Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 11, Number 1 (2016), pp. 59-66

http://www.ripublication.com

An Intuitionistic Fuzzification Technique For Analysing Genetic Disorder

Sharmila. S and I. Arockiarani

Department of Mathematics Nirmala College for women
Coimbatore, Tamilnadu, India
Email: sharmi.skumar@gmail.com

Abstract

We propose a new, lucid and comprehensive approach for medical diagnosis of genetic disorder by employing intuitionistic fuzzy sets. Mining the hereditary disease-genes from human genome is one of the most important tasks in recent research. This paper presents a short survey of some directions in the diagnosis of hereditary disorder, in which estimations containing the Souslin number play the central role. Together with the known facts, the paper contains some new results established by the author.

Keywords: Souslin number, Cardinality, Gene, intuitionistic fuzzy set, distance measures, Normalised Hamming distance.

Mathematics subject Classification 2000: 92Dxx

1 INTRODUCTION

A major task of medical science is to diagnose diseases that are hereditary. It, however, is not a direct and simple task at all, because the information available to the physician about his patient and about medical relationships in general is inherently uncertain [1]. To improve the problem, they have proposed many approaches and theories such as fuzzy set theory and rough set theory. Fuzzy set theory makes it possible to define the inexact medical information as fuzzy sets, therefore, it can be utilized for modelling the diagnostic process. An application of fuzzy set on medical science fields was already proposed by Zadeh in 1969 [30] and Sanchez [24]. The theory of fuzzy sets proposed by Zadeh [29] has attracted wide spread attentions in various fields, especially where conventional mathematical techniques are of limited effectiveness, including biological and social sciences, linguistic, psychology, economics are more generally soft sciences. In such fields, variables are difficult to

quantify and dependencies among variable are so ill-defined that precise characterization in terms of algebraic or differential equations becomes almost impossible. Even in fields where dependencies between variables are well define, it might be necessary or advantageous to employ fuzzy rather than crisp algorithms to arrive at a solution [10]. M. Gupta, R. K. Ragade and R. R. Yager [18] invented a fully developed relationships modelling theory of symptoms and diseases using fuzzy sets.

Out of several higher-order fuzzy sets, intuitionistic fuzzy sets introduced by Atanassov [5, 6] have been found to be well suited to dealing with vagueness. The concept of an intuitionistic fuzzy set can be viewed as an alternative approach to define a fuzzy set in cases where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In general, the theory of intuitionistic fuzzy sets is the generalisation of fuzzy sets. Therefore, it is expected that intuitionistic fuzzy sets could be used to simulate human decision-making processes and any activities requiring human expertise and knowledge which are inevitably imprecise or not totally reliable.

Fuzzy set theory has been utilized in many approaches to model the diagnostic process [1, 3, 11, 14, 28, 31]. Szmit and Kacprzyk [11, 12] considered the use of intuitionistic fuzzy sets for building soft decision- making models with imprecise information, and proposed two solution concepts about the intuitionistic fuzzy core and the consensus winner for group decision making using intuitionistic fuzzy sets. A novel and effective approach to deal with decision making in medical diagnosis using the composition of intuitionistic fuzzy relations was proposed in [20]. However many researchers [27, 28] have applied different techniques to determine the disease of the patients the loss of the information if the disease is genetic is not considered in many studies.

As an extension of previous studies, the proposed method is applied to several patients according to the symptoms and diseases due to deficiency of Vitamin-D and to study a new approach for genetic disorder using Souslins number.

Having too little vitamin D may not be due solely to diet or lack of sunlight, but may be due to our genes. An international consortium of researchers and doctors has identified four gene variants that may play a role in vitamin D deficiency, a condition which may affect up to half of all healthy adults in the developed world. It can contribute to poor musculoskeletal health as well as potentially increase the risk of diabetes, cardiovascular disease, and certain types of common cancers. Vitamin D deficiency is associated with parathyroid hormone. Various symptoms and diseases occur as a result of Vitamin-D deficiency. Here, for our convenience we consider few important symptoms and diseases of Vitamin – D deficiency. As a benchmark we give an approach of the role of genes in this disorder.

2 PRELIMINARIES BRIEF NOTE ON INTUITIONISTIC FUZZY SETS

Definition [30]: Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\mu_A(x)/x \in X\}$ where the function $\mu_A : X \to I$ is the membership function of

the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e having degrees of membership.

Definition [5]: Let X be a nonempty set. An intuitionistic fuzzy set (IFS, in short) A in X is an object having the form $A = \{x, \mu_A(x), \upsilon_A(x) / x \in X\}$ where the functions $\mu_A : X \to I$ and $\upsilon_A : X \to I$ denote the degree of *membership* (namely $\mu_A(x)$) and the degree of *non-membership* (namely $\upsilon_A(x)$) of each element $x \in X$ to the set A on a nonempty set X and $0 \le \mu_A(x) + \upsilon_A(x) \le 1$ for each $x \in X$.

Furthermore, we have $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$ called the intuitionistic fuzzy set index or *hesitation margin* of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ i.e., $\pi_A(x) : X \to [0,1]$ and $0 \le \pi_A \le 1 / x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

Note: $\mu_A(x) + \upsilon_A(x) + \pi_A(x) = 1$

Some distance measures in intuitionistic fuzzy sets:

Definition.[6] Let X be nonempty set such that IFS $A, B, C \in X$. Then the distance measure d between IFS A and B is a mapping $d : X \times X \to [0,1]$; if d(A, B) satisfies the following axioms:

A_1 $0 \le d(A, B) \le 1$

A_2 *d(A, B) = 0 if and only if A=B*

A_3 *d(A, B) = d(B, A)*

A_4 *d(A, C) + d(B, C)* $\ge$ *d(A, B)*

A_5 if $A \subseteq B \subseteq C$, then *d(A, C)* $\ge$ *d(A, B)* and *d(A, C)* $\ge$ *d(B, C)*

Distance measure is a term that describes the difference between intuitionistic fuzzy sets and can be considered a dual concept of similarity measure. We make use of normalised hamming distance proposed in [4] between intuitionistic fuzzy sets, which is partly based on the geometric interpretation of intuitionistic fuzzy sets and have some good geometric properties.

Let $A = \{x, \mu_A(x_i), \upsilon_A(x_i), \pi_A(x_i) / x \in X\}$ and $B = \{x, \mu_B(x_i), \upsilon_B(x_i), \pi_B(x_i) / x \in X\}$

Be two IFSs in $X = \{x_1, x_2, x_3 \ldots x_n\}$ i=1, 2, …n. Based on the geometric interpretation of IFS, Szmidt and Kacprzyk [4, 5] proposed the following distance measures between A and B:

The Hamming distance:

$$d_H(A, B) = \frac{1}{2} \Sigma_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |\upsilon_A(x_i) - \upsilon_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|)$$

The Euclidean distance:

$$d_E(A,B) = \sqrt{\frac{1}{2}\Sigma_{i=1}^{n}[(\mu_A(x_i)-\mu_B(x_i))^2 + (\upsilon_A(x_i)-\upsilon_B(x_i))^2 + (\pi_A(x_i)-\pi_B(x_i))^2]}$$

The normalised Hamming distance:

$$d_{n-H}(A,B) = \frac{1}{2n}\Sigma_{i=1}^{n}(|\mu_A(x_i)-\mu_B(x_i)| + |\upsilon_A(x_i)-\upsilon_B(x_i)| + |\pi_A(x_i)-\pi_B(x_i)|)$$

The normalised Euclidean distance:

$$d_{n-E}(A,B) = \sqrt{\frac{1}{2n}\Sigma_{i=1}^{n}[(\mu_A(x_i)-\mu_B(x_i))^2 + (\upsilon_A(x_i)-\upsilon_B(x_i))^2 + (\pi_A(x_i)-\pi_B(x_i))^2]}$$

An example to show that the normalised Hamming distances gives the best distance measure:

Let us consider the following intuitionistic fuzzy sets A and B in X = {1, 2, 3, 4, 5, 6, 7}

A = (0.5, 0.3, 0.2)/1 + 0.2, 0.6, 0.2)/2 + (0.3, 0.2, 0.5)/4 + (0.2, 0.2, 0.6)/5 + (1, 0, 0)/6,

B = (0.2, 0.6, 0.2)/1 + (0.3, 0.2, 0.5)/4 + (0.5, 0.2, 0.3)/5 + (0.9, 0, 0.1)/7

We use the above distance measures to calculate the distance between A and B.

Hamming distance of A and B i.e $d_H(A, B) = 3$

Euclidean distance of A and B i.e $d_E(A, B) = 1.49$

Normalised Hamming distances of A and B $d_{n-H}(A,B) = 0.43$

Normalised Euclidean distance of A and B $d_{n-E}(A,B) = 0.56$

From these results, we could conclude that the normalised hamming distance gives the best distance measure between A and B. This is because the distance is the shortest or smallest. For this reason, we shall make use of normalised Hamming distance in the applications for its high rate of confidence in terms of accuracy.

Hereditary Property [3]. A fuzzy topological property p is called hereditary (hereditary with respect to closed subspaces, hereditary with respect to open subspaces), iff each subspace (closed subspace, open subspace) of a fuzzy topological space with property p also has property p.

Cardinality[4]. We say that a sets A is of cardinality (or size), and write |A|. We say that a set A has cardinality n, and write |A| = n, if there exists a natural number n such that A={1, 2,..., n}. The cardinality of a finite set is just the number of elements it contains

Souslin Number [1]. The Souslin number c(X) of a topological space X is defined as the least upper bound of cardinalities of disjoint systems of open subsets of the space X. This is a well known Topological invariant.

Souslin number = $\sup\{\tau: \tau = |Y|, Y \subset X, Y - \text{discrete}\}$.
As per the way of R.W. Hansell [19], Souslin sets often inherit topological properties in a strong way. We find the Souslin number and conclude if the patient is inherited with the particular disorder,

3 Model of intuitionistic fuzzy set for genetic disorder using Souslin number

Step 1: Patients name with the symptoms are collected. The symptoms are assigned by a physician.
Step 2: Various distance measures are used to diagnose the disease.
Step 3: Normalised hamming distance gives the best accuracy of all the distance measures.
Step 4: The concept of Souslin number, which is a topological hereditary invariant is calculated to check if the patient is inherited with the diagnosed disorder.

Case Study:
Human reasoning mostly involves the use of variable whose values are uncertain i.e fuzzy in nature. This is the basic concept of linguistic variable, that is, a variable with words values rather than numbers. But in some cases like medical diagnosis, the description by a linguistic variable in terms of membership function alone is not sufficient because there is a chance if the existing of non-membership function. In such a case, IFS is suitable because it uses membership function, non-membership function and the hesitation margin function involved in an uncertain situation.
Let the set of patients be $P = \{P_1, P_2, P_3, P_4\}$; D = {Osteoporosis, Alzheimer's disease, Heart disease, Psoriasis} be the set of diseases and S = {Fatigue, Depression, Muscle cramps, Bone pain} be the set of symptoms. The above mentioned symptoms and diseases arises in the case of Vitamin-D deficiency.
Table 1 is an assumed database of diseases and their symptoms based on medical knowledge in intuitionistic fuzzy nature.

Table 1: Diseases vs Symptoms

	Fatigue	Depression	Muscle cramps	Bone pain
Osteoporosis	(0.5, 0.4, 0.1)	(0.5, 0.3, 0.2)	(0.3, 0.5, 0.2)	(0.2, 0.7, 0.1)
Alzheimer's disease	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.9, 0.0, 0.1)
Heart disease	(0.4, 0.4, 0.2)	(0.2, 0.6, 0.2)	(0.2, 0.7, 0.1)	(0.3, 0.5, 0.2)
Psoriasis	(0.1, 0.8, 0.1)	(0.1, 0.8, 0.1)	(0.8, 0.1, 0.1)	(0.3, 0.6, 0.1)

In table 1, each symptom S_i is described by three numbers i.e., membership μ, non-membership υ and hesitation margin π.
For the diagnosis sake, the samples are taken from patients and analysed. From the analysis, we get the Table 2.

Table 2: Patients vs Symptoms

	Fatigue	Depression	Muscle cramps	Bone pain
P_1	(0.25, 0.5, 0.25)	(0.55, 0.35, 0.1)	(0.15, 0.7, 0.15)	(0.35, 0.45, 0.2)
P_2	(0.75, 0.15, 0.1)	(0.45, 0.5, 0.05)	(0.55, 0.3, 0.15)	(0.65, 0.2, 0.15)
P_3	(0.35, 0.55, 0.1)	(0.25, 0.65, 0.1)	(0.75, 0.05, 0.2)	(0.25, 0.3, 0.45)
P_4	(0.45, 0.5, 0.05)	(0.55, 0.35, 0.1)	(0.15, 0.65, 0.2)	(0.35, 0.4, 0.25)

Using the normalised Hamming distance aforementioned, to calculate the distance between each of the patients in Table 2 and each of the diseases in Table 1.

Table 3: Distance between PATIENTS and DISEASES

	Osteoporosis	Alzheimer's disease	Heart disease	Psoriasis
P_1	8.4×10^{-2}	3.0×10^{-2}	8.6×10^{-2}	3.8×10^{-2}
P_2	7.6×10^{-2}	6.0×10^{-2}	3.6×10^{-2}	6.6×10^{-2}
P_3	3.6×10^{-2}	6.0×10^{-2}	6.4×10^{-2}	8.0×10^{-2}
P_4	8.0×10^{-2}	3.2×10^{-2}	7.0×10^{-2}	4.0×10^{-2}

Table 4: Calculation of Souslin number for each patient

	Osteoporosis	Alzheimer's disease	Heart disease	Psoriasis
P_1	0.084	0.03	0.086	0.038
P_2	0.076	0.060	0.036	0.066
P_3	0.036	0.060	0.064	0.080
P_4	0.080	0.032	0.070	0.040

Hence Souslin number of
Patient P_1= 0.086
Patient P_2= 0.076
Patient P_3= 0.080
Patient P_4= 0.080
i.e., Patient P_1 is inherited with Heart disease,
Patient P_2 is inherited with Osteoporosis
Patient P_3 is inherited with Psoriasis
Patient P_4 is inherited with Osteoporosis.

CONCLUSION:

By employing intuitionistic fuzzy sets in databases we can express a hesitation concerning examined objects. Obviously, the idea of intuitionistic fuzzy sets is of immense significance in decision mathematics because it captures all the possibilities

involve in real life decision problems. The biochemical and genetic in patients has yielded important insights into the structure and function of the receptor in mediating Vitamin-D. Similarly, study of the affected children with vitamin D deficiency continues to provide a more complete understanding of the biological role of Vitamin-D. These studies have been essential to promote the wellbeing of the families with Vitamin-D and in improving the diagnostic and clinical management of this rare genetic disease. This allows us to use flexible ways to simulate real decision situations, thereby building more realistic scenarios describing possible future events. We expect that the method will be improved to be an efficient tool for medical diagnosis and the physician's decision.

REFERENCES:

[1] Adlassnig K. P., 1986, "Fuzzy set theory in medical diagnosis", IEEE Trans. on Systems, Man and Cybernetics, 16, pp. 260-265.

[2] Arhangel'skii, A.V, 1969, "On the cardinality of bicompacta satisfying the first axiom of countability", Soviet Math.Doklady, 10, pp. 951–955.

[3] Arhangel'skii A.V., 1973, " On hereditary properties", General Topology and its applications, 3 pp. 39-46.

[4] Arthur Charlesworth, 1977, "On The Cardinality Of A Topological Space", Proceedings Of The American Mathematical Society, 66, (1).

[5] Atanassov. K, 1986, "Intuitionistic fuzzy sets", Fuzzy Sets and Systems, 20(1), pp. 87-96.

[6] Atanassov. K, 2003, "Intuitionistic fuzzy sets past, present, and future", CLBME-Bulgarian Academy of Science, Sofia.

[7] Atanassova LC., 1995, "Remark on the cardinality of intuitionistic fuzzy sets", Fuzzy Sets and Systems, 75, pp. 399-400.

[8] Bustince. H, Kacprzyk. K, Mohedano.V, 2000, "Intuitionistic fuzzy generators, applications to intuitionistic fuzzy complementation", Fuzzy Sets and Systems, 144, pp. 485-504.

[9] Ejegwa P.A., Akowe S.O., Otene.P.M., Ikyule. J.M., (2001), "An overview on intuitionistic fuzzy sets and systems", 117 (2), pp.209-213.

[10] Ejegwa P.A, Akubo A.J, Joshua O.M, 2014, "Intuitionistic fuzzy set and its application in career determination via normalised Euclidean distance method", European Scientific Journal, 10 (15) pp. s529 – 536.

[11] Eulalia Szmidt and Janusz Kacprzyk, Group decision making via intuitionistic fuzzy sets, FUBEST'96, Sofia, Bulgaria, October 9-11, 1996, pp.107-112

[12] Eulalia Szmidt and Janusz Kacprzyk, On measuring distances between intuitionistic fuzzy sets, Notes on IFS, 3 (4) (1997) 1-3.

[13] Eulalia Szmidt and Janusz Kacprzyk, Distance between intuitionistic fuzzy sets, Fuzzy Sets and Systems, 114 (2000) 505-518.

[14] Eulalia Szmidt and Janusz Kacprzyk, Intuitionistic Fuzzy Sets in some medical applications, NIFS, 7 (2001), 4, 58-64.

[15] Eulalia Szmidt and Janusz Kacprzyk, Medical diagnostic reasoning using a similarity measure for intuitionistic fuzzy sets, Note on IFS, 10 (4) (2004) 61-69.
[16] Eulalia Szmidt, Distances and similarities in intuitionistic fuzzy sets, Springer (2014).
[17] Ghanim M. H., Kerre E. E., Mashhour A. S., 1984, "Separation Axioms, Subspaces and Sums in Fuzzy Topology", Journal of mathematical analysis and applications, 102, pp.189-202.
[18] Gupta. M, Ragade.R.K and Yager R.R., (eds.), 1979, Elsevier Science Ltd.
[19] Hansell R.W., 1986, "Topological Properties of Souslin Subsets", Transactions of the American Mathematical Society, 2, pp. 293.
[20] Hatzimichailidis A.G., Papakostas. G.A., Kaburlasos. V.G., 2012, "A novel distance measures of intuitionistic fuzzy sets and its application to pattern recognition applications", Technological Educational Inst. Of Kavala, Dept. of industrial informatics, 65404, Kavala, Greece.
[21] Li. D, Cheng.C, 2002, "New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions", Pattern Recognition Letters, 23, pp. 221-225.
[22] Mary E. Rudin, 1979, "Hereditary Normality and Souslin Lines", General Topology and its Applications, 10, pp.103-105
[23] Roy A. Johnson, Eliza Wajch, And Wladyslaw Wilczyñski, 1990, Three Cardinal Functions Similar To Net Weight, Proceedings Of The American Mathematical Society, 1, pp. 109.
[24] Sanchez.E, "Medical diagnosis and composite fuzzy relations", Advanvces in Fuzzy Set Theory and Applications
[25] Shapirovskii. B.E., 1994, "The Souslin number in set-theoretic topology", Topology and its Applications, 57, pp.131-150.
[26] Turnali. N and Coker.D, 2001, "Fuzzy connectedness in intuitionistic fuzzy topological spaces", Fuzzy Sets and Systems, 114, pp.505-518.
[27] Wang.W, Xin.X, 2005, "Distance measure between intuitionistic fuzzy sets", Pattern Recognition Letters, 26, pp. 2063-2069.
[28] Ye.J, 2011, "Cosine similarity measures for intuitionistic fuzzy sets and their applications", Mathematics and Computer Modelling, 53, pp. 91-97.
[29] Zadeh L.A., 1965, "Fuzzy sets", Information and Control, 8, pp. 338-353.
[30] Zadeh L. A., 1969, "Biological applications of the theory of fuzzy sets and systems", Proc. Of an International Symposium on Biocybernetics of the Central Nervous System, pp.199-206.
[31] Zeng. W, Li.H, 2006, Note on "Some operations on intuitionistic fuzzy sets", Fuzzy Sets and Systems, 157, pp. 990-991.