Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 11, Number 1 (2016), pp. 25-37

http://www.ripublication.com

Pairwise Fuzzy Extra Resolvable Spaces

G. Thangaraj and P. Vivakanandan

Department of Mathematics Thiruvalluvar University
Vellore-632 115, Tamilnadu, India.
Department of Mathematics Jawahar Science College,
Neyveli-607 803, Tamilnadu, India.

ABSTRACT

In this paper, the concept of pairwise fuzzy extraresolvablespaces is introduced and several characterizations of pairwise fuzzy extraresolvablespaces are studied and the conditions under which fuzzy bitopologicalspacesbecome pairwise fuzzy extraresolvablespaces, are also investigated.

KEY WORDS: Pairwise fuzzy dense set, pairwisefuzzy nowhere dense set, pairwise fuzzy G_δ-set, pairwise fuzzy first category space, pairwise fuzzy Baire space, pairwise fuzzy hyperconnectedspace, pairwise fuzzy nodecspace.

2000 AMS Classification: 54 A 40, 03 E 72.

1. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by **L.A.ZADEH**[16] in 1965, describing fuzziness mathematically for the first time. Among the various fields of Mathematics, the first to be considered in the context of fuzzy sets, was general topology. The concepts of fuzzy topology was defined by **C. L. CHANG** [4] in 1968. The paper of Chang pavedthe way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then, much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. Today fuzzy topology has been firmly establishedas one of the basic disciplines of fuzzy mathematics.

E.HEWIT[6] introduced the concepts of resolvability and irresolvability in topological spaces. **A.G.EL'KIN** [5] introduced open hereditarily irresolvable spaces in classical topology. The concept of almost resolvable spaces was introduced by

RICHARD BOLSTEIN[3] as a generalization of resolvable spaces of E.Hewit. The concept of σ-resolvable spaces was introduced by **BRANISLAV NOVOTNY** [9]in classical topology. In 1989, **KANDIL** [7] introduced the concept of fuzzy bitopological spaces as a generalization of fuzzy topological spaces. Inthis paper, the concept of pairwise fuzzy extraresolvablespaces is introduced and several characterizations of pairwise fuzzy extraresolvablespaces are studied and the conditions under which fuzzy bitopological spaces become pairwise fuzzy extraresolvablespaces, are investigated. For this study, pairwise fuzzy submaximal spaces, pairwise fuzzy first category spaces, pairwise fuzzy Baire spaces, pairwise fuzzy σ-Baire spaces pairwise fuzzy hyperconnected spaces, pairwise fuzzyresolvable spaces, are considered.

2. PRELIMINARIES

Now we introduce some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to CHANG(1968). By a fuzzybitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X.

Definition 2.1: Let λ and μ be fuzzy sets in X. Then, for all $x \in X$,

(i) $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$

(ii) $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$

(iii) $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max\{\lambda(x), \mu(x)\}$

(iv) $\delta = \lambda \wedge \mu \Leftrightarrow \delta(x) = \min\{\lambda(x), \mu(x)\}$

(v) $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$

For a family$\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$, are defined respectively as

(vi) $\psi(x) = sup_i\{\lambda_i(x) \,/\, x \in X\}$

(vii) $\delta(x) = inf_i\{\lambda_i(x) \,/ x \in X\}$

Definition 2.2 [4]: Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The interior Int (λ) and the closure cl (λ) of λ are defined respectively as:

(i). Int (λ) = { μ / μ ≤ λ, μ ∈T } and

(ii) cl (λ) = { μ / λ ≤ μ, 1−μ ∈ T }.

Lemma 2.1 [1]: For a fuzzy set λ of a fuzzy topological space X,

(i). $1 - \text{Int}(\lambda) = \text{Cl}(1 - \lambda)$,

(ii). $1 - \text{Cl}(\lambda) = \text{Int}(1 - \lambda)$.

Definition 2.3 [13]: A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ (i = 1, 2). The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a pairwise fuzzy closed setin (X, T_1, T_2).

Definition 2.4 [10]: A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T_1} cl_{T_2}(\lambda) = cl_{T_2} cl_{T_1}(\lambda) = 1$.

Definition 2.5 [11]: A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $int_{T_1} cl_{T_2}(\lambda) = int_{T_2} cl_{T_1}(\lambda) = 0$.

Definition 2.6 [11]: Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy second category set in (X, T_1, T_2).

Definition 2.7 [11]: If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X, T_1, T_2), then the fuzzy set (1-λ) is called a pairwise fuzzy residual set in (X, T_1, T_2).
Fuzzy G_δ-sets and fuzzy F_σ-sets in fuzzy topological spaces are introduced and studied byG.Balasubramanian in [2]. This idea is extended to fuzzy bitopological spaces in [13].

Definition 2.8 [14]: A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is calleda pairwise fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are ` pairwise fuzzy open sets in (X, T_1, T_2).

Definition 2.9 [14]: A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are pairwise fuzzy closed sets in (X, T_1, T_2).

Definition 2.10[11]: A fuzzybitopological space (X, T_1, T_2) is called pairwise fuzzy first category space if the fuzzy set $\mathbf{1_X}$ is a pairwise fuzzy first category set in (X, T_1, T_2). That is, $1_X = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Otherwise, (X, T_1, T_2) will be called a pairwise fuzzy second category space.

Definition 2.11 [14]: A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy hyperconnected space if each non-null pairwise fuzzy open set λ in (X, T_1, T_2), is a pairwise fuzzy dense set in (X, T_1, T_2). That is, if λ is a pairwise fuzzy open set in a fuzzy bitopological space (X, T_1, T_2), then $cl_{T_1} cl_{T_2}(\lambda) = cl_{T_2} cl_{T_1}(\lambda) = 1$.

Definition 2.12 [15]: A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ-nowhere dense set if λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2) such that $int_{T_1} int_{T_2}(\lambda) = int_{T_2} int_{T_1}(\lambda) = 0$

Definition 2.13[11]: A fuzzy bitopologicalspace (X, T_1, T_2) is said to be a pairwise fuzzyBairespace if $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, $(i = 1, 2)$, where(λ_k)'s are pairwise fuzzy nowhere dense sets in(X, T_1, T_2).

Definition 2.14 [15]: Let (X, T_1, T_2) be a fuzzy bitopological space. Then (X, T_1, T_2) is called a pairwisefuzzy σ-Baire space if $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i= 1, 2), where $(\lambda_k)'$ are pairwise fuzzy σ-nowhere dense setsin(X, T_1, T_2).

Definition 2.15 [10]: A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy resolvable space if there exists a pairwise fuzzy dense set λin (X, T_1, T_2) such that$(1-\lambda)$ is also a pairwise fuzzy dense set λin (X, T_1, T_2). Otherwise (X, T_1, T_2) is called a pairwise fuzzy irresolvable space.

Definition 2.16 [12]: A fuzzy bitopologicalspace (X, T_1, T_2) is said to be a pairwise fuzzy strongly irresolvablespace if $cl_{T_1} int_{T_2}(\lambda) = 1 = cl_{T_2} int_{T_1}(\lambda)$, for each pairwise fuzzy dense setλin(X, T_1, T_2).

Definition 2.17 [12]: A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy almost resolvable space if $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)'$sin (X, T_1, T_2) are such that $int_{T_1} int_{T_2}(\lambda_k) = int_{T_2} int_{T_1}(\lambda_k) = 0$.Otherwise (X, T_1, T_2) is called a pairwise fuzzy almost irresolvable space.

3. PAIRWISE FUZZYEXTRARESOLVABLE SPACES

The concept ofextraresolvable spaces in classical topology was introduced and studied by V.I. Malykhin [8] as a generalization of resolvable spaces of E.Hewitt.Motivated by this, the concept of pairwise fuzzy extraresolvablespaces is introduced and studiedin this section.

Definition 3.1: Afuzzy bitopologicalspace(X, T_1, T_2) is called a pairwise fuzzyextraresolvablespace if $\lambda_m$andλ_n (m≠n), are pairwise fuzzy dense sets in(X, T_1, T_2), then $(\lambda_m \wedge \lambda_n)$is a pairwise fuzzynowhere dense set in (X, T_1, T_2).That is, if$cl_{T_1} cl_{T_2}(\lambda_m) = 1$, $cl_{T_2} cl_{T_1}(\lambda_m) = 1$ and$cl_{T_1} cl_{T_2}(\lambda_n) = 1$, $cl_{T_2} cl_{T_1}(\lambda_n) = 1$, (m ≠n), in (X, T_1, T_2) then $int_{T_1} cl_{T_2}(\lambda_m \wedge \lambda_n) = 0$ and $int_{T_2} cl_{T_1}(\lambda_m \wedge \lambda_n) = 0$, in$(X, T_1, T_2)$.

Example31: Let X={a, b, c}. The fuzzy setsλ, $\mu$and$\gamma$are defined onX asfollows:

λ: X→[o, 1]is defined asλ(a)= 0.6 ; λ(b)= 0.7 ; λ(c)=0.7:

μ: X→ 0,1 is defined asμ(a)= 0.5 ; μ(b)= 0.7 ; μ(c)=0.6;

γ: X → 0,1 is defined as γ(a)= 0.4 ;γ(b)= 0.6 ; γ(c)= 0.5.

Then, clearly $T_1 = \{0, \lambda, \mu, 1\}$ and $T_2 = \{0, \mu, \gamma, 1\}$ are fuzzy topologies defined on X.

Then $cl_{T_1} cl_{T_2}(\lambda) = cl_{T_1}(1) = 1$ and $cl_{T_2} cl_{T_1}(\lambda) = cl_{T_2}(1) = 1$

$cl_{T_1} cl_{T_2}(\mu) = cl_{T_1}(1) = 1$ and$cl_{T_2} cl_{T_1}(\mu) = cl_{T_2}(1) = 1$

$cl_{T_1} cl_{T_2}(\gamma) = cl_{T_1}(1) = 1$ and$cl_{T_2} cl_{T_1}(\gamma) = cl_{T_2}(1) = 1$

This implies that λ, μ, $\gamma$are pairwisefuzzy dense sets in (X, T_1, T_2). Now $\lambda \wedge \mu = \mu$

$int_{T_1} cl_{T_2}(\lambda \wedge \mu) = int_{T_1}(1) = 1 \neq 0$ and $int_{T_2} cl_{T_1}(\lambda \wedge \mu) = int_{T_2}(1) = 1 \neq 0$.This implies that $(\lambda \wedge \mu)$ is notapairwise fuzzy nowhere dense setin (X, T_1, T_2). That is,

λ and μ are pairwise fuzzy dense setsin (X, T_1, T_2), but ($\lambda \wedge \mu$) is not a pairwise fuzzy nowhere dense setin (X, T_1, T_2). Hence (X, T_1, T_2) is not a pairwise fuzzy extraresolvable space.

Proposition 3.1: If$\lambda_m$and λ_n (m $\neq$n)are pairwise fuzzydense sets in a pairwise fuzzy extraresolvablespace (X, T_1, T_2), then$\text{int}_{T_1}\text{int}_{T_2}(\lambda_m \wedge \lambda_n) = 0$and $\text{int}_{T_2}\,\text{int}_{T_1}(\lambda_m \wedge \lambda_n) = 0$.

Proof: Let λ_m and λ_n (m $\neq$n)be pairwise fuzzydense sets in a pairwisefuzzy extraresolvablespace (X, T_1, T_2).Then ($\lambda_m \wedge \lambda_n$)is a pairwise fuzzynowhere denseset in (X, T_1, T_2) and hence $\text{int}_{T_1}\text{cl}_{T_2}(\lambda_m \wedge \lambda_n) = 0$ and $\text{int}_{T_2}\text{cl}_{T_1}(\lambda_m \wedge \lambda_n) = 0$.But, $\text{int}_{T_1}\,\text{int}_{T_2}(\lambda_m \wedge \lambda_n) \leq \text{int}_{T_1}\text{cl}_{T_2}(\lambda_m \wedge \lambda_n)$and $\text{int}_{T_2}\,\text{int}_{T_1}(\lambda_m \wedge \lambda_n) \leq \text{int}_{T_2}\text{cl}_{T_1}(\lambda_m \wedge \lambda_n)$.and hence $\text{int}_{T_1}\text{int}_{T_2}(\lambda_m \wedge \lambda_n) \leq 0$ and $\text{int}_{T_2}\,\text{int}_{T_1}(\lambda_m \wedge \lambda_n) \leq 0$. That is, $\text{int}_{T_1}\text{int}_{T_2}(\lambda_m \wedge \lambda_n) = 0$ and $\text{int}_{T_2}\,\text{int}_{T_1}(\lambda_m \wedge \lambda_n) = 0$.

Theorem3.1 [11]: If λ is a pairwise fuzzy nowhere dense set in a fuzzy bitopologicalspace (X, T_1, T_2), then $(1 - \lambda)$ is a pairwise fuzzydense set in(X, T_1, T_2).

Proposition 3.2: If$\text{int}_{T_i}(\lambda_m \vee \lambda_n) = 0$ (m$\neq$ n), (i = 1, 2) where$\lambda_m$an$\lambda_n$are pairwise fuzzynowhere dense setsin a fuzzy bitopological space (X, T_1, T_2), then, (X, T_1, T_2) is not apairwise fuzzyextraresolvable space.

Proof: Let $\lambda_m$and λ_n(m $\neq$ n),) be pairwise fuzzy nowhere dense setsin (X, T_1, T_2).Then, bytheorem3.1, $(1 - \lambda_m)$and$(1 - \lambda_n)$are pairwise fuzzy dense setsin (X, T_1, T_2).Now, byhypothesis, for m $\neq$ n, $int_{T_i}(\lambda_m \vee \lambda_n) = 0$ (i= 1, 2).Then, $1 - \text{int}_{T_i}(\lambda_m \vee \lambda_n) = 1$, implies that $\text{cl}_{T_i}(1 - (\lambda_m \vee \lambda_n)) = 1$. That is, $\text{cl}_{T_i}[(1 - \lambda_m) \wedge (1 - \lambda_n)]$ =1and henceint$_{T_j}\text{cl}_{T_i}[(1 - \lambda_m) \wedge (1 - \lambda_n)] = \text{int}_{T_j}(1) = 1 \neq 0$(i, j= 1, 2). This implies that $[(1 - \lambda_m) \wedge (1 - \lambda_n)]$ is not apairwise fuzzy nowhere dense set in(X, T_1, T_2), where$(1 - \lambda_m)$ and $(1 - \lambda_n)$ arepairwise fuzzy dense setsin (X, $T_1 T_2$).Hence(X, T_1, T_2) is not apairwise fuzzy extraresolvable space.

Proposition 3.3: If$\lambda_m$andλ_n (m $\neq$n) are pairwise fuzzy dense sets in a pairwise fuzzy extraresolvable space (X, T_1, T_2), then $[(1 - \lambda_m) \vee (1 - \lambda_n)]$ is a pairwise fuzzydense set in(X, T_1, T_2)such that $\text{int}_{T_i}\,\text{int}_{T_j}(1 - \lambda_m) = 0$and $\text{int}_{T_i}\,\text{int}_{T_j}(1 - \lambda_n) = 0$.

Proof: Let λ_m and λ_n(m $\neq$n) be pairwisefuzzy dense sets in (X, T_1, T_2).Since (X, T_1, T_2) is a pairwise fuzzy extraresolvable space, $(\lambda_m \wedge \lambda_n)$ is a pairwise fuzzy nowhere dense set in(X, T_1, T_2).Then, by theorem 3.1, $[(1 - (\lambda_m \wedge \lambda_n)$is a pairwise fuzzydenseset in (X, T_1, T_2). That is, $[(1 - \lambda_m) \vee (1 - \lambda_n)]$ is a pairwise fuzzydense set in(X, T_1, T_2). Since λ_m and λ_n , are pairwise fuzzydense sets in(X, T_1, T_2), $\text{cl}_{T_i}\text{cl}_j(\lambda_m) = 1and\text{cl}_{T_i}\text{cl}_{T_j}(\lambda_n) = 1$(i, j=1, 2).This implies that $1 - \text{cl}_{T_i}\text{cl}_j(\lambda_m) = 0$ and $1 - \text{cl}_{T_i}\text{cl}_{T_j}(\lambda_n) = 0$. Then, we have $\text{int}_{T_i}\,\text{int}_{T_j}(1 - \lambda_m) = 0$ and

$\text{int}_{T_i}\,\text{int}_{T_j}(1-\lambda_n)$= 0.Hence $[(1-\lambda_m)\vee(1-\lambda_n)]$ is a pairwise fuzzy dense set in(X, T_1, T_2) such that $\text{int}_{T_i}\,\text{int}_{T_j}(1-\lambda_m)$= 0 and $\text{int}_{T_i}\,\text{int}_{T_j}(1-\lambda_n)$= 0.

Proposition 3.4: If λ_m and λ_n (m ≠n) are pairwise fuzzy nowhere dense sets inapairwise fuzzyextraresolvablespace (X, T_1, T_2), then$(\lambda_m\vee\lambda_n)$ is a pairwise fuzzy dense set in (X, T_1, T_2).

Proof: Let $\lambda_m$andλ_n(m≠n)be pairwise fuzzy nowhere dense sets in (X, T_1, T_2).Then, by theorem3.1, $(1-\lambda_m)$ and $(1-\lambda_n)$, are pairwise fuzzy dense sets in(X, T_1, T_2).Since (X, T_1, T_2)isa pairwise fuzzy extraresolvablespace, $[(1-\lambda_m)\wedge(1-\lambda_n)]$ is a pairwisefuzzynowheredensesetin(X, T_1, T_2). Thenby theorem 3.1, $(1-[(1-\lambda_m)\wedge(1-\lambda_n)])$is a pairwisefuzzy dense set in (X, T_1, T_2).Then, 1−[1− $(\lambda_m\vee\lambda_n)]$ is a pairwise fuzzy dense set in (X, T_1, T_2).Hence $(\lambda_m\vee\lambda_n)$ is a pairwise fuzzy dense set in(X, T_1, T_2).

Proposition 3.5: If λ_m and λ_n (m ≠ n)are pairwise fuzzy nowhere dense sets ina pairwise fuzzy extraresolvablespace (X, T_1, T_2) such that $(\lambda_m\vee\lambda_n)\neq 1$, then $(\lambda_m\vee\lambda_n)$ (m ≠n), is not a pairwise fuzzy closed set in (X, T_1, T_2).

Proof: Let λ_m and λ_n (m ≠ n) be pairwise fuzzy nowhere dense sets in(X, T_1, T_2).Since(X, T_1, T_2) is apairwise fuzzy extraresolvable space, by proposition 3.4, $(\lambda_m\vee\lambda_n)$is a pairwise fuzzy dense set in(X, T_1, T_2) and hence $\text{cl}_{T_1}\,\text{cl}_{T_2}(\lambda_m\vee\lambda_n)=1$ and $\text{cl}_{T_2}\,\text{cl}_{T_1}(\lambda_m\vee\lambda_n)=1$. Suppose that $(\lambda_m\vee\lambda_n)$is a pairwise fuzzyclosed set.Then, $\text{cl}_{T_1}(\lambda_m\vee\lambda_n)=\lambda_m\vee\lambda_n$ and $\text{cl}_{T_2}(\lambda_m\vee\lambda_n)=\lambda_m\vee\lambda_n$ .Then , $\text{cl}_{T_2}\,\text{cl}_{T_1}(\lambda_m\vee\lambda_n)=\text{cl}_{T_2}(\lambda_m\vee\lambda_n)=\lambda_m\vee\lambda_n$ and $\text{cl}_{T_1}\,\text{cl}_{T_2}(\lambda_m\vee\lambda_n)=\text{cl}_{T_1}(\lambda_m\vee\lambda_n)=\lambda_m\vee\lambda_n$. This will imply that $(\lambda_m\vee\lambda_n)=1$, a contradiction to $(\lambda_m\vee\lambda_n)\neq 1$, in (X, T_1, T_2). Hence$(\lambda_m\vee\lambda_n)$is not a pairwise fuzzyclosed set in (X, T_1, T_2).

Proposition 3.6: If$\lambda_m$and λ_n(m ≠n)are pairwise fuzzyσ-nowheredense sets in a pairwisefuzzyextraresolvablespace (X, T_1, T_2), then $(\lambda_m\vee\lambda_n)$ is a pairwise fuzzy dense and pairwise fuzzyF_σ-setin(X, T_1, T_2).

Proof: Letλ_m andλ_n (m ≠n) be pairwise fuzzyσ-nowhere dense sets inapairwise fuzzyextraresolvable space(X, T_1, T_2).Then, $\lambda_m$andλ_n, arepairwisefuzzy F_σ-sets in (X, T_1, T_2) such that $\text{int}_{T_1}\text{int}_{T_2}(\lambda_m)=0=\text{int}_{T_2}\text{int}_{T_1}(\lambda_m)$ and$\text{int}_{T_1}\text{int}_{T_2}(\lambda_n)=0=\text{int}_{T_2}\text{int}_{T_1}(\lambda_n)$ .Now, $1-\text{int}_{T_1}\text{int}_{T_2}(\lambda_m)=1=1-\text{int}_{T_2}\text{int}_{T_1}(\lambda_m)$ and$1-\text{int}_{T_1}\text{int}_{T_2}(\lambda_n)=1=1-\text{int}_{T_2}\,\text{int}_{T_1}(\lambda_n)$. Then, $\text{cl}_{T_1}\text{cl}_{T_2}(1-\lambda_m)=1=\text{cl}_{T_2}\,\text{cl}_{T_1}(1-\lambda_m)$, $\text{cl}_{T_1}\text{cl}_{T_2}(1-\lambda_n)=1=\text{cl}_{T_2}\text{cl}_{T_1}(1-\lambda_n)$.Hence $(1-\lambda_m)$ and $(1-\lambda_n)$ are pairwise fuzzy dense sets in(X, T_1, T_2).Since(X, T_1, T_2 isa pairwise fuzzyextraresolvable space, $[(1-\lambda_m)\wedge(1-\lambda_n)]$is a pairwise fuzzy nowhere dense set in(X, T_1, T_2).That is, 1 $-(\lambda_m\vee\lambda_n)$is a pairwise fuzzy nowheredense set in (X, T_1, T_2) and hence, by theorem 3.1, $(\lambda_m\vee\lambda_n)$ is a pairwise fuzzydense set in (X,

T_1, T_2).Sinceλ_m and λ_n, are pairwise fuzzyF_σ-sets(X, T_1, T_2), $(\lambda_m \vee \lambda_n)$ is a pairwise fuzzyF_σ-set in (X, T_1, T_2). Hence $(\lambda_m \vee \lambda_n)$ is a pairwise fuzzy dense and pairwise fuzzyF_σ-set in (X, T_1, T_2).

Proposition 3.7: If λ is apairwise fuzzy first category setin a pairwise fuzzy extraresolvablespace (X, T_1, T_2), then λ is apairwise fuzzy denseset in (X, T_1, T_2).

Proof: Letλ be a pairwise fuzzy first category set ina pairwise fuzzy extraresolvablespace (X, T_1, T_2). Then, $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Now $1-\lambda = 1-\vee_{k=1}^{\infty}(\lambda_k)$ and hence $1-\lambda = \wedge_{k=1}^{\infty}(1-\lambda_k)$ ………(1) .Since (λ_k) 's arepairwise fuzzy nowhere dense sets, $(1-\lambda_k)$'s are pairwise fuzzy dense sets in (X, T_1, T_2).Since (X, T_1, T_2) is a pairwise fuzzyextraresolvable space, for m ≠ n, $[(1-\lambda_m) \wedge (1-\lambda_n)]$ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). That is, $\mathrm{int}_{T_i}\mathrm{cl}_{T_j}[(1-\lambda_m) \wedge (1-\lambda_n)] = 0$ (wherei, j = 1, 2)……..(2).Now $\wedge_{k=1}^{\infty}(1-\lambda_k) \leq [(1-\lambda_m) \wedge (1-\lambda_n)]$, impliesthat $\mathrm{int}_{T_i}\mathrm{cl}_{T_j}[\wedge_{k=1}^{\infty}(1-\lambda_k)] \leq \mathrm{int}_{T_i}\mathrm{cl}_{T_j}[(1-\lambda_m) \wedge (1-\lambda_n)]$. Then, from (2), $\mathrm{int}_{T_i}\mathrm{cl}_{T_j}[\wedge_{k=1}^{\infty}(1-\lambda_k)] \leq 0$. Thatis, $\mathrm{int}_{T_i}\mathrm{cl}_{T_j}[\wedge_{k=1}^{\infty}(1-\lambda_k)] = 0$ and hence from(1), $\mathrm{int}_{T_i}\mathrm{cl}_{T_j}(1-\lambda) = 0$.Therefore$(1-\lambda)$is apairwise fuzzy nowhere dense set in (X, T_1, T_2).andhence, by theorem 3.1, λ is a pairwise fuzzy dense set in (X, T_1, T_2).

Proposition3.8: If $\mu \leq \lambda$, where λ is apairwise fuzzy residual set in a pairwise fuzzyextraresolvable space(X, T_1, T_2), then, $\mathrm{int}_{T_1}\mathrm{int}_{T_2}(\mu) = 0$ and $\mathrm{int}_{T_2}\mathrm{int}_{T_1}(\mu) = 0$, in (X, T_1, T_2).

Proof: Suppose that$\mu \leq \lambda$, where λ is apairwise fuzzy residual set in a pairwise fuzzyextraresolvablespace (X, T_1, T_2).Then, wehave $1-\lambda \leq 1-\mu$. Since λ is a pairwisefuzzy residual set in (X, T_1, T_2), $(1-\lambda)$ is apairwise fuzzy first category set in (X, T_1, T_2).Since(X, T_1, T_2) is a pairwisefuzzyextraresolvable space, by proposition 3.7, $(1-\lambda)$ isa pairwise fuzzy dense set in (X, T_1, T_2) and hence $\mathrm{cl}_{T_1}\mathrm{cl}_{T_2}(1-\lambda) = 1$ and $\mathrm{cl}_{T_2}\mathrm{cl}_{T_1}(1-\lambda) = 1$. Now $(1-\lambda) \leq (1-\mu)$ implies that $\mathrm{cl}_{T_1}\mathrm{cl}_{T_2}(1-\lambda) \leq \mathrm{cl}_{T_1}\mathrm{cl}_{T_2}(1-\mu)$ and $\mathrm{cl}_{T_2}\mathrm{cl}_{T_1}(1-\lambda) \leq \mathrm{cl}_{T_2}\mathrm{cl}_{T_1}(1-\mu)$.Hence $1 \leq \mathrm{cl}_{T_1}\mathrm{cl}_{T_2}(1-\mu)$ and $1 \leq \mathrm{cl}_{T_2}\mathrm{cl}_{T_1}(1-\mu)$. That is, $\mathrm{cl}_{T_1}\mathrm{cl}_{T_2}(1-\mu) = 1 = \mathrm{cl}_{T_2}\mathrm{cl}_{T_1}(1-\mu)$. This implies that $1 - \mathrm{int}_{T_1}\mathrm{int}_{T_2}(\mu) = 1$ and $1 - \mathrm{int}_{T_2}\mathrm{int}_{T_1}(\mu) = 1$ and hence$\mathrm{int}_{T_1}\mathrm{int}_{T_2}(\mu) = 0$ and $\mathrm{int}_{T_2}\mathrm{int}_{T_1}(\mu) = 0$.

Proposition3.9: If$\lambda \leq \delta$, whereλ is apairwise fuzzy first category set in a pairwise fuzzyextraresolvable space (X, T_1, T_2), then $\delta$is a pairwise fuzzy dense set in(X, T_1, T_2).

Proof: Letλbe a pairwise fuzzy first category set in(X, T_1, T_2).Then $(1-\lambda)$is a pairwise fuzzy residual setin(X, T_1, T_2).Now $\lambda \leq \delta$, implies that $1-\delta \leq 1-$

λ. Since(X, T_1, T_2) is a pairwise fuzzy extraresolvablespace, by proposition 3.8, $\mathrm{int}_{T_1}\mathrm{int}_{T_2}(1-\delta)$ =0 and $\mathrm{int}_{T_2}\mathrm{int}_{T_1}(1-\delta)$ =0 and hence $\mathrm{cl}_{T_1}\mathrm{cl}_{T_2}(\delta) = 1 = \mathrm{cl}_{T_2}\mathrm{cl}_{T_1}(\delta)$. Therefore δisapairwise fuzzy dense set in (X, T_1, T_2).

4. PAIRWISE FUZZYEXTRARESOLVABLESPACESandOTHER FUZZY TOPOLOGICAL SPACES

Proposition 4.1: If λ_m and λ_n (m ≠ n)are pairwise fuzzy nowheredense sets in a pairwise fuzzy extraresolvablespace (X, T_1, T_2) and if $\wedge_{m,n=1}^{\infty}(\lambda_m \vee \lambda_n) = 0$ (m ≠n), then, (X, T_1, T_2) is a pairwisefuzzy almost resolvable space.

Proof: Let$\lambda_m$andλ_n (m ≠ n) be pairwise fuzzy nowhere dense sets ina pairwise fuzzy extraresolvable space (X, T_1, T_2)such that $\wedge_{m,n=1}^{\infty}(\lambda_m \vee \lambda_n) = 0$ (m ≠ n).Then we have $1 - [\wedge_{m,n=1}^{\infty}(\lambda_m \vee \lambda_n)] = 1.$ That is, $\vee_{m,n=1}^{\infty}[1-(\lambda_m \vee \lambda_n)] = 1$......(1).Since λ_m and λ_n (m ≠ n), are pairwisefuzzy nowhere dense sets in(X, T_1, T_2), $(1-\lambda_m)$and $(1-\lambda_n)$are pairwise fuzzy dense sets in(X, T_1, T_2).Since(X, T_1, T_2) is a pairwise fuzzyextraresolvablespace, byproposition 3.1, $\mathrm{int}_{T_1}\mathrm{int}_{T_2}[(1-\lambda_m) \wedge (1-\lambda_n)] = 0$ and $\mathrm{int}_{T_2}\mathrm{int}_{T_1}[(1-\lambda_m) \wedge (1-\lambda_n)] = 0$……(2).From (1) and (2), $\vee_{m,n=1}^{\infty}[1-(\lambda_m \vee \lambda_n)] = 1$, (m ≠ n), where $\mathrm{int}_{T_1}\mathrm{int}_{T_2}[(1-\lambda_m) \wedge (1-\lambda_n)] = 0$ and $\mathrm{int}_{T_2}\mathrm{int}_{T_1}[(1-\lambda_m) \wedge (1-\lambda_n)] = 0$….Hence (X, T_1, T_2) is apairwise fuzzy almost resolvable space.

Theorem4.1 [13]: If λis a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable space(X, T_1, T_2), then (1—λ) is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Proposition4.2: If λ_m and λ_n(m ≠ n), are pairwise fuzzy dense setsin apairwise fuzzyextra resolvableandpairwise fuzzy strongly irresolvable space(X, T_1, T_2), then$[(1-\lambda_m) \vee (1-\lambda_n)]$, is a pairwise fuzzy dense setin (X, T_1, T_2).

Proof: Let λ_m and λ_n (m ≠ n) be pairwise fuzzy dense setsin apairwise fuzzy extraresolvable andpairwise fuzzy strongly irresolvable space (X, T_1, T_2).Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, for the pairwise fuzzy dense setsλ_m and λ_n(m ≠ n), in(X, T_1, T_2), by theorem 4.1, $(1-\lambda_m)$ and $(1-\lambda_n)$ arepairwise fuzzy nowhere dense setsin (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzyextraresolvablespace, $(\lambda_m \wedge \lambda_n)$(m ≠ n), is a pairwise fuzzy nowhere dense set in(X, T_1, T_2). This implies that $[1-(\lambda_m \wedge \lambda_n)]$, is a pairwise fuzzy dense set in (X, T_1, T_2). Hence $[(1-\lambda_m) \vee (1-\lambda_n)]$is apairwise fuzzy dense set in (X, T_1, T_2), where $(1-\lambda_m)$'s and $(1-\lambda_n)$'s arepairwise fuzzy nowhere dense setsin (X, T_1, T_2).

Proposition 4.3: If each $(\lambda_m \vee \lambda_n)$(m ≠ n), is a pairwise fuzzy dense set, whereλ_m and λ_n are pairwise fuzzy nowhere dense sets in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy extraresolvable space.

Proof: Let λ_m and λ_n (m ≠n) be pairwise fuzzy dense sets ina pairwise fuzzy strongly irresolvable space (X, T_1, T_2). Then, by theorem 4.1, $(1-\lambda_m)$'s and $(1-\lambda_n)'$s are pairwise nowhere fuzzydense sets in (X, T_1, T_2). By hypothesis, $[(1-\lambda_m) \vee (1-\lambda_n)]$isa pairwisefuzzydense set in (X, T_1, T_2).Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, by theorem4.1, $1- [(1-\lambda_m) \vee (1-\lambda_n)]$is apairwise fuzzy nowheredense set in (X, T_1, T_2). That is, $1- [(1-(\lambda_m \wedge \lambda_n)]$is a pairwise fuzzy nowhere dense set in (X, T_1, T_2) and hence $(\lambda_m \wedge \lambda_n)$ is apairwise fuzzy nowheredense setin (X, T_1, T_2), for the pairwise fuzzy dense sets(λ_m) and(λ_n)(m≠n) in (X, T_1, T_2). Therefore (X, T_1, T_2) is a pairwise fuzzy extraresolvablespace.

Proposition 4.4: If λ_m and λ_n (m ≠ n), are pairwise fuzzy nowhere dense sets in a pairwise fuzzy extraresolvable space (X, T_1, T_2) and $\wedge_{m,n=1}^{\infty} (\lambda_m \vee \lambda_n) = 0$ (m ≠ n), then (X, T_1, T_2) is a pairwise fuzzy first category space.

Proof: Let λ_m and λ_n (m ≠n) be pairwise fuzzy nowhere dense sets in (X, T_1, T_2).Then $(1-\lambda_m)$ and $(1-\lambda_n)$ arepairwise fuzzy dense sets in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy extraresolvable space, then $[(1-\lambda_m) \wedge (1-\lambda_n)]$, (for m ≠ n), is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Suppose that $\wedge_{m,n=1}^{\infty} (\lambda_m \vee \lambda_n) = 0$ (m ≠ n), where λ_m and λ_n are pairwisefuzzy nowhere dense sets in (X, T_1, T_2). Then $1 - \wedge_{m,n=1}^{\infty} (\lambda_m \vee \lambda_n) = 1$. That is, $\vee_{m,n=1}^{\infty} [(1-\lambda_m) \wedge (1-\lambda_n)]$ =1.Hence $\vee_{m,n=1}^{\infty} [(1-\lambda_m) \wedge (1-\lambda_n)]$ =1, where $[(1-\lambda_m) \wedge (1-\lambda_n)]$ is apairwise fuzzy nowhere dense set in (X, T_1, T_2). Hence (X, T_1, T_2) is apairwise fuzzy first category space.

Theorem 4.2[11]: Let(X, T_1, T_2) be a fuzzybitopologicalspace.Then the followingare equivalent:

(1) (X, T_1, T_2) is a pairwise fuzzy Baire space.

(2) $int_{T_i}(\lambda) = 0$, (i=1, 2), for every pairwise fuzzy first category set λ in (X, T_1, T_2).

(3) $cl_{T_i}(\mu) = 1$, (i=1, 2), for everypairwise fuzzy residual setμ in (X, T_1, T_2).

Proposition 4.5: If λ_m and λ_n (m ≠ n) are pairwise fuzzydense sets ina pairwise fuzzy extraresolvable and fuzzy Bairespace (X, T_1, T_2), then $(\lambda_m \wedge \lambda_n)$ is not apairwise fuzzy open set in(X, T_1, T_2).

Proof: Let λ_m and λ_n (m ≠n) be pairwise fuzzydense sets in (X, T_1, T_2). Since(X, T_1, T_2)is apairwise fuzzyextraresolvablespace, $(\lambda_m \wedge \lambda_n)$(m ≠ n) isa pairwisefuzzy nowhere dense set in(X, T_1, T_2). Let $\delta_{mn} = (\lambda_m \wedge \lambda_n)$. Then, $\vee_{m,n=1}^{\infty} (\delta_{mn})$ is apairwise fuzzy first category setin (X, T_1, T_2). Since (X, T_1, T_2) is a pairwisefuzzy Baire space, by theorem 4.2, $int_{T_i}[\vee_{m,n=1}^{\infty}(\delta_{mn})] = 0$ (i= 1, 2). But$\vee_{m,n=1}^{\infty} int_{T_i}(\delta_{mn}) \leq int_{T_i}[\vee_{m,n=1}^{\infty}(\delta_{mn})]$. Then $[\vee_{m,n=1}^{\infty} int_{T_i}(\delta_{mn})] = 0$. This implies that $int_{T_i}(\delta_{mn}) = 0$ (i= 1, 2).Hence δ_{mn} is not apairwise fuzzy open set in (X, T_1, T_2).That is, $(\lambda_m \wedge \lambda_n)$is not a pairwise fuzzy open set in(X, T_1, T_2).

Proposition 4.6: If λ is a pairwise fuzzy first category set ina pairwise fuzzy extraresolvable andpairwisefuzzyBairespace (X, T_1, T_2), then $(1 - \lambda)$ is a pairwise fuzzy denseset in (X, T_1, T_2).

Proof: Letλ be apairwise fuzzy first category set ina pairwise fuzzy extraresolvableand pairwise fuzzy Bairespace(X, T_1, T_2).Since$(X, T_1, T_2)$is apairwise fuzzyextraresolvablespace, by proposition 3.7, λ is a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwisefuzzy Baire space, by theorem 4.2, $\mathrm{int}_{T_i}(\lambda) = 0\ (i = 1, 2)$. This implies that $1 - \mathrm{int}_{T_i}(\lambda) = 1$and hence $\mathrm{cl}_{T_i}(1 - \lambda) = 1$. Hence $\mathrm{cl}_{T_1}\mathrm{cl}_{T_2}(1 - \lambda)=\mathrm{cl}_{T_1}(1) = 1$ and$\mathrm{cl}_{T_2}\mathrm{cl}_{T_1}(1 - \lambda)=\mathrm{cl}_{T_2}(1) = 1$. Therefore$(1 - \lambda)$ is a pairwise fuzzy dense setin(X, T_1, T_2).

Proposition 4.7: If λ is a pairwisefuzzyfirst category set ina pairwise fuzzy extraresolvableandpairwise fuzzy Bairespace (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy resolvable space.

Proof: Let λ be a pairwise fuzzy first category set ina pairwise fuzzyextraresolvableandpairwisefuzzy Baire space (X, T_1, T_2).Since$(X, T_1, T_2)$is apairwise fuzzyextraresolvablespace, by proposition3.7, λ is a pairwise fuzzydense set in (X, T_1, T_2).By proposition4.6, $(1 - \lambda)$ isa pairwise fuzzydense set in(X, T_1, T_2).Therefore, $(1 - \lambda)$is a pairwise fuzzydense set in(X, T_1, T_2) for the pairwise fuzzy dense setλ in (X, T_1, T_2) and hence(X, T_1, T_2) is a pairwisefuzzy resolvable space.

Proposition 4.8: If λ_m and λ_n $(m \neq n)$ are pairwise fuzzy first category sets ina pairwise fuzzy extraresolvableand pairwisefuzzy Bairespace (X, T_1, T_2), then$(\lambda_m \vee \lambda_n)$ is apairwisefuzzy denseset in (X, T_1, T_2).

Proof: Let λ_m and λ_n $(m \neq n)$ be pairwise fuzzy first category sets ina pairwise fuzzyBaireand pairwise fuzzyextraresolvable space (X, T_1, T_2).Then, byproposition 4.6, $(1 - \lambda_m)$ and $(1 - \lambda_n)$ are fuzzydense sets in(X, T_1, T_2). Since(X, T_1, T_2) isa pairwise fuzzyextraresolvable space, $[(1 - \lambda_m) \wedge (1 - \lambda_n)]$ is a pairwise fuzzy nowhere dense set in(X, T_1, T_2). Thisimplies that $1 - [(1 - \lambda_m) \wedge (1 - \lambda_n)]$ is a pairwise fuzzy dense set in(X, T_1, T_2) and hence $(1 - [1 - (\lambda_m \vee \lambda_n)])$is apairwise fuzzydense set in (X, T_1, T_2). Hence$(\lambda_m \vee \lambda_n)$is a pairwise fuzzydense set in(X, T_1, T_2).

Proposition4.9: If λ_m and λ_n $(m \neq n)$are pairwise fuzzy dense sets ina pairwise fuzzy extraresolvable and pairwise fuzzy submaximal space (X, T_1, T_2), then$(\lambda_m \wedge \lambda_n)$is a pairwise fuzzy closed set in (X, T_1, T_2).

Proof: Let λ_m and $\lambda_n$$(m \neq n)$bepairwise fuzzydense sets in(X, T_1, T_2). Since(X, T_1, T_2) isa pairwise fuzzyextraresolvablespace, $(\lambda_m \wedge \lambda_n)(m \neq n)$, is a pairwisefuzzynowhere dense set in(X, T_1, T_2).Then $[1 - (\lambda_m \wedge \lambda_n)$ is a pairwise fuzzydense setin (X, T_1, T_2). Since(X, T_1, T_2) is a pairwise fuzzy submaximal space, $[1 -(\lambda_m \wedge \lambda_n)]$ is a pairwise fuzzy open set in (X, T_1, T_2). Then$(\lambda_m \wedge \lambda_n)$ is apairwise fuzzyclosed set in(X, T_1, T_2).

Theorem4.3[15]: If λ is a pairwise fuzzy dense and pairwise fuzzy G_δ-set in a fuzzybitopologicalspace (X, T_1, T_2), then$(1- \lambda)$ is a pairwisefuzzy first category set in (X, T_1, T_2).

Proposition4.10: If λ_m and λ_n $(m \neq n)$are pairwise fuzzydense andpairwise fuzzy G_δ-setsina pairwise fuzzyextraresolvableand pairwisefuzzy Baire space (X, T_1, T_2), then $int_{T_i}(\lambda_m \wedge \lambda_n) = 0, (i = 1, 2)$, in (X, T_1, T_2).

Proof: Let λ_m and λ_n($m \neq n$) be pairwisefuzzy denseandpairwise fuzzy G_δ-setsin(X, T_1, T_2). Since(X, T_1, T_2) is a pairwise fuzzy extraresolvable, space, for the pairwise fuzzydense sets λ_m and λ_n $(m \neq n)$, $(\lambda_m \wedge \lambda_n)$ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Then, $\delta_{mn} = \vee_{m,n=1}^{\infty} (\lambda_m \wedge \lambda_n)$is apairwise fuzzy first category set in (X, T_1, T_2).Since (X, T_1, T_2) is a pairwise fuzzy Baire space, by theorem 4.2, $int_{T_i}(\delta_{mn})=0$ (i= 1, 2). But $int_{T_i}(\delta_{mn})= int_{T_i}\left[\vee_{m,n=1}^{\infty} (\lambda_m \wedge \lambda_n)\right] \geq \vee_{m,n=1}^{\infty} [int_{T_i}(\lambda_m \wedge \lambda_n)]$ and hence $0 \geq \vee_{m,n=1}^{\infty} [int_{T_i}(\lambda_m \wedge \lambda_n)]$. That is, $\vee_{m,n=1}^{\infty} [int_{T_i}(\lambda_m \wedge \lambda_n)] = 0$. This implies that $int_{T_i}(\lambda_m \wedge \lambda_n) = 0, (i = 1, 2)$, in (X, T_1, T_2).

Proposition 4.11: If (X, T_1, T_2) is a pairwise fuzzyBaire space, then it is not a pairwise fuzzyextraresolvablespace.

Proof: Letthe fuzzybitopologicalspace(X, T_1, T_2) be a pairwise fuzzy Baire space. Then, by theorem 4.2, $int_{T_i}\left(\vee_{k=1}^{\infty} (\lambda_k)\right)=0$, (i = 1, 2) where (λ_k) 'sare fuzzy pairwisenowhere dense sets in (X, T_1, T_2).Thisimpliesthat $1 - int_{T_i}\left(\vee_{k=1}^{\infty} (\lambda_k)\right)=1$ and hence we have $cl_{T_i}\left(\wedge_{k=1}^{\infty} (1 - \lambda_k)\right)=1$.Now $\wedge_{k=1}^{\infty} (1 - \lambda_k) .. \leq (1 - \lambda_m) \wedge (1 - \lambda_n)$. Then $cl_{T_i}[\wedge_{k=1}^{\infty} (1 - \lambda_k)] \leq cl_{T_i}[(1 - \lambda_m) \wedge (1 - \lambda_n)]$ and this impliesthat $1 \leq cl_{T_i}[(1 - \lambda_m) \wedge (1 - \lambda_n)]$. That is, $cl_{T_i}[(1 - \lambda_m) \wedge (1 - \lambda_n) = 1$. Then $int_{T_j} cl_{T_i}[(1 - \lambda_m) \wedge (1 - \lambda_n)] = int_{T_j}(1) = 1 \neq 0$, and hence$[(1 - \lambda_m) \wedge (1 - \lambda_n)]$is not a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Also, since (λ_m)'s are pairwise fuzzy nowhere dense sets in(X, T_1, T_2), $(1 - \lambda_m)$'s are pairwise fuzzy dense sets in (X, T_1, T_2). Hence, for thepairwise fuzzy dense sets$(1 - \lambda_m)$and$(1 - \lambda_n)(m \neq n)$ in (X, T_1, T_2), $[(1 - \lambda_m) \wedge (1 - \lambda_n)]$ is not a pairwise fuzzy nowhere dense set in(X, T_1, T_2).Hence(X, T_1, T_2) is not a pairwise fuzzyextraresolvablespace.

Theorem4.4 [15]: In a fuzzybitopologicalspace (X, T_1, T_2), afuzzy set $\lambda$is a pairwise fuzzyσ-nowhere denseif onlyif $(1 - \lambda)$is a pairwise fuzzydenseand pairwise fuzzy G_δ-set in(X, T_1, T_2).

Proposition 4.12: Ifthe fuzzy bitopological space(X, T_1, T_2) is a pairwisefuzzy σ-Bairespace, then (X, T_1, T_2)is not a pairwise fuzzy extraresolvablespace.

Proof: Let (X, T_1, T_2)be apairwisefuzzyσ-Baire space. Then, $\mathrm{int}_{T_i}\left(\vee_{k=1}^{\infty}(\lambda_k)\right) = 0$, (i= 1, 2) where$(\lambda_k)$'s arepairwise fuzzyσ-nowhere dense sets in (X, T_1, T_2) and this implies that$1 - \mathrm{int}_{T_i}\left(\vee_{k=1}^{\infty}(\lambda_k)\right) = 1$. Then, $\mathrm{cl}_{T_i}\left(\wedge_{k=1}^{\infty}(1 - \lambda_k)\right) = 1$.Now$\wedge_{k=1}^{\infty}(1 - \lambda_k) \leq (1 - \lambda_m) \wedge (1 - \lambda_n)$. Then $\mathrm{cl}_{T_i}\left(\wedge_{k=1}^{\infty}(1 - \lambda_k)\right) \leq \mathrm{cl}_{T_i}[(1 - \lambda_m) \wedge (1 - \lambda_n)]$ and hence $1 \leq \mathrm{cl}_{T_i}[(1 - \lambda_m) \wedge (1 - \lambda_n)]$. That is, $\mathrm{cl}_{T_i}[(1 - \lambda_m) \wedge (1 - \lambda_n)] = 1$.Then, $\mathrm{int}_{T_j}\mathrm{cl}_{T_i}[(1 - \lambda_m) \wedge (1 - \lambda_n)] = \mathrm{int}_{T_j}(1) = 1 \neq 0$, implies that $[(1 - \lambda_m) \wedge (1 - \lambda_n)]$ isnot a pairwise fuzzy nowhere dense set in (X, T_1, T_2).
Since(λ_m)'s are pairwise fuzzyσ-nowhere dense sets in (X, T_1, T_2), by theorem 4.5, $(1 - \lambda_m)$'s are pairwise fuzzy denseand pairwise fuzzyG_δ-sets in (X, T_1, T_2). Hence $[(1 - \lambda_m) \wedge (1 - \lambda_n)]$ is not a pairwise fuzzy nowhere dense set in (X, T_1, T_2),.for the pairwise fuzzy dense sets $(1 - \lambda_m)$ and $(1 - \lambda_n)$ in (X, T_1, T_2) .Therefore (X, T_1, T_2)isnot a pairwise fuzzyextraresolvable space.

Proposition 4.13: If$\lambda_m$and $\lambda_n (m \neq n)$ are pairwise fuzzy open sets in a pairwise fuzzyextraresolvableand pairwise fuzzyhyperconnectedspace (X, T_1, T_2) , then$\mathrm{int}_{T_i}\mathrm{int}_{T_j}(\lambda_m \wedge \lambda_n) = 0$ (i =1, 2)in (X, T_1, T_2).

Proof: Let λ_m and $\lambda_n (m \neq n)$ be pairwise fuzzy open sets in a pairwise fuzzy hyperconnected space (X, T_1, T_2).Then λ_m and $\lambda_n$are pairwise fuzzy dense sets in (X, T_1, T_2).Since (X, T_1, T_2)is a pairwise fuzzyextraresolvable, $(\lambda_m \wedge \lambda_n)$is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).This implies that$(1 - (\lambda_m \wedge \lambda_n))$ is a pairwise fuzzy dense sein (X, T_1, T_2) .Then $\mathrm{cl}_{T_i}\mathrm{cl}_{T_j}[1 - (\lambda_m \wedge \lambda_n)] = 1$ and hence$\mathrm{int}_{T_i}\mathrm{int}_{T_j}(\lambda_m \wedge \lambda_n) = 0$ in (X, T_1, T_2).

Definition 4.1[12]: A fuzzy bitopologicalspace (X, T_1, T_2) is called a pairwise fuzzynodec space, if each non-zero pairwise fuzzy nowhere dense set in (X, T_1, T_2), is a pairwise fuzzy closed set in(X, T_1, T_2).

Proposition 4.14: If$\lambda_m$and$\lambda_n (m \neq n)$are pairwise fuzzy dense sets in a pairwise fuzzy extraresolvableand pairwise fuzzynodecspace (X, T_1, T_2), then$(\lambda_m \wedge \lambda_n)$is a pairwise fuzzy closed set in(X, T_1, T_2).

Proof: Let $\lambda_m$and $\lambda_n (m \neq n)$ be pairwise fuzzy dense sets in (X, T_1, T_2). Since (X, T_1, T_2) is apairwise fuzzy extraresolvable, $(\lambda_m \wedge \lambda_n)$is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Since (X, T_1, T_2) is apairwise fuzzynodecspace, $(\lambda_m \wedge \lambda_n)(m \neq n)$is a pairwise fuzzy closed set in(X, T_1, T_2).

REFERENCES

[1] K. K. Azad, OnFuzzy Semicontinuity, Fuzzy Almost Continuity and Fuzzy WeaklyContinuity, J. Math. Anal.Appl., 82 (1981), 14−32.
[2] G.Balasubramanian, Maximal Fuzzy Topologies, Kybernetika, 31(5)(1995), 459 − 464.
[3] R.Bolstein, Sets of Points of Discontinuity, Proc. Amer. Math. Soc., 38 (1973), 193− 197.
[4] C.L. Chang, Fuzzy Topological Spaces, J. Math.Anal. Appl., 24(1968), 182 − 190.
[5] A.G. El'kin, Ultrafiltersand UndecomposableSpaces, MoscowUniv.Math.Bull., 24 (1969), 37 − 40.(Russian original in: VestnikMosk. Gos. Univ., Ser. I, Mat. Mekh. 24 (5) (1969) 51-56.)
[6] E.Hewitt, A Problem in Set-Theoretic Topology, Duke Math. Jour., 10(1943), 309 −333.
[7] A.Kandil., BiproximitiesandFuzzy Bitopological Spaces, Simon Stevin, 63(1989), 45−66.
[8] V.I.Malykhin, Irresolvabilityis not descriptivelygood, Manuscript submitted for publication 1993.
[9] B. Novotny, On Subcontinuity, Real Anal. Exch. Vol.31(2), (2006), 535 − 545.
[10] G.Thangaraj, OnPairwise Fuzzy Resolvable andFuzzy Irresolvable Spaces, Bull. Cal. Math.Soc., 102(1) (2010), 59 −68.
[11] G.ThangarajandS.Sethuraman, OnPairwise Fuzzy BaireBitopological Spaces, Gen. Math.Notes., 20(2) (2014), 12−21.
[12] G.Thangarajand S.Sethuraman, A Note on Pairwise Fuzzy BaireSpaces, Ann. Fuzzy Math.Inform., 8(5) (2014), 729 −737.
[13] G. Thangaraj and S. Sethuraman, Some Remarks on Pairwise Fuzzy BaireSpaces, (To appear in Ann. Fuzzy Math. Inform.,)
[14] G.Thangarajand V.Chandiran, On Pairwise Fuzzy VolterraSpaces, Ann. Fuzzy Math. Inform., 7(6) (2014), 1005−1012.
[15] G.ThangarajandV.Chandiran, A Note on Pairwise Fuzzy VolterraSpaces, Ann. Fuzzy Math. Inform., 9(3) (2015), 365− 372.
[16] L.A. Zadeh, FuzzySets, InformationandControl, Vol.8 (1965), 338−353.