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Q- Intuitionistic fuzzy ordered filters in ordered ternary Γ – semirings

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I. INTRODUCTION

The notion of semiring was introduced by H.S.Vandiver [4] in 1934. The notion of Γ - semirings introduced by N.Nobusawa [3] as a generalization of ring in 1964 M.Murali Krishna Rao [1] introduced the nation of the Γ - semiring which is a generalization of ring. M.murali Krishna rao and B.Venkateswarlu [1] introduced the nation of fuzzy prime ideals and fuzzy filters in gamma semiring and studied their properties. In this paper, we introduce the nation of Q-intutionistic fuzzy prime ideals and Q-Interilionistic fuzzy filters in ordered ternary Γ - semirings.

Definition

A set R together with two associative binary operators called addition and multiplication (defined by + and . respectively) will be called semiring provided.

i. Addition is commutative operation.
ii. Multiplication distributes over addition both from the left and from the right
iii. There exists $0 \in R$ such that $0+y = y$ and $y.0 = 0.y=0$ for every $y \in R$.

Definition

A non – empty set S together with a binary operation called addition and a ternary multiplication denoted by just a position is said to be a ternary semiring if S is additive commutative semigroup satisfying the following conditions.

i. (abc)de= a(bcd)e= ab(cde)
ii. (a+b)cd = acd+bcd
iii. a(b+c)d= abd+acd
iv. ab(c+d) = abc + ade for all a,b,c,d,e, ∈ S

Definition

Let R and Γ be two additive commutative semigroups. R is said to be two additive commutative semigroups. R is said to be ternary Γ- semiring if there exists a mapping from $R\times \Gamma \times R\times \Gamma\times R \rightarrow R$ written as $(x_1, \alpha, x_2, \beta, x_3) \rightarrow [x_1 \alpha, x_2 \beta x_3]$

Satisfying the following conditions.

i. $[\,[a\alpha b\beta c]\,\gamma d\delta e] = [a\alpha[b\beta c\gamma d]\delta e] = [a\alpha b\beta[c\gamma d\delta e]\,]$
ii. $[\,(a+b)\,\alpha c\beta d] = [\,a\alpha c\beta d] +[\,b\,\alpha c\beta d]$
iii. $[a\alpha(b+c)\beta d] = [a\alpha b\beta d] + [\,a\alpha c\beta d]$
iv. $[a\alpha b\beta(c+d)] = [a\alpha b\beta c]+ [a\alpha b\beta d]$

For all a, b,c,d∈R and α,β,γ,δ ∈ Γ

Definition

A ternary Γ-semiring R is said to be an ordered ternary Γ – semiring if there is a partial ordered ≤ on R such that $a \le b$

Implies that

1. $a+c \le b+c$ and $c+a \le c+b$.
2. $[a\alpha c\beta d] \le [b\alpha c\beta d]$, $[c\alpha a\beta d] \le [c\alpha b\beta d]$ and $[c\alpha d\beta a] \le [c\alpha d\beta b]$ for all a,b,c,d∈R and α,β∈ Γ Throughout this paper we denote aαcβd instead of [aαcβd].

Definition

Let R be an ordered ternary Γ-semiring. A non-empty subset E is said to be ordered ternary Γ – subsemiring of R if

i. E is an additive subsemigroup of R [(ie) E +E ⊆ E]
ii. E Γ E Γ E ⊆ S
iii. (E] ⊆ E

Note :

Let R be an ordered ternary Γ – semiring . Let S be non-empty subsit of R

(S] = { t∈ R / t ≤ S for some s ∈ S}

[S)={t∈ R / S ≥ t for some s ∈ S}

Definition

A non – empty subset G of an ordered ternary Γ – semiring R is said to be an ordered left ideal of R.

i. For any a,b, ∈ R, a∈ G implies a+b ∈ G
ii. For any b,c ∈ R, a ∈ G, α,β ∈ Γ implies bαcβa ∈ G.
iii. (G] ⊆ G

A non – empty subset G of an ordered ternary Γ – semiring R is said to be a ordered lateral ideal of R.

i. For any a,b ∈ G implies a+b ∈ G
ii. For any b,c ∈R, a∈G, α,β ∈ Γ implies bαaβc ∈ G.
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A non – empty subset G of an ordered ternary Γ – semiring R is said to be an ordered right ideal of R.

i. For any a,b∈G implies a+b∈ G.
ii. For any b,c∈ R, a∈ G, α,β∈ Γ, aαbβc∈ G
iii. (G] ⊆ G

A non empty subset G is of an ordered ternary (Γ-semiring R is said to be an ordered ideal if G is ordered right ideal ordered left ideal and ordered lateral ideal of R.

Definition

Let R be an ordered ternary Γ - semiring.

An ordered Γ subsemiring A of R is called an ordered right filter of R.

i. For any a,b,c∈ R and α,β∈ Γ, aαbβc∈ A implies a∈A

ii. For all a,bc∈ R , a ≤ b and a∈ A and imply b∈A

Let R be an ordered ternary Γ – semiring. An ordered Γ – subsemiring A of R is called an ordered left filter of R.

i. For any α,b,c ∈ R and α,β∈ Γ, bαaβc∈ A imply a∈ A
ii. For all a,b∈ R, a≤b and a∈ A imply b ∈ A.

Let R be an ordered ternary Γ –semiring. An ordered Γ – subsemiring A of R is called an ordered lateral filter of R.

1. For any a,b,c∈R, and αa β ∈ Γ, ba βc∈A imply a∈A
2. For all a,b∈ R, a≤b and a∈ A imply b∈A.

A said to be ordered filter of R if it is an ordered left filter, ordered right filter and ordered lateral filter of R.

Definition

Let R be an ordered ternary Γ- semiring. An ordered ideal A of R is called an ordered prime ideal of R if for any x,y,z ∈ R , α,β ∈ Γ, and xαyβz implies x ∈ A or y ∈ A or z ∈ A.

Definition

Let Q and R be a set and group respectively. A mapping μ:R×Q→[0,1] is called Q-fuzzy set in R.

Definition

Let R be an ordered ternary Γ Semiring. An Q-interitiencitic fuzzy set A=< μ_A, ν_A> in R is called Q-interitionistic fuzzy ordered ternary Γ-subsemiring of R if

1. $\mu_A(x+y,q) \geq \min\{ \mu_A(x,q) , \mu_A(y,q)\}$.
2. $\mu_A(x\alpha y\beta z,q) \geq \min\{ \mu_A(x,q) ,\mu_A(y,q), \mu_A(z,q)\}$
3. x≤y implies $\mu_A(x,q) \geq \mu_A(y,q)$ and $\nu_A (x,q) \leq \nu_A (y,q)$
4. $\nu_A(x+y,q) \leq \max\{ \nu_A(x,q), \nu_A(y,q)\}$
5. $\nu_A(x\alpha y\beta z,q) \leq \max\{ \nu_A(x,q), \nu_A(y,q) ,\nu_A(z,q)\}$

for all x,y,z∈R, α,β, ∈ Γ and q∈Q.

Definition

Let R be an ordered ternary Γ-semiring. An Q-interitionistic fuzzy set A= < μ_A, ν_A> in R is called Q-interitionistic fuzzy ordered ideal of R if

1. $\mu_A (x+ y,q) \geq \min\{ \mu_A(x,q), \mu_A(y,q)\}$.

2. μ_A (x α yβz,q)≥ max { μ_A(x,q), μ_A(y,q) μ_A(z,q) }.
3. x≤y implies μ_A(x,q)≥ μ_A(y,q)and ν_A(x,y)≤ ν_A(y,q).
4. ν_A (x+ y,q)≤ max { ν_A (x,q), ν_A (y,q)}
5. ν_A (x α yβz,q)≤ min { ν_A (x,q), ν_A (y,q) ν_A (z,q) } for all x,y,z∈R, αβ∈ Γ and q∈ Q.

Definition

Let R be an ordered ternary Γ – semiring. A. Q – interitionestic fuzzy ordered ternary Γ – subsemiring A=< μ_A ,ν_A> is R is called Q-interitionistic fuzzy ordered filter of R if

1. μ_A (x α yβz,q)= min { μ_A(x,q), μ_A(y,q) μ_A(z,q) }.
2. ν_A(x α yβz,q)= max { ν_A(x,q), ν_A(y,q), ν_A(z,q)}.
3. x≤y, implies μ_A(x,q)≤ μ_A(y,q) and ν_A(x,q)≤, ν_A(y,q)

Definition

A Proper Q - intuitionistic fuzzy order ideal of R is called Q - intuitionistic fuzzy order ideal of R.

μ_A (xαyβz,q) = max { μ_A(x,q), μ_A(y,q) μ_A(z,q) }

ν_A (xαyβz,q)= min{ ν_A(x,q), ν_A(y,q), ν_A(z,q)}

for all x,y,z ∈ R , α,β ∈ Γ, and q ∈ Q.

Theorem

A=< μA ,νA> is R is called Q-intuitionistic fuzzy ordered filter of on ordered ternary Γ – semiring R iff it is non - empty level subset A<t,s> = {x ∈ A/ μA(x,q)≥t and νA(x,q)≤s } for any t,s ∈ [0,1] and t+s≤1 is an ordered filter of R.

Proof

Suppose A is a Q –intuitionistic fuzzy ordered ternery Γ- semiring R.

Let t,s ∈[0,1] and t+s ≤ 1 such that $A_{<t,s>}$ is a ternery Γ-sub semiring of R.

Let x,y,z ∈ R , α,β ∈ Γ, q ∈ Q and xαyβz∈$A_{<t,s>}$ then

μ_A (xαyβz,q)≥t and ν_A (xαyβz,q)≤s

⇨ min { μ_A(x,q), μ_A(y,q) μ_A(z,q) }≥ t and

max { ν_A(x,q), ν_A(y,q), ν_A(z,q)}≤ s.

⇨ μ_A(x,q), μ_A(y,q) μ_A(z,q)≥ t and

$\nu_A(x,q), \nu_A(y,q), \nu_A(z,q) \leq s.$

⇨ $x,y,z \in A_{<t,s>}$

Hence $A_{<t,s>}$ is ordered filter of ordered ternery Γ- semiring

Conversely suppose that its level subset $A_{<t,s>}$ for any $t \in [0,1]$ is an ordered filter of R

Let $x,y,z \in R$, $\alpha,\beta \in \Gamma$, $q \in Q$

suppose $t = \min\{ \mu_A(x,q), \mu_A(y,q)\ \mu_A(z,q) \}$ and

$s = \max\{ \nu_A(x,q), \nu_A(y,q), \nu_A(z,q)\}$.

⇨ $\mu_A(x,q) \geq t$, $\mu_A(y,q) \geq t$ $\mu_A(z,q) \geq t$ and

$\nu_A(x,q) \leq s$, $\nu_A(y,q) \leq s$, $\nu_A(z,q) \leq s$.

Therefore $x,y,z \in A_{<t,s>}$

⇨ $\mu_A(x+y,q) \geq t$ $\mu_A(x\alpha y\beta z,q) \geq t$ and

$\nu_A(x+y,q) \leq s$ $\nu_A(x\alpha y\beta z,q) \leq s$

$\mu_A(x+y,q) \geq t = \min\{ \mu_A(x,q), \mu_A(y,q) \}$

and $\nu_A(x+y,q) \leq s = \max\{ \nu_A(x,q), \nu_A(y,q), \}$

Also $\mu_A(x\alpha y\beta z,q) \geq t = \min\{ \mu_A(x,q), \mu_A(y,q)\ \mu_A(z,q) \}$

and $\nu_A(x\alpha y\beta z,q) \leq s = \max\{ \nu_A(x,q), \nu_A(y,q), \nu_A(z,q)\}$.

Let $x, \leq y$ and $\mu_A(y,q) = t_1$ and $\nu_A(y,q) = s_1$

Then $y \in A_{<t1,s1>}$. This implies $x \in A_{<t,s>}$

Then $\mu_A(x,q) \geq t = \mu_A(y,q)$ and $\nu_A(x,q) \leq s = \nu_A(y,q)$

Hence A is a Q –intuitionistic fuzzy ordered ternery Γ- subsemiring R.

Let $x,y \in R$, $\alpha,\beta \in \Gamma$, $q \in Q$ and $x\alpha y\beta z \in A_{<t,s>}$ then

$\mu_A(x\,\alpha\,y\beta z,q) = t$ and $\nu_A(x\,\alpha\,y\beta z,q) = s$

Then $x\alpha y\beta z \in A_{<t,s>}$

⇨ $X,y,z \in A_{<t,s>}$

⇨ $\mu_A(x,q), \mu_A(y,q)\ \mu_A(z,q) \geq t$ and

$\nu_A(x,q), \nu_A(y,q), \nu_A(z,q) \leq s$

⇨ $\min\{ \mu_A(x,q), \mu_A(y,q)\ \mu_A(z,q) \} \geq t$

$\max\{ \nu_A(x,q), \nu_A(y,q), \nu_A(z,q)\} \leq s$

⇨ $\min\{ \mu_A(x,q), \mu_A(y,q)\ \mu_A(z,q) \} \geq \mu_A(x\,\alpha\,y\beta z,q)$

$\max\{ \nu_A(x,q), \nu_A(y,q), \nu_A(z,q)\} \leq \nu_A(x\,\alpha\,y\beta z,q)$

since A is a Q –intuitionistic fuzzy ordered ternary Γ- subsemiring of R , $\mu_A(x\,\alpha\,y\beta z,q) = \min\{ \mu_A(x,q), \mu_A(y,q)\ \mu_A(z,q) \}$.

$\nu_A(x\,\alpha\,y\beta z,q) = \max\{ \nu_A(x,q), \nu_A(y,q), \nu_A(z,q)\}$.

Therefore $A_{<t,s>}$ is a Q –intuitionistic fuzzy ordered filter of an ordered Γ- semiring of R.

Theorem

A is a Q –intuitionistic fuzzy prime ordered ideal of an ordered ternary Γ- semiring iff for any t ∈[0,1] such that $A_{<t,s>}$ is an ordered prime ideal of R .

Proof:

Suppose A is an Q –intuitionistic fuzzy ordered prime ideal of an ordered ternary Γ-semiring of R.

Let t,s ∈[0,1] such that $A_{<t,s>}$ is an proper ordered ideal of an ordered ternary Γ-semiring of R.

Let X,y,z $\in A_{<t,s>}$

Then $\mu_A(x,q)$, $\mu_A(y,q)$ $\mu_A(z,q) \geq t$ and

$\nu_A(x,q)$, $\nu_A(y,q)$, $\nu_A(z,q) \leq s$.

This implies min { $\mu_A(x,q)$, $\mu_A(y,q)$ $\mu_A(z,q)$ }$\geq$ t and

max { $\nu_A(x,q)$, $\nu_A(y,q)$, $\nu_A(z,q)$}$\leq$ s for all q ∈ Q.

Also

μ_A (x+y,q) ≥ min { $\mu_A(x,q)$, $\mu_A(y,q)$ $\mu_A(z,q)$ }≥ t and

$\nu_A \leq$ (x+y,q) max { $\nu_A(x,q)$, $\nu_A(y,q)$, $\nu_A(z,q)$}≤ s for all q ∈ Q.

Thus x+y$\in A_{<t,s>}$

Clearly x,y,z $\in A_{<t,s>}$ implies xαyβz, $\in A_{<t,s>}$ for all α, β ∈ Γ.

Suppose x $\in A_{<t,s>}$ and z,y $\notin A_{<t,s>}$ for α, β ∈ Γ.

Therefore μ_A (x,y) ≥ t μ_A (y,q)<t and μ_A (z,q)< t and

ν_A (x,y) ≤ s ν_A (y,q)>s and ν_A (z,q)>s for all q ∈ Q.

This implies that μ_A (x,y) ≥ t >μ_A (y,q) and μ_A (x,y) ≥ t >μ_A (z,q)

Also ν_A (x,y) ≤ s < ν_A (y,q) and ν_A (x,y) ≤ s < ν_A (z,q)

So μ_A (xαyβz,q)= max { $\mu_A(x,q)$, $\mu_A(y,q)$ $\mu_A(z,q)$ }≥ t

and ν_A (xαyβz,q)= min{ $\nu_A(x,q)$, $\nu_A(y,q)$, $\nu_A(z,q)$}≤ s

This implies that xαyβz $\in A_{<t,s>}$.

Let y$\in A_{<t,s>}$, x ∈ R and x ≤ y then $\mu_A(y,q) \geq t$ and $\nu_A(y,q) \leq s$

Then we have x$\in A_{<t,s>}$

Hence $A_{<t,s>}$ is an ordered ideal of an ordered ternary Γ- semiring of R.

Let x,y,z ∈ R, α,β ∈ Γ xαyβz ∈ $A_{<t,s>}$ and q ∈ Q.

Then μ_A (xαyβz,q) ≥ t and ν_A (xαyβz,q) ≤ s

Therefore μ_A (xαyβz,q) = max { $\mu_A(x,q)$, $\mu_A(y,q)$ $\mu_A(z,q)$ }≥ t

ν_A (xαyβz,q)= max{ $\nu_A(x,q)$, $\nu_A(y,q)$, $\nu_A(z,q)$}≤ s

⇨ $\mu_A(x,q) \geq t$, $\mu_A(y,q) \geq t$,$\mu_A(z,q) \geq t$ and
$\nu_A(x,q) \leq s$, $\nu_A(y,q) \leq s$, $\nu_A(z,q) \leq s$

hence x,y,z ∈ $A_{<t,s>}$ hence $A_{<t,s>}$ is an ordered prime ideal of R.

Conversely suppose that $A_{<t,s>}$ is an ordered prime ideal for any t ∈ [o,1].

Let x,y,z ∈ R, α,β ∈ Γ , q ∈ Q.

and min { $\mu_A(x,q)$, $\mu_A(y,q)$ $\mu_A(z,q)$ }= t
max{ $\nu_A(x,q)$, $\nu_A(y,q)$, $\nu_A(z,q)$}= s

⇨ $\mu_A(x,q) \geq t$, $\mu_A(y,q) \geq t$,$\mu_A(z,q) \geq t$ and
$\nu_A(x,q) \leq s$, $\nu_A(y,q) \leq s$, $\nu_A(z,q) \leq s$.
Therefore x,y,z ∈ $A_{<t,s>}$

⇨ x+y ∈ $A_{<t,s>}$ for every x,y $A_{<t,s>}$

Therefore $\mu_A(x+y,q) \geq t$=min{$\mu_A(x,q)$, $\mu_A(y,q)$} and
$\nu_A(x+y,q) \leq s$= max{ $\nu_A(x,q)$, $\nu_A(y,q)$}

Let max { $\mu_A(x,q)$, $\mu_A(y,q)$ $\mu_A(z,q)$ }= u
and min{ $\nu_A(x,q)$, $\nu_A(y,q)$, $\nu_A(z,q)$}=v.

⇨ $\mu_A(x,q)$ =u or $\mu_A(y,q)$=u or $\mu_A(z,q)$ }= u
And $\nu_A(x,q)$=v or $\nu_A(y,q)$=v or $\nu_A(z,q)$}=v

⇨ x or y or z ∈ $A_{<u,v>}$

⇨ xαyβz, ∈ $A_{<u,v>}$ for all α,β ∈ Γ

⇨ μ_A (xαyβz,q) ≥ u and ν_A (xαyβz,q) ≤ v for all q ∈ Q.

⇨ μ_A (xαyβz,q) ≥ max { $\mu_A(x,q)$, $\mu_A(y,q)$ $\mu_A(z,q)$ }
ν_A (xαyβz,q) ≤ min{ $\nu_A(x,q)$, $\nu_A(y,q)$, $\nu_A(z,q)$} for all q ∈ Q.

Let x ≤ y μ_A (u,v)=p and ν_A (u,v)=q

⇨ Then y ∈ $A_{<p,q>}$ by our assumption x ($A_{<p,q>}$)
Hence $\mu_{A(}$ x,q) ≥ p = $\mu_{A(}$ y,q) and $\nu_{A\,(}$ x,q) ≤ q = $\nu_{A\,(}$ y,q)

Hence A is an Q intuitionistic fuzzy ordered ideal of an ordered ternery Γ-semiring.

Let x,y ∈ R , α,β ∈ Γ, q ∈ Q.
μ_A (xαyβz,q)=m and ν_A (xαyβz,q)=n

Then xαyβz ∈ $A_{<m,n>}$ this gives x ∈ $A_{<m,n>}$ or y ∈ $A_{<m,n>}$ or z ∈ $A_{<m,n>}$.

This becomes $\mu_A(x,q) \geq m$ or $\mu_A(y,q) \geq m$ or $\mu_A(z,q) \geq m$ and

$\nu_A(x,q) \leq n$ or $\nu_A(y,q) \leq n$ or $\nu_A(z,q) \leq n$.

Hence max { $\mu_A(x,q)$, $\mu_A(y,q)$ $\mu_A(z,q)$ } $\geq m = \mu_A(x\alpha y\beta z,q)$ and

min{ $\nu_A(x,q)$, $\nu_A(y,q)$, $\nu_A(z,q)$} $\leq n = \nu_A(x\alpha y\beta z,q)$.

Hence A is a Q- intuitionistic fuzzy ordered prime ideal of R.

Theorem

Let R be an ordered ternary Γ- semiring. Then P is an ordered prime ideal of R iff the Q- intuitionistic fuzzy P= < χ_P , $\chi_P{}^c$ > is a Q- intuitionistic fuzzy ordered prime ideal of R.

Proof:

Let P be an ordered prime ideal of an ordered ternary Γ- semiring R.

Obviously P=< χ_P , $\chi_P{}^c$ > is a Q- intuitionistic fuzzy ordered prime ideal of R.

Since P is an ordered prime ideal, $x \in P$ or $y \in P$.

This gives $\chi_P(x,q) = 1$ or $\chi_P(y,q) = 1$ and

$\chi_P{}^c(x,q) = 0$ or $\chi_P{}^c(y,q) = 0$.

This gives $\chi_P(x\alpha y\beta z,q)$ =max { $\chi_P(x,q)$, $\chi_P(y,q)$ $\chi_P(z,q)$} and

$\chi_P{}^c(x\alpha y\beta z,q)$ =min{ $\chi_P{}^c(x,q)$, $\chi_P{}^c(y,q)$, $\chi_P{}^c(z,q)$}.

Let $x \leq y$, $x \in R$ and $y \in P$. Then $x \in P$.

This becomes $\chi_P(x,q) = 1 = \chi_P(y,q)$

$\chi_P{}^c(x,q) = 0 = \chi_P{}^c(y,q)$.

Hence P =< χ_P , $\chi_P{}^c$ > is a Q- intuitionistic fuzzy ordered prime ideal of R.

Suppose P is a Q- intuitionistic fuzzy ordered prime ideal of R. Let $x,y,z \in R$, $\alpha, \beta \in \Gamma$, $q \in Q$ such that $x\alpha y\beta z \in P$. We have

$\chi_P(x\alpha y\beta z,q)$ =max { $\chi_P(x,q)$, $\chi_P(y,q)$ $\chi_P(z,q)$} and

$\chi_P{}^c(x\alpha y\beta z,q)$ =min{ $\chi_P{}^c(x,q)$, $\chi_P{}^c(y,q)$, $\chi_P{}^c(z,q)$}.

⇨ $\chi_P(x,q) = 1$or $\chi_P(y,q) =1$ or $\chi_P(z,q) =1$and

$\chi_P{}^c(x,q)=0$ or $\chi_P{}^c(y,q)=0$ or $\chi_P{}^c(z,q)=0$

⇨ $x \in p$ or $y \in p$ or $z \in p$.

Hence P is an ordered prime ideal of an ordered ternary Γ- semiring R.

Theorem

Let R be an ordered ternary Γ- semiring. Then a non – empty subset A of or is an ordered filter of R iff the Q - intuitionistic fuzzy subset A=< χ_A , $\chi_A{}^c$ > is a Q- intuitionistic fuzzy ordered filter of R.

Proof:

Suppose A is an ordered filter of ordered ternary Γ- semiring R.

Obviously A is a non -empty Q- intuitionistic subset of R and Q- intuitionistic fuzzy ordered ternary Γ- subsemiring of R.

Let x,y,z ∈ R, α, β ∈ Γ , q∈ Q

suppose xαyβz ∉ A

⇨ x ∉ A, y ∈ A, z ∈ A

⇨ χ_A (xαyβz,q) = 0 , χ_A (x,q) =0, χ_A (y,q) =1 , χ_A (z,q) =1 and
χ_{A^c} (xαyβz,q)=1, χ_{A^c} (x,q)= 1, χ_{A^c} (y,q)=0 or χ_{A^c} (z,q)=0

⇨ χ_A (xαyβz,q) =min { χ_A(x,q), χ_A (y,q) χ_A (z,q)} and
χ_{A^c} (xαyβz,q) =max{ χ_{A^c} (x,q), χ_{A^c} (y,q), χ_{A^c} (z,q)}.

Let x =y, x ∈ R and x ∈ A . Then y ∈ A.

⇨ χ_A (x) =1= χ_A (y) and χ_{A^c} (x)= 0= χ_{A^c} (y).

Hence A is Q intuitionistic fuzzy ordered filter of R.

Conversely suppose that A=< χ_A , χ_{A^c} > is a Q- intuitionistic fuzzy ordered filter of R.

Obviously A is a non –empty ordered ternary Γ- subsemiring of R.

Let xαyβz ∈ A, x,y,z ∈ R, α, β ∈ Γ , q∈ Q since A is a Q- intuitionistic fuzzy ordered filter of R.

We have χ_A (xαyβz,q) =max { χ_A (x,q), χ_A (y,q) χ_A (z,q)} =1
χ_{A^c} (xαyβz,q) =min{ χ_{A^c} (x,q), χ_{A^c} (y,q), χ_{A^c} (z,q)}=0.

⇨ χ_A (x,q) = 1or χ_A (y,q) =1 or χ_A(z,q) =1and
χ_{A^c} (x,q)=0 or χ_{A^c} (y,q)=0 or χ_{A^c} (z,q)=0

⇨ x,y,z ∈ A.

Hence A is an ordered filter of an ordered ternary Γ- semiring of R.

Theorem

If A is a proper maximal Q - intuitionistic fuzzy order ideal of ordered ternary Γ-semiring of R. Then A in is an Q- intuitionistic fuzzy ordered prime ideal of an ordered ternary Γ- semiring of R.

Proof:

Suppose A is an proper and maximal Q - intuitionistic fuzzy order ideal of R.

Let t ∈ [0,1] such that $A_{<t,s>}$ is proper ordered ideal of R .

Let B be an ordered ideal of R such that $A_{<t,s>} \leq B$ suppose $B \neq R$. Then there exists $a \in R$ such that $a \notin B$. Therefore $a \notin A_{<t,s>}$.

⇨ $\mu_A(a,q) < t$ and $\nu_A(a,q)>s$ for all $q \in Q$.

Of R defined by $\mu_C(x,q)= \mu_C(y,q)$

$\nu_C(x,q)= \nu_C(y,q)$ if $x \neq a$ $\mu_C(x,q)= t$ and $\nu_C(x,q)=s$ if x=a for all $q\in Q$.

Then B=R.

Therefore $A_{<t,s>}$ is maximal ordered ideal of R.

⇨ $A_{<t,s>}$ is an ordered ideal of R.

Let $x,y \in R$, $\alpha,\beta \in \Gamma$, $q \in Q$.

$\mu_A (x\alpha y\beta z,q)=u$ and $\nu_A (x\alpha y\beta z,q)=v$

⇨ Then $x\alpha y\beta z \in A_{<u,v>}$ this gives x or y or $z \in A_{<u,v>}$.

⇨ $\mu_A(x,q) \geq u$ or $\mu_A(y,q) \geq u$ or $\mu_A(z,q) \geq u$ and
$\nu_A(x,q) \leq v$ or $\nu_A(y,q) \leq v$ or $\nu_A(z,q) \leq v$.

⇨ $\max \{ \mu_A(x,q), \mu_A(y,q) \mu_A(z,q) \} \geq \mu_A (x\alpha y\beta z,q)$ and
$\min\{ \nu_A(x,q), \nu_A(y,q), \nu_A(z,q)\} \leq \nu_A (x\alpha y\beta z,q)$.

⇨ Hence A is a Q- intuitionistic fuzzy ordered ideal of R.

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