{21}^3)},\\ \frac{\alpha_{35}^2(\lambda_1^2+\lambda_2^2\alpha_{43}^2)+\alpha_{45}^2(\lambda_2^2+\lambda_1^2\alpha_{34}^2)}{\mu_5^2(1-\alpha_{34}^2\alpha_{43}^2)}+\frac{\alpha_{15}^2[(\lambda_{1H}^2+\lambda_{2H}^2\alpha_{21}^2)+(\lambda_{1L}^2+\lambda_{2L}^2\alpha_{21}^2)]+\alpha_{25}^2[(\lambda_{2H}^2+\lambda_{1H}^2\alpha_{12}^2)+(\lambda_{2L}^2+\lambda_{1L}^2\alpha_{12}^2)]}{\mu_5^2(1-\alpha_{12}^2\alpha_{21}^2)},\\ \frac{\alpha_{35}^3(\lambda_1^3+\lambda_2^3\alpha_{43}^3)+\alpha_{45}^3(\lambda_2^3+\lambda_1^3\alpha_{34}^3)}{\mu_5^1(1-\alpha_{34}^1\alpha_{43}^1)}+\frac{\alpha_{15}^3[(\lambda_{1H}^3+\lambda_{2H}^3\alpha_{21}^3)+(\lambda_{1L}^3+\lambda_{2L}^3\alpha_{21}^3)]+\alpha_{25}^3[(\lambda_{2H}^3+\lambda_{1H}^3\alpha_{12}^3)+(\lambda_{2L}^3+\lambda_{1L}^3\alpha_{12}^3)]}{\mu_5^1(1-\alpha_{12}^1\alpha_{21}^1)}\end{array}\right)$$

$$\widetilde{\gamma_6}=\left(\begin{array}{c}\frac{\mu_{1L}^1(\lambda_{1H}^1+\lambda_{2H}^1\alpha_{21}^1)+\mu_{1H}^1(\lambda_{1L}^1+\lambda_{2L}^1\alpha_{21}^1)}{\mu_{1L}^3\mu_{1H}^3(1-\alpha_{12}^3\alpha_{21}^3)},\\ \frac{\mu_{1L}^2(\lambda_{1H}^2+\lambda_{2H}^2\alpha_{21}^2)+\mu_{1H}^2(\lambda_{1L}^2+\lambda_{2L}^2\alpha_{21}^2)}{\mu_{1L}^2\mu_{1H}^2(1-\alpha_{12}^2\alpha_{21}^2)},\\ \frac{\mu_{1L}^3(\lambda_{1H}^3+\lambda_{2H}^3\alpha_{21}^3)+\mu_{1H}^3(\lambda_{1L}^3+\lambda_{2L}^3\alpha_{21}^3)}{\mu_{1L}^1\mu_{1H}^1(1-\alpha_{12}^1\alpha_{21}^1)}\end{array}\right)$$

$$\widetilde{\gamma_7}=\left(\begin{array}{c}\frac{\mu_{2L}^1(\lambda_{2H}^1+\lambda_{1H}^1\alpha_{12}^1)+\mu_{2H}^1(\lambda_{2L}^1+\lambda_{1L}^1\alpha_{12}^1)}{\mu_{2L}^3\mu_{2H}^3(1-\alpha_{12}^3\alpha_{21}^3)},\\ \frac{\mu_{2L}^2(\lambda_{2H}^2+\lambda_{1H}^2\alpha_{12}^2)+\mu_{2H}^2(\lambda_{2L}^2+\lambda_{1L}^2\alpha_{12}^2)}{\mu_{2L}^2\mu_{2H}^2(1-\alpha_{12}^2\alpha_{21}^2)},\\ \frac{\mu_{2L}^3(\lambda_{2H}^3+\lambda_{1H}^3\alpha_{12}^3)+\mu_{2H}^3(\lambda_{2L}^3+\lambda_{1L}^3\alpha_{12}^3)}{\mu_{2L}^1\mu_{2H}^1(1-\alpha_{12}^1\alpha_{21}^1)}\end{array}\right)$$

Numerical Evaluation

Customers in Queues	Arrival rate	Probabilities	Service rate
$\widetilde{m_{1L}}=(2,3,4)$	$\widetilde{\lambda_{1L}}=(1,2,3)$	$\widetilde{\alpha_{12}}=(.4,.5,.6)$	$\widetilde{\mu_{1L}}=(22,23,24)$
$\widetilde{m_{1H}}=(1,2,3)$	$\widetilde{\lambda_{1H}}=(3,5,7)$	$\widetilde{\alpha_{15}}=(.3,.5,.7)$	$\widetilde{\mu_{1H}}=(28,29,30)$
$\widetilde{m_{2L}}=(2,2,2)$	$\widetilde{\lambda_{2L}}=(1,2,3)$	$\widetilde{\alpha_{21}}=(.7,.8,.9)$	$\widetilde{\mu_{2L}}=(18,19,20)$
$\widetilde{m_{2H}}=(3,4,5)$	$\widetilde{\lambda_{2H}}=(2,3,4)$	$\widetilde{\alpha_{25}}=(.1,.2,.3)$	$\widetilde{\mu_{2H}}=(25,26,27)$
$\widetilde{m_3}=(2,4,6)$	$\widetilde{\lambda_1}=(4,5,6)$	$\widetilde{\alpha_{34}}=(.3,.5,.7)$	$\widetilde{\mu_3}=(26,27,28)$
$\widetilde{m_4}=(4,5,6)$	$\widetilde{\lambda_2}=(2,4,6)$	$\widetilde{\alpha_{43}}=(.5,.6,.7)$	$\widetilde{\mu_4}=(24,25,26)$
$\widetilde{m_5}=(5,6,7)$		$\widetilde{\alpha_{35}}=(.4,.5,.6)$	$\widetilde{\mu_5}=(40,41,42)$
		$\widetilde{\alpha_{45}}=(.2,.4,.6)$	

Using these numerical values, the utilization factor of the servers is

$$\widetilde{\gamma_1} = (.3416, .4568, .5662)$$
$$\widetilde{\gamma_2} = (.2576, .3526, .4556)$$
$$\widetilde{\gamma_3} = (.3501, .3915, .4615)$$
$$\widetilde{\gamma_4} = (.2413, .3714, .5)$$
$$\widetilde{\gamma_5} = (.2417, .5122, .8916)$$
$$\widetilde{\gamma_6} = (.4360, .6862, .9591)$$
$$\widetilde{\gamma_7} = (.3728, .6393, .9062)$$
$$\widetilde{L_1} = \frac{\widetilde{\gamma_1}}{1-\widetilde{\gamma_1}} = (.6255, .8409, 1.0368)$$
$$\widetilde{L_2} = \frac{\widetilde{\gamma_2}}{1-\widetilde{\gamma_2}} = (.4004, .5446, .7081)$$
$$\widetilde{L_3} = \frac{\widetilde{\gamma_3}}{1-\widetilde{\gamma_3}} = (.5892, .6434, .7767)$$
$$\widetilde{L_4} = \frac{\widetilde{\gamma_4}}{1-\widetilde{\gamma_4}} = (.3834, .5908, .7944)$$
$$\widetilde{L_5} = \frac{\widetilde{\gamma_5}}{1-\widetilde{\gamma_5}} = (.5577, 1.0500, 2.0575)$$
$$\widetilde{L_6} = \frac{\widetilde{\gamma_6}}{1-\widetilde{\gamma_6}} = (1.4416, 2.1867, 3.1711)$$
$$\widetilde{L_7} = \frac{\widetilde{\gamma_7}}{1-\widetilde{\gamma_7}} = (1.0341, 1.7724, 2.5137)$$
$$\tilde{L} = (5.0319, 7.6288, 11.0583)$$
$$\tilde{\lambda} = (13, 21, 29)$$
$$E(\widetilde{W}) = (.2396, .3633, .5266)$$

On using Robust Ranking method i.e., if $\tilde{G} = (g_1, g_2, g_3)$ be a TFN, then Robust Ranking is defined as

$R(\tilde{G}) = \int_0^1 (0.5)\,(g_\alpha^L, g_\alpha^U)\,d\alpha, where$ $(g_\alpha^L, g_\alpha^U) = \{(g_2 - g_1)\alpha + g_1, g_3 - (g_3 - g_2)\alpha\}$ is the $\alpha - cut$ representation of fuzzy number $\tilde{G} = (g_1, g_2, g_3)$.

The crisp values of mean queue length, arrival rate and expected waiting time are

$$R(\tilde{L}) = 8.0451$$
$$R(\tilde{\lambda}) = 21$$
$$R\left(E(\widetilde{W})\right) = .3831$$

Thus, above calculation evidently shows average queue length and waiting time are fuzzy in nature. The most possible values of queue length & waiting time are 7.6288 & $.3633$ which falls between support values $(5.0319, 11.0583)$ & $(.2396, .5266)$ respectively. The utilization of server by low priority at C_{11} is 68% and at C_{12} is 63% which is higher than utilization factor of high priority customer at these subsystems.

CONCLUSION

In the present study, the queue performance of the system in fuzzy environment have been analyzed by utilizing Zadeh's principle on triangular fuzzy numbers. α-cut approach is more suitable to derive the membership values of queue characteristics. Robust Ranking method have been used to change fuzzy data into crisp numbers. Both input and output data are fuzzy in nature. The utilization of service mechanism at biserial server by low priority customers are high as compared to others, so that analyst can take the decision about the best optimum utilizations of servers. Numerical illustrations for TFN are described well the existence of the purposed model. The study can be further extended with more parallel servers and batch arrival.

REFERENCE

[1] R. E. Bellman and L. A. Zadeh (1970), Decision-making in a fuzzy environment, Management Science 17, pp.141-164.

[2] Li, R.J., and Lee, E.S. (1989). Analysis of fuzzy queues. Computers and Mathematics with Applications, 17(7), pp. 1143- 1147.

[3] C.Kao, C.C.Li and S.P.Chen(1999) , Parametric programming to the analysis of fuzzy queues, Fuzzy Sets and Systems, 107, pp. 93-100

[4] S. P. Chen (2005), Parametric nonlinear programming approach to fuzzy queues with bulk service, European Journal of Operational Research 163, pp. 434-444.

[5] S. P. Chen (2006), A Mathematics programming approach to the machine interference problem with fuzzy parameters, Applied Mathematics and Computation 174, pp.374-387.

[6] Singh, T.P., and Pardeep (2009). Serial queue model with blocking under fuzzy environment. JMASS, 5(2), pp.86-94.

[7] Robert, L., and Ritha, W. (2010). Machine interference problem with fuzzy environment. I. J. Contemporary Math. Science, 5(39), pp. 1905-1912.

[8] Seema, Gupta, D. and Sharma, S. (2013). Analysis of biserial servers linked to a common server in fuzzy environment, International Journal of Computer Applications, 68(6), pp. 26-32.

[9] Sharma Sameer (2017), "Fuzzy Analysis of Synergistic Collaboration of Biserial and Parallel Servers with a Common Server", Advances in Fuzzy Mathematics. Vol.12(2). pp. 283-296.

[10] Mittal M., Singh T.P. and Gupta D. (2015). "Threshold Effect on a Fuzzy Queue Model with Batch Arrival." Aryabhatta Journal of Mathematics & Informatics 7(1), pp. 109-118. [Google Scholar] [Publisher Link]

[11] T.P.Singh, M.Mittal, D.Gupta (2016), Modelling of a bulk queue system in Triangular fuzzy numbers using α-cut, International Journal of IT and Engineering, 4(9), pp. 72-79.

[12] T.P.Singh, M.Mittal, D.Gupta (2016), Priority queue model along intermediate queue under fuzzy environment with application, International Journal of physical & Applied Sciences, 3(4), pp. 102-113.

[13] W.Ritha and S.B. Menon (2011), Fuzzy n policy queues with infinite capacity, Journal of Physical Sciences, 15, pp.73-82.

[14] W. Ritha, Josephine S. (2019), "Analysis of Fuzzy Tandem Queues by Flexible α -Cuts Method", International Journal of Research in Advent Technology (IJRAT) Special Issue, pp. 156-161.

[15] Saini A, Gupta D and Tripathi A. K. (2023), "Analysis of Fuzzy Priority Queuing System with Heterogeneous Servers**",** Aryabhatta Journal of Mathematics and Informatics (AJMI). Vol. 15(1), pp. 111-120

[16] Saini A, Gupta D and Tripathi A. K. (2023), "Priority Biserial Queues in Fuzzy Environment", Journal of Emerging Technologies and Innovative Research. Vol. 10(4), pp. 156-161

[17] Selvakumaria K., Revathi S. (2021), "Analysis of Fuzzy Non-preemptive Priority Queuing Model with Unequal Service Rate", Turkish Journal of Computer and Mathematics Education Vol.12(5), pp.1457-1460.

[18] Saini A, Gupta D and Tripathi A. K. (2023), "Analysis of Bi-Tandem Priority Queue System in Stochastic Environment", International Journal of Mathematics Trends and Technology. Vol. 69(5), pp. 54-69.