Advances in Fuzzy Mathematics.
ISSN 0973-533XVolume 11, Number 2 (2016), pp. 245-264

http://www.ripublication.com

On Fuzzy Simply* Continuous Functions

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Abstract

In this paper, the concepts of fuzzy simply* open sets, fuzzy simply* continuous and fuzzy simply* open functions are introduced and some of their basic properties are studied. The inter-relations between fuzzy simply* continuous functions and fuzzy simply continuous functions, are obtained.

Keywords : Fuzzy dense set, fuzzy nowhere dense set, fuzzy simply open set, fuzzy G_δ-set, fuzzy hyper-connected space, fuzzy Baire space, fuzzy strongly irresolvable space, fuzzy resolvable space.

2000 AMS Classification: 54 A 40, 03 E 72.

1. INTRODUCTION

In 1965, **L.A. Zadeh** [19] introduced the concept of fuzzy sets as a new approach for modeling uncertainties. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. Topology provided the most natural framework for the concepts of fuzzy sets to flourish. The concept of fuzzy topological space was introduced by **C. L. Chang** [5] in 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

Continuity is one of the most important and fundamental properties that have been widely used in Mathematical Analysis. In the recent years, there has been a growing trend among many topologists to introduce and study different forms of open sets and a considerable amount of research has been done on many types of continuity in general topology. In 1969, **N. Biswas** [4] introduced the concept of simply continuity by means of the notion of simply open sets and investigated some of its properties.

J.**Ewert [7]** and **A**.**Neubrunnova [10]** used simply open set to define the concept of simply continuity in classical topology. **Josef Dobos** [6] gave a complete characterizations of stationary sets for the class of simply continuous functions. **Araf A. Nasef** and **R.Mareay [1]** established several characterizations of simply open sets in terms of the ideal of nowhere dense set. **Mona S Bakry** and **Rodyna A.Hosny** [9] introduced a new class of sets in topological spaces, namely simply* open sets and used this class of sets to introduce new classes of functions called simply* continuous, simply * -irresolute functions in topological space. The purpose of this paper is to introduce and study fuzzy simply* open sets in fuzzy topological spaces and fuzzy simply* continuous functions, fuzzy simply* open functions between fuzzy topological spaces. The inter-relations between fuzzy simply* continuous functions and fuzzy simply continuous functions, are also obtained.

2. PRELIMINARIES

In order to make the exposition self-contained, we give some basic notions and results used in the sequel. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to CHANG (1968). Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I.

Definition 2.1[5] : Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The fuzzy interior and the fuzzy closure of λ respectively are defined as follows :

(i). $\operatorname{int}(\lambda) = \vee \{ \mu / \mu \leq \lambda, \mu \in T \}$

(ii). $\operatorname{cl}(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1-\mu \in T \}$.

Lemma 2.1 [2] : For a fuzzy set λ of a fuzzy topological space X,

(i). $1 - \operatorname{Int}(\lambda) = \operatorname{Cl}(1-\lambda)$,

(ii). $1 - \operatorname{Cl}(\lambda) = \operatorname{Int}(1-\lambda)$.

Lemma 2.2 [18] : Let A, B be two fuzzy sets in a fuzzy topological space X.

(i). $\operatorname{int}(A) \leq A$, $\operatorname{int}(\operatorname{int}(A)) = \operatorname{int}(A)$.

(ii). $\operatorname{int}(A) \leq \operatorname{int}(B)$, wherever $A \leq B$.

(iii). $\operatorname{int}(A \wedge B) = \operatorname{int}(A) \wedge \operatorname{int}(B)$, $\operatorname{int}(A \vee B) \geq \operatorname{int}(A) \vee \operatorname{int}(B)$,

(iv). $A \leq \operatorname{cl}(A)$, $\operatorname{cl}(\operatorname{cl}(A)) = \operatorname{cl}(A)$.

(v). $\operatorname{cl}(A) \leq \operatorname{cl}(B)$, wherever $A \leq B$.

(vi). $\operatorname{cl}(A \wedge B) \leq \operatorname{cl}(A) \wedge \operatorname{cl}(B)$, $\operatorname{cl}(A \vee B) = \operatorname{cl}(A) \vee \operatorname{cl}(B)$.

Definition 2.2 [13] : A fuzzy set λ in a fuzzy topological space (X,T), is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, cl $(\lambda) = 1$, in (X,T).

Definition 2.3 [13] : A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu <$ cl (λ). That is, int cl (λ) = 0, in (X,T).

Definition 2.4 [11] : Let λ be a fuzzy set in a fuzzy topological space (X,T). The fuzzy boundary of λ is defined as Bd $(\lambda) =$ cl $(\lambda) \wedge$ cl $(1 - \lambda)$.

Definition 2.5 [17] : A fuzzy set λ in a fuzzy topological space (X,T), is called a fuzzy simply open set if Bd (λ) is a fuzzy nowhere dense set in (X,T). That is, λ is a fuzzy simply open set in (X,T) if cl $(\lambda) \wedge$ cl $(1 - \lambda)$, is a fuzzy nowhere dense set in (X,T).

Definition 2.6 [13]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy first category if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category**.**

Definition 2.7 [16] : Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Then 1 – λ is a fuzzy residual set in (X,T).

3. FUZZY SIMPLY* OPEN SETS

Definition 3.1 : A fuzzy set λ in a fuzzy topological space (X,T), is called a fuzzy simply* open set if $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T) and $1 - \lambda$ is called a fuzzy simply* closed set in (X,T).

Example 3.1 : Let X = {a, b, c }. The fuzzy sets α, β, δ, λ, μ, η and υ are defined on X as follows :

α : X → [0,1] is defined as α(a) = 0.5 ; α(b) = 0.4 ; α(c) = 0.6.

β : X → [0,1] is defined as β(a) = 0.6 ; β(b) = 0.5 ; β(c) = 0.7.

υ : X → [0,1] is defined as υ(a) = 0.4 ; υ(b) = 0.6 ; υ(c) = 0.3.

δ : X → [0,1] is defined as δ(a) = 0.3 ; δ(b) = 0.1 ; δ(c) = 0.7.

λ : X → [0,1] is defined as λ(a) = 0.5 ; λ(b) = 0.6 ; λ(c) = 0.6.

μ : X → [0,1] is defined as μ(a) = 0.5 ; μ(b) = 0,5 ; μ(c) = 0.6.

η : X → [0,1] is defined as η(a) = 0.6 ; η(b) = 0,6 ; η(c) = 0.7.

Then, T = { 0, α, β, 1 } is a fuzzy topology on X. On computation, the fuzzy nowhere dense sets in (X,T), are υ, $1-\alpha$, $1-\beta$, $1-\lambda$, $1-\mu$ and $1-\eta$. and the fuzzy simply* open sets in (X,T) are α, β, λ, μ and η.

Remarks 3.1 :(1).Each fuzzy open set is a fuzzy simply* open set in a fuzzy topological space. But the converse need not be true. For, in example 3.1, λ, μ and η, are fuzzy simply* open sets in (X,T) but are not fuzzy open sets in (X,T).

(2).If $\lambda_i = \mu_i \vee \delta$, where $\mu_i \in T$ and int cl(δ) = 0, in (X,T), then $\bigvee_{i=1}^{\infty}(\lambda_i)$ is a fuzzy simply* open set in (X,T).For, $\bigvee_{i=1}^{\infty}(\lambda_i) = \bigvee_{i=1}^{\infty}(\mu_i \vee \delta) = [\bigvee_{i=1}^{\infty}(\mu_i)] \vee \delta$ and $\mu_i \in T$ implies that $\bigvee_{i=1}^{\infty}(\mu_i) \in T$.

(3). If $\lambda_i = \mu \vee \delta_i$, where $\mu \in T$ and int cl (δ_i) = 0, in (X,T), then $\bigvee_{i=1}^{\infty}(\lambda_i)$ need not be a fuzzy simply* open set in (X,T), since $\bigvee_{i=1}^{\infty}(\delta_i)$ need not be a fuzzy nowhere dense set in (X,T).

(4). If $\lambda_i = \mu_i \wedge \delta$, where $\mu_i \in T$ and int cl(δ) = 0, in (X,T), then $\bigwedge_{i=1}^{\infty}(\lambda_i)$ need not be a fuzzy simply* open set in (X,T). For, $\bigwedge_{i=1}^{\infty}(\lambda_i) = \bigwedge_{i=1}^{\infty}(\mu_i \wedge \delta = [\bigwedge_{i=1}^{\infty}(\mu_i)] \wedge \delta$ and $\bigwedge_{i=1}^{\infty}(\mu_i)$ need not be fuzzy open in (X,T).

(5). If $\lambda_i = \mu_i \wedge \delta$, where $\mu_i \in T$ and int cl(δ) = 0, in a fuzzy P-space (X,T), then $\bigwedge_{i=1}^{\infty}(\lambda_i)$ is a fuzzy simply* open set in (X,T). For, $\bigwedge_{i=1}^{\infty}(\lambda_i) = \bigwedge_{i=1}^{\infty}(\mu_i \wedge \delta) = [\bigwedge_{i=1}^{\infty}(\mu_i)] \wedge \delta$ and $\bigwedge_{i=1}^{\infty}(\mu_i)$ is a fuzzy open in a fuzzy P-space (X,T).

Proposition 3.1 : If λ is a fuzzy simply* open set in a fuzzy topological space (X,T), then λ is not a fuzzy nowhere dense set in (X,T).

Proof : Let λ be a fuzzy simply* open set in (X,T). Then $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T). Then int cl(λ) = int cl($\mu \vee \delta$) = int [cl(μ) $\vee$ cl(δ)] $\geq$ int cl(μ) $\vee$ int cl(δ). Since δ is a fuzzy nowhere dense set in (X,T), int cl(δ) = 0, in (X,T) and hence int cl(λ) $\geq$ int cl(μ) $\vee$ 0 = int cl(μ) $\geq$ int (μ) = $\mu \neq 0$. Thus, int cl(λ) $\neq 0$, in (X,T). Therefore λ is not a fuzzy nowhere dense set in (X,T).

Remarks 3.2 : If λ is a fuzzy simply* open set in a fuzzy topological space (X,T) and $\lambda = \mu \vee \delta$, where $\mu \in T$ and int cl(δ) = 0 in (X,T), then $\boldsymbol{\mu \not\leq \delta}$.

For, if $\mu \leq \delta$, then μ = int (μ) $\leq$ int cl(μ) $\leq$ int [cl(δ)] = 0, implies that $\mu = 0$, a contradiction.

Proposition 3.2 : If λ is a fuzzy simply* open set in a fuzzy topological space (X,T), then $1-\lambda = \alpha \wedge \beta$, where α is a fuzzy closed set in (X,T) and β is a fuzzy dense set

Proof : Let λ be a fuzzy simply* open set in (X,T). Then $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T). Then, $1 - \lambda = 1 - (\mu \vee \delta) = (1 - \mu) \wedge (1 - \delta)$. Since μ is a fuzzy open set in (X,T), $1 - \mu$ is fuzzy closed set. Now δ is a fuzzy nowhere dense set in (X,T), implies that int cl $(\delta) = 0$ in (X,T). and hence $1 -$ int cl $(\delta) = 1 - 0 = 1$. Then cl int $(1 - \delta) = 1$, in (X,T). Let $\alpha = 1 - \mu$ and $\beta = 1 - \delta$, in (X,T). Then, α is a fuzzy closed set and cl int $(\beta) = 1$, in (X,T). Now cl int $(\beta) \leq$ cl (β), in (X,T), implies that $1 \leq$ cl (β), That is, cl$(\beta) = 1$, in (X,T). Thus, $1 - \lambda = \alpha \wedge \beta$, where α is a fuzzy closed set and β is a fuzzy dense set in (X,T).

Proposition 3.3 : If λ is a fuzzy simply* open set in a fuzzy topological space (X,T), then $1 - \lambda$ is not a fuzzy dense set in (X,T).

Proof : Let λ be a fuzzy simply* open set in (X,T). Then, by proposition 3.2, $1 - \lambda = \alpha \wedge \beta$, where α is a fuzzy closed set and β is a fuzzy dense set in (X,T) and hence cl $(1 - \lambda) =$ cl $(\alpha \wedge \beta)$, in (X,T). Thus cl $(1 - \lambda) \leq$ cl $(\alpha) \wedge$ cl $(\beta) =$ cl $(\alpha) \wedge 1 =$ cl $(\alpha) = \alpha < 1$ and hence cl $(1 - \lambda) \leq \alpha < 1$. Therefore $1 - \lambda$ is not a fuzzy dense set in (X,T).

Proposition 3.4 : If λ is a fuzzy simply* open set and η is a fuzzy open set in a fuzzy topological space (X,T), then $\lambda \wedge \eta$ is a fuzzy simply* open set in (X,T).

Proof : Let λ be a fuzzy simply* open set in (X,T). Then $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T). Now $\lambda \wedge \eta = [\mu \vee \delta] \wedge \eta = [\mu \wedge \eta] \vee [\delta \wedge \eta]$, in (X,T). Since μ and η are fuzzy open sets in (X,T), $\mu \wedge \eta$ is a fuzzy open set in (X,T). Also int cl $[\delta \wedge \eta] \leq$ int (cl $(\delta) \wedge$ cl (η)) and since int (cl $(\delta) \wedge$ cl (η)) = int cl $(\delta) \wedge$ int cl (η), int cl $[\delta \wedge \eta] \leq$ int cl $(\delta) \wedge$ int cl $(\eta) = 0 \wedge$ int cl $(\eta) = 0$, (since δ is a fuzzy nowhere dense set in (X,T), int cl $(\delta) = 0$, in (X,T),). That is, int cl $[\delta \wedge \eta] = 0$ and hence $\delta \wedge \eta$ is a fuzzy nowhere dense set in (X,T). Thus, $\lambda \wedge \eta = [\mu \wedge \eta] \vee [\delta \wedge \eta]$, where $\mu \wedge \eta$ is a fuzzy open set and $\delta \wedge \eta$ is a fuzzy nowhere dense set in (X,T), implies that $\lambda \wedge \eta$ is a fuzzy simply* open set in (X,T).

Proposition 3.5 : If λ is a fuzzy simply* open set in a fuzzy topological space (X,T), then int $(\lambda) \neq 0$, in (X,T).

Proof : Let λ be a fuzzy simply* open set in (X,T). Then $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T). Now $\lambda = \mu \vee \delta$, implies that $\lambda > \delta$ and hence int $(\lambda) >$ int (δ), in (X,T). Since δ is a fuzzy nowhere dense set in (X,T), int cl $(\delta) = 0$, in (X,T) and int $(\delta) <$ int cl(δ), implies that int $(\delta) = 0$ and hence int $(\lambda) > 0$. Therefore int $(\lambda) \neq 0$, in (X,T).

Proposition 3.6 : If λ is a fuzzy simply* open set in a fuzzy topological space (X,T), then λ is not a fuzzy simply open set in (X,T).

Proof : Let λ be a fuzzy simply* open set in (X,T). Then, by proposition 3.5, int $(\lambda) \neq 0$, in (X,T).

Case I: Suppose that cl$(\lambda) \neq 1$, in (X,T). Then, $1 - \text{cl}(\lambda) \neq 0$, in (X,T).

Now int cl (Bd (λ)) = int cl [cl$(\lambda) \wedge$ cl $(1 - \lambda)$]

$\geq$ int cl cl [$\lambda \wedge (1 - \lambda)$] (by lemma 2.2)

$\geq$ int cl [$\lambda \wedge (1 - \lambda)$]

$\geq$ int [$\lambda \wedge (1 - \lambda)$]

= int $(\lambda) \wedge$ int [$(1 - \lambda)$] (by lemma 2.2)

= int $(\lambda) \wedge (1 - \text{cl}(\lambda))$

Thus, int cl (Bd (λ)) $\neq 0$ and hence Bd (λ) is not a fuzzy nowhere dense set in (X,T). Therefore λ is not a fuzzy simply open set in (X,T).

Case II: Suppose that cl$(\lambda) = 1$, in (X,T). Then, $1 - \text{cl}(\lambda) = 0$, in (X,T).

Now int cl (Bd (λ)) = int cl [cl$(\lambda) \wedge$ cl $(1 - \lambda)$] = int cl [$1 \wedge$ cl$(1 - \lambda)$] = int cl [cl $(1 - \lambda)$] = int cl $(1 - \lambda) = 1 -$ cl int $(\lambda) > 1 - \text{cl}(\lambda) = 0$, in (X,T) and hence int cl (Bd (λ)) > 0, in (X,T). Thus, int cl (Bd (λ)) $\neq 0$ implies that Bd (λ) is not a fuzzy nowhere dense set in (X,T). Therefore λ is not a fuzzy simply open set in (X,T).

Proposition 3.7 : If λ is a fuzzy simply* open set in a fuzzy topological space (X,T) such that cl int $(\lambda) = 1$, then λ is a fuzzy simply open set in (X,T).

Proof : Let λ be a fuzzy simply* open set in (X,T). Then, cl int $(\lambda) \leq$ cl (λ) and cl int $(\lambda) = 1$, implies that cl $(\lambda) = 1$, in (X,T). Now int cl (Bd (λ)) = intcl[cl$(\lambda) \wedge$ cl $(1 - \lambda)$] = int cl [$1 \wedge$ cl$(1 - \lambda)$] = int cl[cl $(1 - \lambda)$] = int cl $(1 - \lambda)$ = $1 -$ cl int $(\lambda) = 1 - 1 = 0$, in (X,T) and hence λ is a fuzzy simply open set in (X,T).

Theorem 3.1 [17] : If $\lambda = \mu \vee \delta$, where μ is a fuzzy open and fuzzy dense set and δ is a fuzzy nowhere dense set in a fuzzy topological space (X,T), then λ is a fuzzy simply open set in (X,T).

Definition 3.2 [8] : A fuzzy topological space (X,T) is said to be fuzzy hyper-connected if every non- null fuzzy open subset of (X,T)is fuzzy dense in (X,T). That is, a fuzzy topological space (X,T) is fuzzy hyper-connected if cl $(\mu_i) = 1$, for all $\mu_i \in T$.

Proposition 3.8 : If λ is a fuzzy simply* open set in a fuzzy hyper- connected space (X,T), then λ is a fuzzy simply open set in (X,T).

Proof : Let λ be a fuzzy simply* open set in (X,T). Then $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T). Since (X,T) is a fuzzy hyper connected space, the fuzzy open set μ is a fuzzy dense set in (X,T) and hence μ is a fuzzy open and fuzzy dense set in (X,T). Thus $\lambda = \mu \vee \delta$, where μ is a fuzzy open and fuzzy dense set and δ is a fuzzy nowhere dense set in (X,T) implies, by theorem 3.1, that λ is a fuzzy simply open set in (X,T).

Definition 3.3 [15] : A fuzzy topological space (X, T) is called a fuzzy strongly irresolvable space if for every fuzzy dense set λ in (X,T), cl int $(\lambda) = 1$. That is, cl $(\lambda) = 1$ implies that cl int $(\lambda) = 1$ in (X,T).

Proposition 3.9 : If λ is a fuzzy simply* open and fuzzy dense set in a fuzzy strongly irresolvable space (X,T), then λ is a fuzzy simply open set in (X,T).

Proof : Let λ be a fuzzy simply* open and fuzzy dense set in (X,T).Since (X, T) is a fuzzy strongly irresolvable space, for the fuzzy dense set in (X,T), we have cl int $(\lambda) = 1$, in (X,T).Thus λ is a fuzzy simply* open set in (X,T) such that cl int$(\lambda) = 1$.Then, by theorem 3.7, λ is a fuzzy simply open set in (X,T).

4. FUZZY SIMPLY* CONTINUOUS FUNCTIONS

Definition 4.1 : Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function f : (X, T) → (Y, S) is called a fuzzy simply* continuous if $f^{-1}(\lambda)$ is a fuzzy simply* open set in (X,T), for each fuzzy open set λ in (Y,S).

Clearly each fuzzy continuous function, is fuzzy simply continuous, since each fuzzy open set is fuzzy simply* open set in fuzzy topological spaces. But the converse need not be true. For, consider the following example:*

Example 4.1 : Let X = {a, b, c }. The fuzzy sets λ, μ, υ, A and B are defined on X as follows :

$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.6$; $\lambda(c) = 0.6$.

$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.5$; $\mu(b) = 0,5$; $\mu(c) = 0.6$.

$\upsilon : X \to [0,1]$ is defined as $\upsilon(a) = 0.4$; $\upsilon(b) = 0.6$; $\upsilon(c) = 0.3$.

$A : X \to [0,1]$ is defined as $A(a) = 0.5$; $A(b) = 0.4$; $A(c) = 0.6$.

$B: X \to [0,1]$ is defined as $B(a) = 0.6$; $B(b) = 0.5$; $B(c) = 0.7$.

Then, T = { 0, λ, μ, 1 } and S = { 0, A, B, 1 } are fuzzy topologies on X. The fuzzy nowhere dense sets in (X,T) are $1 - \lambda$, $1 - \mu$ and υ. Now the fuzzy simply* open sets in (X,T), are λ, μ, $\lambda \vee \upsilon$, $\mu \vee \upsilon$, $\lambda \vee (1 - \lambda)$ $\mu \vee (1 - \lambda)$, $\lambda \vee (1 - \mu)$, $\mu \vee (1 - \mu)$. On computation we see that $\lambda \vee \upsilon = \lambda \vee (1 - \lambda)$; $\mu \vee \upsilon = \mu \vee (1 - \lambda)$; $\mu \vee (1 - \mu) = \mu$ and let $\alpha = \lambda \vee \upsilon$, $\beta = \mu \vee \upsilon$, $\rho = \lambda \vee (1 - \mu)$ in (X,T). Thus α, β and ρ are the fuzzy sets defined on X such that

α(a) = 0.5 ; α(b) = 0.6 ; α(c) = 0.6.

β(a) = 0.6 ; β(b) = 0.6 ; β(c) = 0.7.

ρ(a) = 0.3 ; ρ(b) = 0.1 ; ρ(c) = 0.7.

and the fuzzy simply* open sets in (X,T) are λ, μ, α, β and ρ. Define a function f : (X,T) → (X,S) by f (a) = b ; f (b) = c ; f (c) = a. On computation, we see that for the fuzzy open sets A, B in (X,S), $f^{-1}(A) = \alpha$ and $f^{-1}(B) = \beta$, in (X,T) and α, β are fuzzy simply* open sets in (X,T), implies that f : (X, T) → (X, S) is a fuzzy simply * continuous function. But f is not a fuzzy continuous function, since $\alpha \notin T$ and $\beta \notin T$.

Proposition 4.1 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then int [$f^{-1}(\lambda)$] ≠ 0, in (X,T), for a fuzzy open set λ in (Y,S).

Proof : Let λ be a fuzzy open set in (Y,S). Since f : (X, T) → (Y, S) is a fuzzy simply* continuous function, $f^{-1}(\lambda)$ is a fuzzy simply* open set in (X,T). Then, by proposition 3.3, $1 - f^{-1}(\lambda)$ is not a fuzzy dense set in (X,T). That is, cl[$1 - f^{-1}(\lambda)$] ≠ 1, in (X,T). Then $1 -$ int[$f^{-1}(\lambda)$] ≠ 1 and hence int [$f^{-1}(\lambda)$] ≠ 0, in (X,T).

Definition 4.2 [13]: A function f : (X,T) → (Y,S) from a fuzzy topological space (X,T) into another fuzzy topological space (Y, S) is called somewhat fuzzy continuous if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$ implies that there exist a fuzzy open set δ in (X,T) such that $\delta \neq 0$ and $\delta \leq f^{-1}(\lambda)$.

Proposition 4.2 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then f is a somewhat fuzzy continuous function from (X,T) into (Y,S).

Proof : Let f be a fuzzy simply* continuous function from (X,T) into (Y,S). Then, by proposition 4.1, int [$f^{-1}(\lambda)$] ≠ 0, for a fuzzy open set λ in (Y,S) and hence f is a somewhat fuzzy continuous function.

Proposition 4.3 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then for a fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is not a fuzzy nowhere dense set in (X,T).

Proof : Let λ be a fuzzy open set in (Y,S). Since f : (X, T) → (Y, S) is a fuzzy simply* continuous function, $f^{-1}(\lambda)$ is a fuzzy simply* open set in (X,T). Then, by proposition 4.1, int [$f^{-1}(\lambda)$] ≠ 0, in (X,T) and int [$f^{-1}(\lambda)$] ≤ int cl [$f^{-1}(\lambda)$], implies that int cl [$f^{-1}(\lambda)$] ≠ 0, in (X,T) and hence $f^{-1}(\lambda)$ is not a fuzzy nowhere dense set in (X,T).

Proposition 4.4 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S), then f is a fuzzy simply continuous function.

Proof : Let f be a fuzzy simply* continuous function from (X,T) into (Y,S). Then, $f^{-1}(\lambda)$ is a fuzzy simply* open set in (X,T), for a fuzzy open set λ in (Y,S). Since (X,T) is a fuzzy hyper- connected space, by proposition 3.8, $f^{-1}(\lambda)$ is a fuzzy simply open set in (X,T). Hence, for a fuzzy open set λ in (Y,S),, $f^{-1}(\lambda)$ is a fuzzy simply open set in (X,T), implies that f is a fuzzy simply continuous function.

Proposition 4.5 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) and cl int (μ) $= 1$, for each fuzzy simply* open set in (X,T), then f is a fuzzy simply continuous function.

Proof : Let f be a fuzzy simply* continuous function from (X,T) into (Y,S). Then, $f^{-1}(\lambda)$ is a fuzzy simply* open set in (X,T), for a fuzzy open set λ in (Y,S). Since cl int (μ) $= 1$, for each fuzzy simply* open set μ in (X,T),

cl int [$f^{-1}(\lambda)$] $=$ 1 in (X,T). Then, by proposition 3.7,, $f^{-1}(\lambda)$ is a fuzzy simply open set in (X,T) and hence f is a fuzzy simply continuous function.

Proposition 4.6 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy strongly irresolvable space (X,T) into a fuzzy topological space (Y,S) and each fuzzy simply* open set is fuzzy dense in (X,T), then f is a fuzzy simply continuous function.

Proof : Let f be a fuzzy simply* continuous function from (X,T) into (Y,S). Then, $f^{-1}(\lambda)$ is a fuzzy simply* open set in (X,T), for a fuzzy open set λ in (Y,S). Since each fuzzy simply* open set is fuzzy dense in (X,T),

cl [$f^{-1}(\lambda)$] $= 1$ in (X,T). Thus, $f^{-1}(\lambda)$ is the fuzzy simply* open and fuzzy dense set in (X,T). Then, by proposition 3.9, $f^{-1}(\lambda)$ is a fuzzy simply open set in (X,T) and hence f is a fuzzy simply continuous function.

Proposition 4.7 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then cl[$f^{-1}(\lambda)$] $\neq$ 1 in (X,T), for a fuzzy closed set λ in (Y,S).

Proof : Let λ be a fuzzy closed set in (Y,S). Then, $1-\lambda$ is a fuzzy open set in (Y,S). Since f : (X, T) → (Y, S) is a fuzzy simply* continuous function, $f^{-1}(1-\lambda)$ is a fuzzy simply* open set in (X,T). Then, $1-f^{-1}(\lambda)$ is a fuzzy simply* open set in (X,T). By proposition 3.3,$1-[1-f^{-1}(\lambda)]$ is not a fuzzy dense set in (X,T). That is, cl $f^{-1}(\lambda)]$ $\neq$ 1, in (X,T).

Proposition 4.8 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then int cl [f^{-1}(int (λ))] $\neq$ 0 in (X,T), for a fuzzy set λ defined on Y.

Proof : Let λ be a fuzzy set defined on Y. Then, int(λ) is a fuzzy open set in (Y,S). Since f : (X, T) → (Y, S) is a fuzzy simply* continuous function, by proposition 4.3, f^{-1}(int(λ)) is not a fuzzy nowhere dense set in (X,T) and hence int cl [f^{-1}(int (λ))] $\neq$ 0 in (X,T).

Proposition 4.9 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy hyper- connected space (X,T) into a fuzzy topological space (Y,S), then for a fuzzy open set λ in (Y,S), $f^{-1}(1-\lambda)$ is a fuzzy nowhere dense set in (X,T).

Proof : Let λ be a fuzzy open set in (Y,S). Since f is a fuzzy simply* continuous function from (X,T) into (Y,S), by proposition 4.1, int [$f^{-1}(\lambda)$] $\neq$ 0, in (X,T). Now int [$f^{-1}(\lambda)$] is a non-zero fuzzy open set in (X,T). Since (X,T) is a fuzzy hyper-connected space, int [$f^{-1}(\lambda)$] is a fuzzy dense set in (X,T). Then, cl (int [$f^{-1}(\lambda)$]) $=$ 1 and hence $1-$ cl (int [$f^{-1}(\lambda)$]) $=0$, in (X,T). This implies that int cl $[1-f^{-1}(\lambda)]=0$ and hence int cl $[f^{-1}(1-\lambda)]=0$. Therefore $f^{-1}(1-\lambda)$ is a fuzzy nowhere dense set in (X,T).

Proposition 4.10 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy hyper- connected space (X,T) into a fuzzy topological space (Y,S), then for a fuzzy G_δ set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy residual set in (X,T).

Proof : Let λ be a fuzzy G_δ set in (Y,S). Then, $=\wedge_{k=1}^{\infty}(\lambda_k)$, where $\lambda_k \in S$. Now $f^{-1}(\lambda)=f^{-1}(\wedge_{k=1}^{\infty}(\lambda_k)) = (\wedge_{k=1}^{\infty} f^{-1}(\lambda_k)$. Since the function f is fuzzy simply* continuous from the fuzzy hyper-connected space (X,T) into a fuzzy topological space (Y,S), by proposition 4.9, [$f^{-1}(1-\lambda_k)]'s$ are fuzzy nowhere dense sets in (X,T). Then, $\vee_{k=1}^{\infty}$ $[f^{-1}(1-\lambda_k)]$ is a fuzzy first category set in (X,T) and then $[1-\wedge_{k=1}^{\infty} f^{-1}(\lambda_k)]$ is a fuzzy first category set in (X,T). Thus,

$\wedge_{k=1}^{\infty} f^{-1}(\lambda_k)$ is a fuzzy residual set in (X,T). Therefore $f^{-1}(\lambda)$ is a fuzzy residual set in (X,T).

Definition 4.3 [16] : Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy Baire space if $\mathrm{int}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T).

Theorem 4.1 [16] : Let (X,T) be a fuzzy topological space. Then the following are equivalent :

(1) (X,T) is a fuzzy Baire space.

(2) int (λ) = 0, for every fuzzy first category set λ in (X,T).

(3) cl (μ) = 1, for every fuzzy residual set μ in (X,T).

Proposition 4.11 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy hyper- connected and fuzzy Baire space (X,T) into a fuzzy topological space (Y,S), then for a fuzzy G_δ set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy dense set in (X,T).

Proof : Let λ be a fuzzy G_δ set in (Y,S). Since the function f is fuzzy simply* continuous from a fuzzy hyper- connected space (X,T) into a fuzzy topological space (Y,S), by proposition 4.10, $f^{-1}(\lambda)$ is a fuzzy residual set in (X,T). Since (X,T) is a fuzzy Baire space, by theorem 4.1, $\mathrm{cl}(f^{-1}(\lambda)) = 1$ for the fuzzy residual set λ in (X,T). Therefore $f^{-1}(\lambda)$ is a fuzzy dense set in (X,T).

Definition 4.4 [12] : A fuzzy topological space (X,T) is called a fuzzy P-space if countable intersection of fuzzy open sets in (X,T) is fuzzy open. That is, every non-zero fuzzy G_δ -set in (X,T), is fuzzy open in (X,T).

Proposition 4.12 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy topological space (X,T) into a fuzzy P-space (Y,S), then $\mathrm{int}[f^{-1}(\lambda)] \neq 0$, for a fuzzy G_δ set λ in (Y,S).

Proof : Let λ be a fuzzy G_δ set in (Y,S). Since (Y,S) is a fuzzy P-space, the fuzzy G_δ set λ is fuzzy open in (Y,S). Now f : (X, T) → (Y, S) is a fuzzy simply* continuous function and λ is fuzzy open in (Y,S), by proposition 4.1, $\mathrm{int}[f^{-1}(\lambda)] \neq 0$, in (X,T).

Proposition 4.13 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous from a fuzzy hyper-connected space (X,T) into a fuzzy P-space (Y,S), then for a fuzzy G_δ set λ in (Y,S), $f^{-1}(\lambda)$ is a fuzzy dense set in (X,T).

Proof : Let λ be a fuzzy G_δ set in (Y,S). Since (Y,S) is a fuzzy P-space, the fuzzy G_δ set λ is fuzzy open in (Y,S). Now f : (X, T) → (Y, S) is a fuzzy simply* continuous function and λ is fuzzy open in (Y,S), by proposition 4.1, $\mathrm{int}[f^{-1}(\lambda)] \neq 0$, in (X,T). Since (X,T) is a fuzzy hyper- connected space, $\mathrm{int}[f^{-1}(\lambda)]$ is a fuzzy dense set in

(X,T).Then, cl (int $[f^{-1}(\lambda)]$) $= 1$, in (X,T). But cl (int $[f^{-1}(\lambda)]$) $\leq$ cl ($f^{-1}(\lambda)$), implies that $1 \leq$ cl ($f^{-1}(\lambda)$) and hence cl ($f^{-1}(\lambda)$) $= 1$, in (X,T). Therefore $f^{-1}(\lambda)$ is a fuzzy dense set in (X,T).

Proposition 4.14 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous function from a fuzzy hyper- connected space (X,T) into a fuzzy topological space (Y,S) and cl $[\bigwedge_{k=1}^{\infty} (f^{-1}(\lambda_k))] = 1$ where $\lambda_k \in S$, then (X,T) is a fuzzy Baire space.

Proof : Let $(\lambda_k)'s$ (k = 1 to ∞) be fuzzy open sets in (Y,S). Since the function f : (X, T) → (Y,S) is a fuzzy simply* continuous function from a fuzzy hyper- connected space (X,T) into a fuzzy topological space (Y,S), then by proposition 4.9, for the fuzzy open sets $(\lambda_k)'s$ in (Y,S), $[f^{-1}(1 - \lambda_k)]'s$ are fuzzy nowhere dense sets in (X,T).Now cl $[\bigwedge_{k=1}^{\infty} (f^{-1}(\lambda_k))] = 1$, implies that $1 -$ cl $[\bigwedge_{k=1}^{\infty} (f^{-1}(\lambda_k))] = 0$ and hence int $[1 - [\bigwedge_{k=1}^{\infty} (f^{-1}(\lambda_k))] = 0$, in (X,T). Then, int $[\bigvee_{k=1}^{\infty}(f^{-1}(1 - \lambda_k)] = 0$, where $[f^{-1}(1 - \lambda_k)]'s$ are fuzzy nowhere dense sets in (X,T), implies that (X,T) is a fuzzy Baire space.

Proposition 4.15 : If f : (X, T) → (Y,S) is a fuzzy simply* continuous function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) and g : (Y, S) → (Z,W) is a fuzzy continuous function from (Y,S) into a fuzzy topological space (Z,W), then g ○ f : (X, T) → (Z,W) is a fuzzy simply* continuous function from (X,T) into (Z,W).

Proof : Let λ be a non-zero fuzzy open set in (Z, W). Since g is a fuzzy continuous function from (Y,S) into (Z, W), $g^{-1}(\lambda)$ is a fuzzy open set in (Y,S). Since f is a fuzzy simply* continuous function from (X,T) into (Y,S), $f^{-1}(g^{-1}(\lambda))$ is a fuzzy simply* open set in (X,T). Hence $(g \circ f)^{-1}(\lambda)$ is a fuzzy simply * open set in (X,T), for the fuzzy open set λ in (Z, W). Therefore, g ○ f is a fuzzy simply* continuous function from (X,T) into (Z,W).

Theorem 4.2 [13]: Let (X,T) and (Y,S) be any two fuzzy topological spaces. Let f : (X,T) → (Y,S) be a function. Then the following are equivalent.

(1). f is somewhat fuzzy continuous.

(2). If λ is a fuzzy closed set of (Y,S) such that $f^{-1}(\lambda) \neq 1$, then there exists a proper fuzzy closed set μ of (X,T) such that $\mu > f^{-1}(\lambda)$.

(3). If λ is a fuzzy dense set in (X,T), then f (λ) is a fuzzy dense set in (Y,S).

Definition 4.5 [14]: A fuzzy topological space (X,T) is called a fuzzy resolvable space if there exists a fuzzy dense set λ in (X,T) such that $1-\lambda$ is also a fuzzy dense set in (X,T). Otherwise (X,T) is called a fuzzy irresolvable space.

Proposition 4.16 : If f : (X,T) → (Y, S) is a fuzzy simply* continuous and one- to-one function from a fuzzy resolvable space (X,T) onto a fuzzy topological space (Y,S), then (Y, S) is a fuzzy resolvable space.

Proof : Let (X,T) be a resolvable space. Then, there exists a fuzzy set λ in (X,T) such that cl (1− λ) = 1, in (X,T). Thus, λ and 1− λ are fuzzy dense sets in (X,T). Since f : (X, T) → (Y, S) is a fuzzy simply* continuous function from (X,T) into (Y,S), by proposition 4.2, f is a somewhat fuzzy continuous function from (X,T) into (Y,S). Then, by theorem 4.2, f (λ) and f (1− λ) are fuzzy dense sets in (Y,S). Also, since the function f is one - to -one and onto, then f (1 – λ) = 1 – f(λ), in (Y,S). Then, f (λ) and 1− f (λ) are fuzzy dense sets in (Y,S). Thus, cl [f (λ)] = 1 and cl [1 – f(λ)] = 1, in (Y,S), implies that (Y, S) is a fuzzy resolvable space.

Definition 4.6 [13]: A fuzzy topological space (X,T) is called fuzzy first category if $1_X = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). A topological space which is not of fuzzy first category, is said to be of fuzzy second category**.**

Proposition 4.17 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous, fuzzy open and one - to- one function from a fuzzy hyper- connected space (X,T) into a fuzzy topological space (Y,S), then for a fuzzy open set λ in (X,T), $1-\lambda$ is a fuzzy nowhere dense set in (X,T).

Proof : Let λ be a fuzzy open set in (X,T). Since f : (X, T) → (Y, S) is a fuzzy open function, f (λ) is a fuzzy open set in (Y,S). Also since f is a fuzzy simply* continuous and one-to-one function from fuzzy hyper- connected space (X,T) onto a fuzzy topological space (Y,S), for the fuzzy open set f (λ) in (Y,S), by proposition 4.9, $f^{-1}(1-f(\lambda))$ is a fuzzy nowhere dense set in (X,T). That is, int cl [$f^{-1}(1-f(\lambda)) = 0$, in (X,T). Then, int cl [$(1-f^{-1}f(\lambda))] = 0$, in (X,T). Since f is a one-to -one function, $f^{-1}f(\lambda)=\lambda$ and then int cl $[1-\lambda] = 0$, in (X,T). Therefore $1-\lambda$ is a fuzzy nowhere dense set in (X,T).

Proposition 4.18 : If f : (X, T) → (Y, S) is a fuzzy simply* continuous, fuzzy open and one - to- one function from a fuzzy hyper- connected space (X,T) into a fuzzy topological space (Y,S) and $\bigwedge_{k=1}^{\infty}(\lambda_k) \neq 0$, where $\lambda_k \in T$, then (X,T) is a fuzzy second category space.

Proof : Let λ_k (k = 1 to ∞) be fuzzy open sets in (X,T). Since the function f is a fuzzy simply* continuous, fuzzy open and one -to-one function from fuzzy hyper-connected space (X,T) onto a fuzzy topological space (Y,S), by proposition4.17, $[1-\lambda_k]'s$ are fuzzy nowhere dense sets in(X,T). Now $\bigvee_{k=1}^{\infty} [1 - \lambda_k] = 1 - \bigwedge_{k=1}^{\infty}(\lambda_k)\ 1 - 0 \neq 1$, in (X,T) and hence $\bigvee_{k=1}^{\infty} [1 - \lambda_k] \neq 1$, where $[1 - \lambda_k]'s$ are fuzzy nowhere dense sets in (X,T), implies that (X,T) is a fuzzy second category space.

5. FUZZY SIMPLY* OPEN FUNCTIONS

Definition 5.1 : Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function f : (X, T) → (Y, S) is called a fuzzy simply* open function if f (λ) is a fuzzy simply* open set in (Y,S), for each fuzzy open set λ in (X,T).

Clearly a fuzzy open function is fuzzy simply open, since each fuzzy open set is fuzzy simply* open set in fuzzy topological spaces. But the converse need not be true. For, consider the following example:*

Example 5.1 : Let X = {a, b, c }. The fuzzy sets λ, μ, υ, A and B are defined on X as follows :

λ : X → [0,1] is defined as λ(a) = 0.5 ; λ(b) = 0.6 ; λ(c) = 0.6.

μ : X → [0,1] is defined as μ(a) = 0.5 ; μ(b) = 0,5 ; μ(c) = 0.6.

υ : X → [0,1] is defined as υ(a) = 0.4 ; υ(b) = 0.6 ; υ(c) = 0.3.

A : X → [0,1] is defined as A(a) = 0.5 ; A(b) = 0.4 ; A(c) = 0.6.

B: X → [0,1] is defined as B(a) = 0.6 ; B(b) = 0.5 ; B(c) = 0.7.

Then, T = { 0, A, B, 1 } and S = { 0, λ, μ, 1 } are fuzzy topologies on X.

The fuzzy nowhere dense sets in (X,S) are $1 - \lambda$, $1 - \mu$ and υ. Now the fuzzy simply* open sets in (X,T), are λ, μ, $\lambda \vee \upsilon$, $\mu \vee \upsilon$, $\lambda \vee (1 - \lambda)$, $\mu \vee (1 - \lambda)$, $\lambda \vee (1 - \mu)$, $\mu \vee (1 - \mu)$. On computation we see that $\lambda \vee \upsilon = \lambda \vee (1 - \lambda)$; $\mu \vee \upsilon = \mu \vee (1 - \lambda)$; $\mu \vee (1 - \mu) = \mu$ andlet $\alpha = \lambda \vee \upsilon$, $\beta = \mu \vee \upsilon$, $\rho = \lambda \vee (1 - \mu)$ in (X,T). Thus α, β and ρ are the fuzzy sets defined on X such that

α(a) = 0.5 ; α(b) = 0.6 ; α(c) = 0.6.

β(a) = 0.6 ; β(b) = 0.6 ; β(c) = 0.7.

ρ(a) = 0.3 ; ρ(b) = 0.1 ; ρ(c) = 0.7.

and the fuzzy simply* open sets in (X,S) are λ, μ, α, β and ρ. Define a function f : (X,T) → (X,S) by f (a) = c ; f (b) = a; f (c) = b. On computation, we see that for the fuzzy open sets A, B in (X,T), $f(A) = \alpha$ and $f(B) = \beta$, in (X,S) and α, β are fuzzy simply* open sets in (X,S), implies that f : (X, T) → (X, S) is a fuzzy simply * open function. But f is not a fuzzy open function, since $\alpha \notin$ Sand $\beta \notin$ S.

Proposition 5.1 : If f : (X, T) → (Y, S) is a fuzzy simply* open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then int [f (λ)] ≠ 0 in (Y,S), for a fuzzy open set λ in (X,T).

Proof : Let λ be a fuzzy open set in (X,T). Since f : (X, T) → (Y, S) is a fuzzy simply* open function, f (λ) is a fuzzy simply* open set in (Y,S). Then, by proposition 3.3, 1 − f (λ) is not a fuzzy dense set in (Y,S). That is, cl [1 − f (λ)] ≠ 1, in (Y,S). Then 1 − int[f (λ)] ≠ 1 and hence int [f(λ)] ≠ 0, in (Y,S).

Proposition 5.2 : If f : (X, T) → (Y, S) is a fuzzy simply* open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then f is a somewhat fuzzy open function from (X,T) into (Y,S).

Proof : Let f be a fuzzy simply* open function from (X,T) into (Y,S). Then, by proposition 5.1, int [f (λ)] ≠ 0 in (Y,S), for a fuzzy open set λ in (X,T) and hence f is a somewhat fuzzy open function from (X,T) into (Y,S).

Proposition 5.3 : If f : (X, T) → (Y, S) is a fuzzy simply* open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then for a fuzzy open set λ in (X,T), f (λ) is not a fuzzy nowhere dense set in (Y,S).

Proof : Let λ be a fuzzy open set in (X,T). Since f : (X, T) → (Y, S) is a fuzzy simply* open function, f (λ) is a fuzzy simply* open set in (Y,S). Then, by proposition 5.1, int [f (λ)] ≠ 0, in (Y,S).Now int [f (λ)] ≤ int cl [f(λ)], implies that int cl [f(λ)] ≠ 0, in (Y,S) and hence f (λ) is not a fuzzy nowhere dense set in (Y,S).

Proposition 5.4 : If f : (X, T) → (Y, S) is a fuzzy simply* open function from a fuzzy topological space (X,T) into a fuzzy hyper- connected space (Y,S), then f is a fuzzy simply open function from (X,T) into (Y,S).

Proof : Let f be a fuzzy simply* open function from (X,T) into (Y,S). Then, f (λ) is a fuzzy simply* open set in (Y,S), for a fuzzy open set λ in (X,T). Since (Y,S) is a fuzzy hyper- connected space, by proposition 3.8, f (λ) is a fuzzy simply open set in (Y,S). Hence, for a fuzzy open set λ in (X,T), f (λ) is a fuzzy simply open set in (Y,S), implies that f is a fuzzy simply open function from (X,T) into (Y,S).

Proposition 5.5 : If f : (X, T) → (Y, S) is a fuzzy simply* open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) and cl int (μ) = 1, for each fuzzy simply* open set in (Y,S), f is a fuzzy simply open function from (X,T) into (Y,S).

Proof : Let f be a fuzzy simply* open function from (X,T) into (Y,S). Then, f (λ) is a fuzzy simply* open set in (Y,S), for a fuzzy open set λ in (X,T). By hypothesis, for the fuzzy simply* open set f (λ) in (Y,S),cl int [f (λ)] = 1 in (Y,S). Then, by proposition 3.7, f (λ) is a fuzzy simply open set in (Y,S) and hence f is a fuzzy simply open function from (X,T) into (Y,S).

Proposition 5.6 : If f : (X, T) → (Y, S) is a fuzzy simply* open function from a fuzzy topological space (X,T) into a fuzzy strongly irresolvable space (Y,S) and each fuzzy simply* open set is fuzzy dense in (Y,S), then f is a fuzzy simply open function from (X,T) into (Y,S).

Proof : Let f be a fuzzy simply* open function from (X,T) into (Y,S). Then, f (λ) is a fuzzy simply* open set in (Y,S), for a fuzzy open set λ in (X,T). Since each fuzzy simply* open set is fuzzy dense in (Y,S), cl [f λ)] = 1 in (Y,S). Thus, f (λ) is the fuzzy simply* open and fuzzy dense set in the fuzzy strongly irresolvable space (Y,S). Then, by proposition 3.9, f (λ) is a fuzzy simply open set in (Y,S) and hence f is a fuzzy simply open function from (X,T) into (Y,S).

Proposition 5.7 : If f :(X, T) → (Y, S) is a fuzzy simply* open and one-to-one function from a fuzzy topological space (X,T) onto a fuzzy topological space (Y,S), then cl[f λ)] ≠ 1 in (Y,S), for a fuzzy closed set λ in (X,T).

Proof : Let λ be a fuzzy closed set in (X,T). Then, $1 - \lambda$ is a fuzzy open set in (X,T). Since f : (X, T) → (Y, S) is a fuzzy simply* open function, f ($1- \lambda$) is a fuzzy simply* open set in (Y,S). Since f is one - to -one and onto, f ($1 - \lambda$) = 1 – f(λ), in (Y,S). Then, 1 − f (λ) is a fuzzy simply* open set in (Y,S). By proposition 3.3, 1 − [1 − f (λ)] is not a fuzzy dense set in (Y,S). That is, cl [f λ)] ≠ 1, in (Y,S).

Proposition 5.8 : If f : (X, T) → (Y, S) is a fuzzy simply* open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then int cl [f(int (λ))] ≠ 0 in (Y,S), for a fuzzy set λ defined on X.

Proof : Let λ be a fuzzy set defined on X. Then, int (λ) is a fuzzy open set in (X,T). Since f : (X, T) → (Y, S) is a fuzzy simply* open function, by proposition 5.3, f (int(λ)) is not a fuzzy nowhere dense set in (Y,S) and hence int cl [f (int(λ)) ≠ 0 in (X,T).

Proposition 5.9 : If f : (X, T) → (Y, S) is a fuzzy simply* open and one-to-one function from a fuzzy topological space (X,T) onto a fuzzy hyper- connected space (Y,S), then for a fuzzy open set λ in (X,T), f ($1 - \lambda$) is a fuzzy nowhere dense set in (Y,S).

Proof : Let λ be a fuzzy open set in (X,T). Since f is a fuzzy simply* open function from (X,T) onto (Y,S), by proposition 5.1, int [f (λ)] ≠ 0, in (Y,S). Now int [f (λ)] is a non-zero fuzzy open set in (Y,S). Since (Y,S) is a fuzzy hyper- connected space, int [f (λ)] is a fuzzy dense set in (Y,S). Then, cl (int [f (λ)]) = 1 and hence 1 − cl (int [f (λ)]) = 0, in (Y,S). This implies that int cl [1 − f (λ)]) = 0 in (Y,S). Since f is one - to -one and onto, f (1 – λ) = 1 – f(λ), in (Y,S) and hence int cl [f (1 − λ)]) = 0. Therefore f (1− λ) is a fuzzy nowhere dense set in (Y,S).

Proposition 5.10 : If f : (X, T) → (Y, S) is a fuzzy simply* open and one-to-one function from a fuzzy topological space (X,T) onto a fuzzy hyper- connected space (Y,S), then for a fuzzy G_δ set λ in (X,T), f (λ) ≤ μ,where μ is a fuzzy residual set in (YS).

Proof : Let λ be a fuzzy G_δ set in (X,T). Then, $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where $\lambda_k \in T$. Now $f(\lambda) = f(\wedge_{k=1}^{\infty}(\lambda_k)) \leq (\wedge_{k=1}^{\infty} f(\lambda_k))$. Since the function f is fuzzy simply* open from the fuzzy topological space (X,T) onto the fuzzy hyper-connected space (Y,S), by proposition 5.9, $[f(1-\lambda_k)]$'s are fuzzy nowhere dense sets in (Y,S). Then, $[\vee_{k=1}^{\infty} f(1-\lambda_k)]$ is a fuzzy first category set in (Y,S). Since f is one -to-one and onto, $f(1-\lambda_k) = 1 - f(\lambda_k)$, in (Y,S) and hence $\vee_{k=1}^{\infty}[1 - f(\lambda_k)]$ is a fuzzy first category set in (Y,S). Then $[1 - \wedge_{k=1}^{\infty} f(\lambda_k)]$ is a fuzzy first category set in (Y,S) and hence $\wedge_{k=1}^{\infty} f(\lambda_k)]$ is a fuzzy residual set in (YS). Let $\mu = \wedge_{k=1}^{\infty} f(\lambda_k)$. Thus $f(\lambda) \leq \mu$, where μ is a fuzzy residual set in (YS).

Proposition 5.11 : If f : (X, T) → (Y, S) is a fuzzy simply* open function from a fuzzy P-space (X,T) into a fuzzy topological space (Y,S), then int [f (λ)] ≠ 0, for a fuzzy G_δ set λ in (X,T).

Proof : Let λ be a fuzzy G_δ set in (X,T). Since (X,T) is a fuzzy P-space, the fuzzy G_δ set λ is fuzzy open in (X,T). Now f : (X, T) → (Y, S) is a fuzzy simply* open function and λ is fuzzy open in (X,T), by proposition 5.1, int [f (λ)] ≠ 0, in (Y,S).

Proposition 5.12 : If f : (X, T) → (Y, S) is a fuzzy simply* open function from a fuzzy P-space (X,T) into a fuzzy hyper-connected space (Y,S), then, for a fuzzy G_δ set λ in (X,T), f (λ) is a fuzzy dense set in (Y,S).

Proof : Let λ be a fuzzy G_δ set in (X,T). Since (X,T) is a fuzzy P-space, the fuzzy G_δ set λ is fuzzy open in (X,T). Now f : (X, T) → (Y, S) is a fuzzy simply* open function and λ is fuzzy open in (X,T), by proposition 5.1, int [f (λ)] ≠ 0, in (Y,S). Since (Y,S) is a fuzzy hyper- connected space int [f (λ)] is a fuzzy dense set in (Y,S).Then, cl (int [f (λ)]) = 1, in (Y,S). But cl (int [f (λ)]) ≤ cl (f (λ)),implies that 1 ≤ cl (f(λ)) and hence cl (f (λ)) = 1, in (Y,S). Therefore f (λ) is a fuzzy dense set in (Y,S).

Proposition 5.13 : If f : (X, T) → (Y, S) is a fuzzy simply* open and one-to-one function from a fuzzy topological space (X,T) onto a fuzzy hyper- connected space (Y,S) and cl [$\wedge_{k=1}^{\infty}$ (f (λ_k))] = 1 where $\lambda_k \in T$, then (Y,S) is a fuzzy Baire space.

Proof : Let $(\lambda_k)'s$ (k = 1 to ∞) be fuzzy open sets in (X,T). Since the function f : (X, T) → (Y,S) is a fuzzy simply* open function from a fuzzy topological space (X,T) into a fuzzy hyper- connected space (Y,S), by proposition 5.9, for the fuzzy open sets $(\lambda_k)'s$ in (X,T), [f $(1-\lambda_k)$]'s are fuzzy nowhere dense sets in (Y,S).Now cl [$\wedge_{k=1}^{\infty}$ (f (λ_k))] = 1, implies that $1-$ cl [$\wedge_{k=1}^{\infty}$ (f (λ_k))] = 0 and hence int $[1-[\wedge_{k=1}^{\infty}$ (f (λ_k))] = 0, in (Y,S). Then, int [$\vee_{k=1}^{\infty}$($1-$ f (λ_k))] = 0 in (Y,S). Since f is one - to -one and onto, f $(1-\lambda_k) = 1-$ f(λ_k), in (Y,S) and hence int [$\vee_{k=1}^{\infty}$(f $(1-\lambda_k)$] = 0, where [f $(1-\lambda_k)$]'s are fuzzy nowhere dense sets in (Y,S), implies that (Y,S) is a fuzzy Baire space.

Proposition 5.14 : If f : (X, T) → (Y, S) is a fuzzy continuous, fuzzy simply* open and one - to- one function from a fuzzy topological space (X,T) onto a fuzzy hyper-connected space (Y,S) and if λ is a fuzzy open set in (Y,S), then $1-\lambda$ is a fuzzy nowhere dense set in (Y,S).

Proof : Let λ be a fuzzy open set in (Y,S). Since f : (X, T) → (Y, S) is a fuzzy continuous function, $f^{-1}(\lambda)$ is a fuzzy open set in (X,T). Also since f is a fuzzy simply* open and one-to-one function from (X,T) onto a fuzzy hyper- connected space (Y,S), for the fuzzy open set $f^{-1}(\lambda)$ in (X,T), by proposition 5.9, f $(1-f^{-1}(\lambda))$ is a fuzzy nowhere dense set in (Y,S). That is, int cl [f $(1-f^{-1}(\lambda)) = 0$, in (Y,S). Then, int cl [f $(f^{-1}(1-\lambda))] = 0$. Since f is an onto function, f $f^{-1}(1-\lambda) = 1-\lambda$ and then int cl [$1-\lambda$] = 0, in (Y,S). Therefore $1-\lambda$ is a fuzzy nowhere dense set in (Y,S).

Proposition 5.15 : If f : (X, T) → (Y, S) is a fuzzy continuous, fuzzy simply* open and one -to-one function from a fuzzy topological space (X,T) onto a fuzzy hyper-connected space (Y,S) and $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$, where $\lambda_k \in S$, in (Y,S) then (Y,S) is a fuzzy second category space.

Proof : Let λ_k (k = 1 to ∞) be fuzzy open sets in (Y,S). Since the function f is a fuzzy continuous, fuzzy simply* open and one -to-one function from (X,T) onto a fuzzy hyper-connected space (Y,S), by proposition 5.14, $[1-\lambda_k]'s$ are fuzzy nowhere dense sets in (Y,S). Now $\vee_{k=1}^{\infty}[1-\lambda_k] = 1-\wedge_{k=1}^{\infty}(\lambda_k) \neq 1-0 \neq 1$, in (Y,S) and hence $\vee_{k=1}^{\infty}[1-\lambda_k] \neq 1$, where $[1-\lambda_k]'s$ are fuzzy nowhere dense sets in (Y,S), implies that (Y,S) is a fuzzy second category space.

Theorem 5.1 [13]: Suppose (X,T) and (Y,S) be fuzzy topological spaces. Let f : (X,T) → (Y,S) be an onto function. Then the following conditions are equivalent.

(1) . f is somewhat fuzzy open.

(2) . If λ is a fuzzy dense set in (Y,S), then $f^{-1}(\lambda)$ is a fuzzy dense set in (X,T).

Proposition 5.16 : If f : (X,T) → (Y, S) is a fuzzy simply* open and one- to-one function from a fuzzy topological space (X,T) onto a fuzzy resolvable space (Y,S), then (X,T) is a fuzzy resolvable space.

Proof : Let (Y,S) be a resolvable space. Then, there exists a fuzzy set μ in (Y,S) such that cl (1− μ) = 1, in (Y,S). Thus, μ and 1 − μ are fuzzy dense sets in (Y,S). Since f : (X, T) → (Y, S) is a fuzzy simply* open function from (X,T) into (Y,S), by proposition 5.2, f is a somewhat fuzzy open function from (X,T) into (Y,S). Then, by theorem 5.1, $f^{-1}(\mu)$ and $f^{-1}(1-\mu)$ are fuzzy dense sets in (Y,S). Now $f^{-1}(1-\mu) = 1 - f^{-1}(\mu)$, in (X,T). Then, $f^{-1}(\mu)$ and $1 - f^{-1}(\mu)$ are fuzzy dense sets in (X,T). Thus, cl $[f^{-1}(\lambda)] = 1$ and cl $[1 - f^{-1}(\mu)] = 1$, in (X,T), implies that (X,T) is a fuzzy resolvable space.

Proposition 5.17 : If f : (X, T) → (Y,S) is a fuzzy open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) and g : (Y, S) → (Z,W) is a fuzzy simply* open function from (Y,S) into a fuzzy topological space (Z,W), then g ○ f : (X, T) → (Z,W) is a fuzzy simply* open function from (X,T) into (Z,W).

Proof : Let λ be a non-zero fuzzy open set in (X,T). Since f is a fuzzy open function from (X,T) into (Y,S), f (λ) is a fuzzy open set in (Y,S). Since g is a fuzzy simply* open function from (Y,S) into (Z,W), g(f (λ)) is a fuzzy simply* open set in (Z,W). Hence g ○ f is a fuzzy simply* open function from (X,T) into (Z,W).

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