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Smarandache Q-fuzzy semigroups

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Abstract

In this paper, we introduce the concept of Smarandache Q-fuzzy semigroups, Smarandache Q-fuzzy normal semigroups and investigate some of their properties.

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1. Introduction

In 1965, L. A. Zadeh [11] Mathematically formulated the concept of fuzzy subset. He defined fuzzy subset of a nonempty set as a collection of objects with grade of membership value assigned between 0 and 1 by a membership function. In 1971, A. Rosenfeld [7] introduced the notion of fuzzy group. Padilla Raul [5] introduced the notion of Smarandache semigroup in the year 1998. Smarandache fuzzy semigroups were studied in 2003 by W. B. Vasantha kandasamy [9,10]. In 2008, A. Solairaju and R. Nagarajan [8] introduced a new algebraic structure namely Q-fuzzy groups. T. Priya and et al. [4] introduced the concept of Q-fuzzy normal subgroups in 2013. In this paper we introduce Smarandache Q-fuzzy semigroups, Smarandache Q-fuzzy normal semigroups and study some of their properties.

2. Preliminaries

Definition 2.1. Let X be a non empty set. A *fuzzy subset* μ of the set X is a function $\mu : X \to [0, 1]$.

Definition 2.2. Let H be a Semigroup. H is said to be *Smarandache semigroup (S-semigroup)* if H has a proper subset G such that G is a Group under the operation of H.

Definition 2.3. Let X and Q be non empty sets. A *Q-fuzzy subset* A of X is a function $A : X \times Q \rightarrow [0, 1]$.

Definition 2.4. Let G be a group and Q be any non empty set. A Q-fuzzy subset A of G is said to be *Q-fuzzy group* of G if

(i) $A(xy, q) \geq min\{A(x, q), A(y, q)\}$

(ii) $A(x^{-1}, q) = A(x, q)$, for all $x, y \in G$ and $q \in Q$.

3. Smarandache Q-fuzzy semigroups

Definition 3.1. Let H be an S-semigroup. A Q-fuzzy subset A of H is said to be a *Smarandache Q-fuzzy semigroup* if $A : H \times Q \rightarrow [0, 1]$ is such that A is restricted to at least one proper subset G of H which is a group and the restriction map $A_G : G \times Q \rightarrow [0, 1]$ is a Q-fuzzy group, that is for all $x, y \in G$ and $q \in Q$, $A_G(xy, q) \geq \min\{A_G(x, q), A_G(y, q)\}$ and $A_G(x, q)$=$A_G(x^{-1}, q)$.

Example 3.2. Consider an S-semigroup Z_8 under multiplication modulo 8.
Let Q={1}.
Let $A : Z_8 \times Q \rightarrow [0, 1]$ be defined by,

$$A(x) = \begin{cases} 1 & \text{if } x\text{=(0,1)} \\ 0.7 & \text{if } x\text{=(2,1),(4,1),(6,1)} \\ 0.3 & \text{if } x\text{=(1,1),(3,1),(5,1),(7,1)} \end{cases}$$

Clearly A is a Q-fuzzy subset of Z_8. Consider $P = \{1, 3\}$ which is a proper subset of Z_8 and is also a group in Z_8 under the operation of Z_8. It can be easily verified that $A_P : P \times Q \rightarrow [0, 1]$ is a Q-fuzzy group. Therefore A is a Smarandache Q-fuzzy semigroup.

Remark 3.3. Throughout this paper we mention Smarandache Q-Fuzzy Semigroup as S-QFS.

Proposition 3.4. If A is an S-QFS of an S-semigroup H relative to a group G and Q is a non empty set, then

(i) $A_G(e, q) \geq A_G(x, q)$, where *e* is the identity element of G.

(ii) $A_G(x^{-1}, q) \geq A(x, q)$ for all $x \in G$ and $q \in Q$.

Proof. Let A be an S-QFS of an S-semigroup H. Then A is restricted to at least one proper subset G of H which is a group and $A_G : G \times Q \to [0, 1]$ is a Q-fuzzy group. Therefore $A_G(x, q) = A(x, q), \forall x \in G$ and $q \in Q$
(i) Let $e \in G$, where e is the identity element of G. Now,

$$\begin{aligned} A_G(e, q) &= A_G(xx^{-1}, q) \\ &\geq \min\{A_G(x, q), A_G(x^{-1}, q)\} \\ &= \min\{A_G(x, q), A_G(x, q)\} \\ &= A_G(x, q) \end{aligned}$$

Therefore $A_G(e, q) \geq A_G(x, q)$.

$$\begin{aligned} \text{(ii) } A_G(x^{-1}, q) &= A_G(ex^{-1}, q) \\ &\geq \min\{A_G(e, q), A_G(x^{-1}, q)\} \\ &= \min\{A_G(e, q), A_G(x, q)\} \\ &\geq A_G(x, q)(\text{ by (i)}) \\ A_G(x^{-1}, q) &\geq A_G(x, q) \end{aligned}$$

Therefore $A_G(x^{-1}, q) \geq A_G(x, q)$. ■

Theorem 3.5. If A is an S-QFS of an S-semigroup H and Q is a non empty set, then $A_G(xy^{-1}, q) = A_G(e, q) \Longrightarrow A_G(x, q) = A_G(y, q)$, for all $x, y \in G \subset S$, where G is a group and A_G is the restriction of A, $q \in Q$ and e is the identity element of G.

Proof. Let A be an S-QFS of an S-semigroup H and let Q be a non empty set. Then A is restricted to at least one proper subset G of H which is a group and $A_G : G \times Q \to [0, 1]$ is a Q-fuzzy group. Therefore $A_G(x, q) = A(x, q), \forall x \in G$ and $q \in Q$.
Let $x, y \in G$ and $q \in Q$. Assume that $A_G(xy^{-1}, q) = A_G(e, q)$.

$$\begin{aligned} A_G(x, q) &= A_G(xy^{-1}y, q) \\ &\geq \min\{A_G(xy^{-1}, q), A_G(y, q)\} \\ &= \min\{A_G(e, q), A_G(y, q)\} \\ &= A_G(y, q)(\text{ by Proposition 3.4(i)}) \\ A_G(x, q) &\geq A_G(y, q) \end{aligned}$$

Now,

$$
\begin{aligned}
A_G(y,q) &= A_G(yx^{-1}x,q)\\
&\geq \min\{A_G(yx^{-1},q), A_G(x,q)\}\\
&= \min\{A_G((yx^{-1})^{-1},q), A_G(x,q)\}\\
&= \min\{A_G(xy^{-1},q), A_G(x,q)\}\\
&= \min\{A_G(e,q), A_G(x,q)\}\\
&= A_G(x,q)\\
A_G(y,q) &\geq A_G(x,q)\\
\text{Therefore } A_G(x,q) &= A_G(y,q) \text{for all } x, y \in G \text{ and } q \in \mathrm{Q}
\end{aligned}
$$

■

Theorem 3.6. Let H be an S-semigroup, Q any nonempty set and let G be a proper subset of H which is a group in H. $\mu : H \times Q \to [0,1]$ is an S-QFS relative to G if and only if

$$\mu_G(xy^{-1},q) \geq \min\{\mu_G(x,q), \mu_G(y,q)\},$$

for all $x, y \in G$ and $q \in Q$.

Proof. Assume that $\mu : H \times Q \to [0,1]$ is an S-QFS relative to G. Then μ is restricted to G and $\mu_G : G \times Q \to [0,1]$ is a Q-fuzzy group. Then $\mu_G(x,q) = \mu(x,q)$, $\forall x \in G$ and $q \in Q$.
Let $x, y^{-1} \in G$ and $q \in Q$.

$$
\begin{aligned}
\text{Then } \mu_G(xy^{-1},q) &\geq \min\{\mu_G(x,q), \mu_G(y^{-1},q)\}\\
&= \min\{\mu_G(x,q), \mu_G(y,q)\}
\end{aligned}
$$

Therefore,

$$\mu_G(xy^{-1},q) \geq \min\{\mu_G(x,q), \mu_G(y,q)\}$$

Conversely, assume that

$$\mu_G(xy^{-1},q) \geq \min\{\mu_G(x,q), \mu_G(y,q)\} \quad \text{(i)}$$
$$\text{for all } x, y \in G \text{ and } q \in Q$$

Put $y = x$ in (i). Then

$$
\begin{aligned}
\mu_G(xx^{-1},q) &\geq \min\{\mu_G(x,q), \mu_G(x,q)\}\\
\mu_G(e,q) &\geq \mu_G(x,q) \quad \text{(ii)}
\end{aligned}
$$

iii) Now,

$$
\begin{aligned}
\mu_G(y^{-1},q) &= \mu_G(ey^{-1},q)\\
&\geq \min\{\mu_G(e,q), \mu_G(y,q)\} \text{ (by (i))}\\
\mu_G(y^{-1},q) &\geq \mu_G(y,q) \text{ (by(ii))}
\end{aligned}
$$

Also,

$$\begin{aligned}\mu_G(y,q) &= \mu_G(e(y^{-1})^{-1},q)\\ &\geq \min\{\mu_G(e,q),\mu_G(y^{-1},q)\} \text{ (by (i))}\\ \mu_G(y,q) &\geq \mu_G(y^{-1},q) \text{ (by (ii))}\end{aligned}$$

Therefore $\mu_G(y,q) = \mu_G(y^{-1},q)$
iv)

$$\begin{aligned}\mu_G(xy,q) &= \mu_G(xy)^{-1},q)\\ &= \mu_G(y^{-1}x^{-1},q)\\ &\geq \min\{\mu_G(y^{-1},q),\mu_G(x^{-1},q)\}\\ &\geq \min\{\mu_G(y,q),\mu_G(x,q)\} \text{ (by (i))}\\ \mu_G(xy,q) &\geq \min\{\mu_G(x,q),\mu_G(y,q)\}\end{aligned}$$

From (iii) and (iv) μ is an S-QFS relative to a group G. ■

Theorem 3.7. Let A be an S-QFS of an S-semigroup H relative to a group G. If $A_G(xy^{-1},q) = 1$, then $A_G(x,q) = A_G(y,q)$ for every $x, y \in G \subset S$ and $q \in Q$.

Proof. Let A be an S-QFS of an S-semigroup H relative to a group G. Then $A_G : G \times Q \to [0,1]$ is a Q-fuzzy group. Therefore $A_G(x,q) = A(x,q)$, $\forall x$ in G and q in Q. Let $x, y \in G$ and $q \in$ Q. Assume that $A_G(xy^{-1},q) = 1$

$$\begin{aligned}A_G(x,q) &= A_G(xy^{-1}y,q)\\ &\geq \min\{A_G(xy^{-1},q),A_G(y,q)\}\\ &= \min\{1,A_G(y,q)\}\\ &= A_G(y,q)\\ A_G(x,q) &\geq A_G(y,q)\end{aligned}$$

Now,

$$\begin{aligned}A_G(y,q) &= A_G(y^{-1},q)\\ &= A_G(x^{-1}xy^{-1},q)\\ &\geq \min\{A_G(x^{-1},q),A_G(xy^{-1},q)\}\\ &= \min\{A_G(x^{-1},q),1\}\\ &= A_G(x^{-1},q)\\ &= A_G(x,q)\\ A_G(y,q) &\geq A_G(x,q)\end{aligned}$$

Therefore $A_G(x, q) = A_G(y, q)$ for all $x, y \in G$ and $q \in Q$. ■

Definition 3.8. Let X and Q be any non empty sets. The *intersection of two Q-fuzzy subsets* A and B of X is defined as $(A \cap B)(x, q) = \min\{A(x, q), B(x, q)\}$, for all $x \in X$ and $q \in Q$.

Theorem 3.9. The intersection of two S-QFS of an S-semigroup H relative to a group G is also an S-QFS of H.

Proof. Let A and B be any two S-QFS of an S-semigroup H relative to a group G. Then $A_G : G \times Q \to [0, 1]$ and $B_G : G \times Q \to [0, 1]$ are Q-fuzzy groups. Let $x, y \in G$ and $q \in Q$.

$$\begin{aligned}
\text{(i) } (A_G \cap B_G)(xy, q) &= \min\{A_G(xy, q), B_G(xy, q)\} \\
&\geq \min\{\min\{A_G(x, q), A_G(y, q)\}, \min\{B_G(x, q), B_G(y, q)\}\} \\
&= \min\{\min\{A_G(x, q), B_G(x, q)\}, \min\{A_G(y, q), B_G(y, q)\}\} \\
&= \min\{(A_G \cap B_G)(x, q), (A_G \cap B_G)(y, q)\} \\
(A_G \cap B_G)(xy, q) &\geq \min\{(A_G \cap B_G)(x, q), (A_G \cap B_G)(y, q)\}
\end{aligned}$$

$$\begin{aligned}
\text{(ii) } (A_G \cap B_G)(x^{-1}, q) &= \min\{A_G(x^{-1}, q), B_G(x^{-1}, q)\} \\
&= \min\{(A_G(x, q), B_G(x, q)\} \\
(A_G \cap B_G)(x^{-1}, q) &= (A_G \cap B_G)(x, q)
\end{aligned}$$

From (i) and (ii), the intersection of two S-QFS is also an S-QFS of H relative a group G. ■

Definition 3.10. Let X and Q be any non empty sets. The *complement of a Q-fuzzy subset* A of X is denoted by A^c and is defined as $A^c(x, q) = 1 - A(x, q)$, for $x \in X$ and $q \in Q$.

Theorem 3.11. If A is an S-QFS of an S-semigroup H relative to a group G, then A^c is also an S-QFS of H.

Proof. Let A be an S-QFS of an S-semigroup H relative to a group G. Then the restriction map $A_G : G \times Q \to [0, 1]$ is a Q-fuzzy group. Therefore $A_G(x, q) = A(x, q)$, $\forall x \in G$ and $q \in Q$. For any $x, y \in G, q \in Q$,
(i)

$$\begin{aligned}
A^c_G(xy, q) &= 1 - A_G(xy, q) \\
&\leq 1 - \min\{A_G(x, q), A_G(y, q)\} \\
&= \max\{1 - A_G(x, q), 1 - A_G(y, q)\} \\
A^c_G(xy, q) &\leq \max\{A^c_G(x, q), A^c_G(y, q)\}
\end{aligned}$$

(ii)

$$\begin{aligned} A_G^c(x^{-1}, q) &= 1 - A_G(x^{-1}, q) \\ &= 1 - A_G(x, q) \\ &= A_G^c(x, q) \end{aligned}$$

From (i) and (ii), A_G^c is an S-QFS of H. ■

4. Smarandache Q-fuzzy normal semigroups

Definition 4.1. Let A be an S-QFS of an S-semigroup H relative to a group G. Then A is called *Smarandache Q-fuzzy normal semigroup(S-QFNS)* if $A_G(xy, q) = A_G(yx, q)$, that is $A_G(x, q) = A_G(yxy^{-1}, q)$ for all $x, y \in G \subset H$ and $q \in Q$.

Theorem 4.2. Let A be an S-QFNS of an S-semigroup H relative to a group G. Then for all $x, y \in G$, $A_G(x^{-1}y^{-1}xy, q) = A_G(e, q)$, where e is the identity element of G.

Proof. Let A be an S-QFNS of an S-semigroup H relative to a group G. Since A is an S-QFNS of H relative to G, we have $A_G(x, q) = A_G(yxy^{-1}, q)$, for all $x, y \in G$, $q \in Q$. Replacing x by y^{-1} and y by x^{-1}, we get

$$\begin{aligned} A_G(y^{-1}, q) &= A_G(x^{-1}y^{-1}x, q) \\ A_G(y^{-1}, q) &= A_G(x^{-1}y^{-1}xe, q) \\ A_G(y^{-1}, q) &= A_G(x^{-1}y^{-1}xyy^{-1}, q) \\ A_G(e, q) &= A_G(x^{-1}y^{-1}xy, q). \end{aligned}$$

■

Theorem 4.3. Let μ be an S-QFNS of an S-semigroup H relative to a group G. Then for any $x, y \in G$ and $q \in Q$, $\mu_G(yxy^{-1}, q) = \mu_G(y^{-1}xy, q)$.

Proof. Let μ be an S-QFNS of an S-semigroup H relative to a group G. Then $\mu_G : G \times Q \to [0, 1]$ is a Q-fuzzy group. For any $x, y \in G$ and $q \in Q$, we have

$$\begin{aligned} \mu_G(yxy^{-1}, q) &= \mu_G(x, q) \\ &= \mu_G(xyy^{-1}, q) \\ &= \mu_G(y^{-1}xy, q) \end{aligned}$$

Therefore $\mu_G(yxy^{-1}, q) = \mu_G(y^{-1}xy, q)$. ■

Theorem 4.4. The intersection of two S-QFNS of an S-semigroup H relative to a group G is also an S-QFNS of H.

Proof. Let λ and μ be two S-QFNS of H relative to a same group G in S-semigroup H. Then the restriction maps $\lambda_G : G \times Q \to [0, 1]$ and $\mu_G : G \times Q \to [0, 1]$ are Q-fuzzy groups. By Theorem 3.9, $\lambda_G \cap \mu_G$ is an S-QFS of H relative to G. For all $x, y \in G$, we have

$$\begin{aligned}(\lambda_G \cap \mu_G)(xyx^{-1}, q) &= \min\{\lambda_G(xyx^{-1}, q), \mu_G(xyx^{-1}, q)\} \\ &= \min\{\lambda_G(y, q), \mu_G(y, q)\} \\ &= (\lambda_G \cap \mu_G)(y, q)\end{aligned}$$

Therefore

$$(\lambda_G \cap \mu_G)(xyx^{-1}, q) = (\lambda_G \cap \mu_G)(y, q).$$

Hence $(\lambda_G \cap \mu_G)$ is an S-QFNS of H relative to G. ■

Remark 4.5. In this paper the Cartesian Product of two sets G_1 and G_2 is defined as

$$(a_1, b_1).(a_2, b_2) = (a_1a_2, b_1b_2),$$

$$\forall (a_1, b_1), (a_2, b_2) \in G_1 \times G_2.$$

Theorem 4.6. Let A and B be any two S-QFNS of an S-semigroup of H relative to the groups R and S respectively. Then $A_R \times B_S$ is an S-QFNS relative to $R \times S$.

Proof. Let A and B be any two S-QFNS of an S-semigroup H relative to the groups R and S respectively. Since A and B are S-QFS relative to the groups R and S respectively, it is clear that $A_R \times B_S$ is an S-QFS relative to $R \times S$. Let $x_1, x_2 \in R$, $y_1, y_2 \in S$ & $q \in Q$. Then (x_1, y_1) and (x_2, y_2) are in $R \times S$. Now,

$$\begin{aligned}(A_R \times B_S)((x_1, y_1)(x_2, y_2), q) &= (A_R \times B_S)((x_1x_2, y_1y_2), q) \\ &= \min\{A_R(x_1x_2, q), B_S(y_1y_2, q)\} \\ &= \min\{A_R(x_2x_1, q), B_S(y_2y_1, q)\} \\ &= (A_R \times B_S)((x_2x_1, y_2y_1), q) \\ &= (A_R \times B_S)((x_2, y_2)(x_1y_1), q)\end{aligned}$$

Therefore

$$(A_R \times B_S)((x_1, y_1)(x_2, y_2), q) = (A_R \times B_S)((x_2, y_2)(x_1y_1), q)$$

Hence $A_R \times B_S$ is an S-QFNS of $R \times S$. ■

5. Conclusion

In this paper, we have made an attempt to study the results on Smarandache Q-fuzzy semigroups and Smarandache Q-fuzzy normal semigroups with their properties.

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