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On Fuzzy Wing Graphs and their properties

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Abstract

The wing graph W (G) of a simple, finite undirected graph G has the edges of G as vertices and two vertices of W (G) are adjacent if they are non adjacent edges of an induced path of length three in G. In this paper, we introduce the notion of the fuzzy wing graph of (σ, μ), a partial fuzzy subgraph of G. We show that a fuzzy wing graph cannot be isomorphic to a complete fuzzy bipartite graph. We also show that a partial fuzzy subgraph of any connected graph is vertex isomorphic and weakly line isomorphic to a subgraph of a fuzzy wing graph. We establish a necessary and sufficient condition for a fuzzy graph to be the fuzzy wing graph of some fuzzy graph. Finally, we demonstrate how to construct a fuzzy graph on G such that its radius is the same as that of its wing graph.

Keywords: Wing graph, fuzzy graph, fuzzy wing graph, isomorphism, connected fuzzy graph, radius.
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1. INTRODUCTION

Graph Theory is a branch of Mathematics that finds applications in several diverse fields, ranging from computer science to medicine and even logistics and especially information technology. The advent of integrated circuits was a huge leap forward in that it facilitated the construction of many large interconnection networks, whose topological structures are basically graphs. Graph Theory has proved to be a powerful tool in the design and study of interconnection networks, as seen in [9]. The study of 'graph operators' has received wide attention since Ore's work [13] on the line graph operator. The line graphical method is widely used in the design of interconnection networks, a detailed study of which is available in [19].A detailed survey on various kinds of graph operators and their properties are included in [11].Another graph operator, the 'wing graph operator' was introduced by Hoang in [6], as part of the quest to break the perfect graph conjecture. Further studies on the wing graph were made in [7, 8]. The notion of fuzzy graphs was developed by Rosenfeld [15] . Since then, the study of fuzzy graphs has witnessed tremendous growth – since graphs model real life situations and in real life, information is 'imprecise' or 'fuzzy'. Connectivity is of prime importance in an interconnection network. In the case of fuzzy graphs, connectivity differs widely from the situation in crisp graphs. Bhutani and Rosenfeld [2, 3, 4] and Mathew and Sunitha [17, 18] have made major contributions in the study of connectivity problems in fuzzy graphs. Studies on fuzzy graphs, fuzzy trees, complements of fuzzy graphs, domination in fuzzy graphs and distance concepts in fuzzy graphs are established in [1, 16].

In [10], Mordeson introduced the concept of a 'fuzzy line graph' and obtained several results regarding their structure. The wing graph of a graph and some of its properties are studied by Pramada in [12]. Motivated by these works , the 'fuzzy wing graph' is defined and results are obtained. All the graphs considered here are finite, undirected and simple. For all basic concepts and notations regarding graph theory and graph operators we refer [5] and [14] respectively. The seminal book [11] is a primary reference for all concepts regarding fuzzy graph theory.

Throughout this paper, a graph G = (V, E) or simply G, is chosen such that it is finite, simple and undirected. The vertices of G are denoted by v_i , $1 \leq i \leq n$ and its edges by $v_i - v_j$, $i \neq j$. The vertex set and edge set are also represented by V (G) and E (G). Unless otherwise stated, the vertex in the wing graph of G, W (G), corresponding to the edge $v_i - v_j$ is denoted by $v_i - v_j$. A fuzzy subgraph of G, (V, σ, μ) is chosen such that (supp σ, supp μ) is G itself. For convenience, we denote the fuzzy subgraph of G by (σ, μ) or simply G itself. Its fuzzy wing graph is denoted by (σ_W, μ_W) or simply W(G).

2. PRELIMINARIES

In this section, we see definitions and results that will be needed in the sequel.

Definition 2. 1. [14] A *graph dynamical system* is a pair (Γ, Φ) where Γ is a set of graphs and $\Phi : \Gamma \rightarrow \Gamma$ is a mapping called a *(graph) operator*.

Definition 2. 2. [14] The *wing graph* W(G) of a graph G has all its edges as vertices and two edges of G are adjacent vertices in W(G) if they are non incident edges of some induced 4-vertex path in G.

Following is an illustration of a graph and its wing graph:

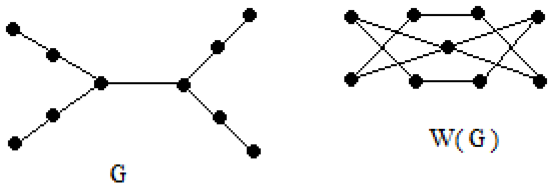


Fig. 1

Observation2. 3. W (C_{2k+1}) = C_{2k+1}, $k \geq 2$

Definition2. 4. [11] A *fuzzy graph* G = (σ, μ) with the underlying set V is a nonempty set V together with a pair of functions $\sigma : V \to [0, 1]$ and $\mu: V \times V \to [0, 1]$ such that

$$\mu(u, v) \leq \sigma(u) \wedge \sigma(v), \quad \forall\, u, v \text{ in } V.$$

We say the fuzzy graph (σ_1, μ_1) is a *(partial) fuzzy subgraph* of the fuzzy graph (σ_2, μ_2) if

$\sigma_1 \subseteq \sigma_2$ and $\mu_1 \subseteq \mu_2$

To distinguish this concept from the following definition, we refer to (σ_1, μ_1) as simply a fuzzy subgraph of (σ_2, μ_2).

Definition2.5. [11] A *partial fuzzy subgraph* (σ, μ) *of a graph G = (V, E)* consists of the fuzzy subsets 'σ' of V and 'μ' of V X V respectively such that (σ, μ) is a fuzzy graph and supp μ is contained in E.

Definition2. 6. [11] Let (σ, μ) and (σ', μ') be partial fuzzy subgraphs of G and G' respectively. A one - one onto mapping 'f ' : V → V' is called a *vertex isomorphism* of

(σ, μ) onto (σ', μ') iff. $\sigma(v) = \sigma'[f(v)]\ \forall\, v \in V$.

Definition2. 7. [11] Let (σ, μ) and (σ', μ') be partial fuzzy subgraphs of G and G' respectively. A one - one onto mapping 'f': E → E' is called a *line isomorphism* of (σ, μ) onto (σ', μ') iff. $\mu(u, v) = \mu'[f(u),f(v)]\ \forall\, u, v \in V$.

Definition2. 8. [11] Let (σ, μ) and (σ', μ') be partial fuzzy subgraphs of G = (V, E). A one - one onto mapping 'f ' : V → V' is called an *isomorphism* of (σ, μ) onto (σ', μ') iff. it is both a line isomorphism and vertex isomorphism.

Theorem2. 9. [11] Let (σ, μ) and (σ', μ') be partial fuzzy subgraphs of G and G' respectively. If 'f' is a weak isomorphism from (σ, μ) onto (σ', μ') , then f is an isomorphism of (supp σ, supp μ) onto (supp σ', supp μ') .

Definition2. 10. [15] A *path* in a fuzzy graph (V, σ, μ) is a sequence of distinct vertices $v_0, v_1, \ldots, v_n$ (except possibly v_0 and v_n) such that $\mu(v_{i-1}, v_i) > 0$, $1 \leq i \leq n$. The *strength* of the path is $\overset{n}{\underset{i=1}{\wedge}} \mu(v_{i-1}, v_i)$. The fuzzy graph is said to be *connected* iff. there is a path with non zero strength between every pair of vertices in the graph.

Definition2. 11. [15] The *μ- length* of a path $P = v_0, v_1, ., v_n$ in a fuzzy graph is given by $l(P) = \sum_{i=1}^{n} \frac{1}{\mu(v_{i-1}, v_i)}$

Definition2. 12. [1] The *μ- distance* δ(u, v) between any two vertices u and v of a fuzzy graph is the minimum of the μ- lengths of all paths joining them.

Definition2. 13. [1] The *eccentricity* of a vertex 'u' in a fuzzy graph is the maximum of the μ- distances δ(u, v) for all vertices v.

Definition2. 14. [1] The *radius r (G)* of a connected fuzzy graph G is the minimum of the eccentricities of all vertices of the graph.

Theorem21.15 [12] For any graph G, W (G) will never be isomorphic to a complete bipartite graph.

Theorem 2.16 [12] Given any connected graph H, there exists a connected graph G such that H is an induced subgraph of W (G).

Theorem2.17 [12] If a path is the wing graph of some graph G, then it is of length five, six or seven.

3. THE FUZZY WING GRAPH

In this section, we define the fuzzy wing graph. We arrive at several results regarding the structure of the fuzzy wing graph and the relation between a fuzzy graph and its fuzzy wing graph.

Consider a graph G = (V, E) with wing graph W (G) = (V_W, E_W). Let (σ, μ) be a partial fuzzy subgraph of G. We define the fuzzy subsets σ_W and μ_W of V_W and E_W as follows:

$$\sigma_W (v_i - v_j) = \mu(v_i, v_j), \forall\, v_i - v_j \in V_W$$

and

$$\mu_W (v_i - v_j , v_k - v_l) = \mu (v_i - v_j) \wedge \mu (v_k - v_l) , \forall (v_i - v_j , v_k - v_l) \in E_W$$

Then we have

Proposition3.1. *(σ_W, μ_W) is a fuzzy subgraph of $W(G)$.*

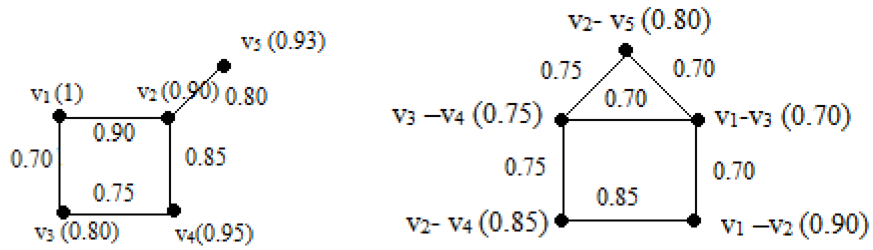
Proof. $\mu_W (v_i - v_j , v_k - v_l) = \mu (v_i - v_j) \wedge \mu (v_k - v_l) , \forall (v_i - v_j , v_k - v_l) \in E_W$

$$= \sigma_W (v_i - v_j) \wedge \sigma_W (v_k - v_l) , \forall (v_i - v_j , v_k - v_l) \in E_W$$

The definition satisfies the conditions for the fuzzy subset to be a fuzzy graph

Definition 3.2 The fuzzy subgraph (σ_W, μ_W) of W (G) as defined above is called the *fuzzy wing graph* of (σ, μ) .

Illustrated below is a fuzzy graph and its fuzzy wing graph. The membership degrees of the vertices are indicated in brackets:



G W(G)

Fig.2

Theorem 3.3. *If (σ_W, μ_W) is the fuzzy wing graph of (σ, μ) then $(supp\ \sigma_W, supp\ \mu_W)$ is the wing graph of $(supp\ \sigma, supp\ \mu)$.*

Proof. Given that (σ_W, μ_W) is the fuzzy wing graph corresponding to (σ, μ).

$Supp\ \sigma_W = \{v_i - v_j / \sigma_W (v_i - v_j) > 0\}$

Hence $v_i - v_j \in Supp\ \sigma_W \Leftrightarrow \sigma_W (v_i - v_j) > 0$

$\Leftrightarrow \mu_W (v_i, v_j) > 0$

$\Leftrightarrow v_i - v_j \in Supp\ \mu$

Again, $Supp\ \mu_W = \{(v_i - v_j ,v_k - v_l) / \mu_W (v_i - v_j , v_k - v_l) > 0\}$

Hence $(v_i - v_j ,v_k - v_l) \in Supp\ \mu_W \Leftrightarrow \mu_W (v_i - v_j , v_k - v_l) > 0$

$\Leftrightarrow \mu (v_i , v_j) \wedge \mu (v_k ,v_l) > 0$

$\Leftrightarrow$ $v_i - v_j$ and $v_k - v_l$ both $\in Supp\ \mu$

$\Leftrightarrow$ the end vertices v_i , v_j , v_k and v_l all $\in Supp\ \sigma$

Recalling the construction of the wing graph of a given graph, we see that (supp σ_W, supp μ_W) is the wing graph of (supp σ, supp μ).

Definition 3.5 A fuzzy graph $G = (V, \sigma, \mu)$ is said to be *complete bipartite* if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that $\mu(v_i, v_j) = 0$ if v_i and v_j both belong to V_1 or V_2 and $\mu(v_i, v_j) \geq \sigma(v_i) \wedge \sigma(v_j)$ if $v_i \in V_1$ and $v_j \in V_2$ We denote it by K_{σ_1,σ_2}, σ_i being the restriction of σ to V_i.

Remark: In [16], a fuzzy graph $G = (V, \sigma, \mu)$ is said to be *complete bipartite* if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that $\mu(v_i, v_j) = 0$ if v_i and v_j both belong to V_1 or V_2 and $\mu(v_i, v_j) = \sigma(v_i) \wedge \sigma(v_j)$ if $v_i \in V_1$ and $v_j \in V_2$
Here, we remove the restriction on μ and arrive at a more generalized definition.

Observation 3.6 The support of a complete bipartite fuzzy graph is a complete bipartite graph in the crisp sense.

Theorem 3.7 *The fuzzy wing graph is non isomorphic to a complete bipartite fuzzy graph.*

Proof Let (σ_W, μ_W) be the fuzzy wing graph corresponding to (σ, μ) .Suppose there exists an isomorphism 'f' between (σ_W, μ_W) and a complete bipartite fuzzy graph, say, $K_{\sigma_1,\sigma_2} = (\sigma_K, \mu_K)$ .Then by definition, 'f 'is also a weak isomorphism. By Theorem 2.9, 'f' is an isomorphism of (Supp σ_W, Supp μ_W) onto (Supp σ_K, Supp μ_K). By proposition 3.3 and observation 3.6, this shows that the wing graph (Supp σ_W, Supp μ_W) is isomorphic to a complete bipartite graph. This contradicts Theorem 2.15. So there cannot exist an isomorphism between a fuzzy wing graph and a complete bipartite fuzzy graph. □

Theorem 3.8 *Any partial fuzzy subgraph on a connected graph is vertex isomorphic and weakly line isomorphic to a subgraph of a fuzzy wing graph.*

Proof Let (σ, μ) be a partial fuzzy subgraph on a connected graph H with Supp $\sigma = \{v_0, v_1, .., v_n\}$. For each v_i , introduce a vertex w_i . We define the fuzzy sets σ_G and μ_G as follows:

$\sigma_G(v_i) = \sigma(v_i)$ and $\sigma_G(w_i) = 1$

$\mu_G(w_i, v_i) = \sigma(v_i)$ and $\mu_G(w_i, w_j) = 0$

Let G = (Supp σ_G, Supp μ_G) .It can be easily verified that (σ_G, μ_G) is a partial fuzzy subgraph of G.

Consider the fuzzy wing graph of G, W (G) = (σ_W, μ_W). Denote by u_i the vertex in W (G) corresponding to the edge w_i - v_i. Then it follows from Theorem 2.16 that the subgraph induced by the u_is, H', is isomorphic to H. Let $\sigma_{H'}$ and $\mu_{H'}$, respectively, be the restrictions of σ_W and μ_W to H'.

Define 'f ': V (H) → V (H') as f(v_i) = u_i

Then f is obviously a bijection.

Now, $\sigma_{H'}[f(v_i)] = \sigma_W[f(v_i)] = \sigma_W(u_i) = \mu_G(w_i - v_i) = \sigma(v_i)$. Hence f is a vertex isomorphism.

Again, $\mu_{H'}[f(v_i), f(v_j)] = \mu_{H'}(u_i, u_j) = \mu_W(u_i, u_j)$

$$= \sigma_W(u_i) \wedge \sigma_W(u_j)$$

$$= \mu_G(w_i, v_i) \wedge \mu_G(w_j, v_j)$$

$$= \sigma(v_i) \wedge \sigma(v_j)$$

By definition, $\mu(v_i, v_j) \leq \sigma(v_i) \wedge \sigma(v_j)$. Hence $\mu(v_i, v_j) \leq \mu_{H'}(u_i, u_j)$. Thus f is a weak edgeisomorphism.

Theorem 3.9 *Let (σ, μ) be a partial fuzzy subgraph on an odd cycle of length at least 5. Then it is isomorphic to its fuzzy wing graph iff. σ and μ are fuzzy sets that take the same constant value.*

Proof Let G = (σ, μ) be a partial fuzzy subgraph on the odd cycle C_{2k+1}, $k \geq 2$ and W (G) = (σ_W, μ_W) be the corresponding fuzzy wing graph. Let Supp σ = V = {v_1, v_2, ……, v_{2k+1}/ v_{2k+2}= v_1} and Supp σ_W = V_W.

Suppose there exists an isomorphism 'f': V → V_W such that $\sigma(v_i) = \sigma_W[f(v_i)]$

and $\mu(v_i, v_j) = \mu_W[f(v_i), f(v_j)]$

Consider the vertex $v_i - v_j$ in W(G). Then $\sigma_W(v_i - v_j) = \mu(v_i, v_j)$

$= \mu_W[f(v_i), f(v_j)]$, $\forall$ $v_i - v_j \in V_W$.

Now let $\sigma(v_i) = s_i$, $\mu(v_i, v_{i+1}) = r_{i+1}$

Since $\sigma(v_i) = \sigma_W[f(v_i)]$,there exists a permutation between the s_i s and r_i s, say π, so that $s_i = r_{\pi(i)}$ – (A)

By the property of fuzzy graphs, $r_i \leq s_i \wedge s_{i+1}$

Again, since $\mu(v_i, v_j) = \mu_W[f(v_i), f(v_j)]$, there is a permutation between the r_i s and the grades $r_i \wedge r_{\pi(i)}$, so that $r_i = r_{\pi(i)} \wedge r_{\pi(i+1)}$

$\Rightarrow r_i = s_i \wedge s_{i+1}$ by (A).

Further, $r_i = r_{\pi(i)} \wedge r_{\pi(i+1)} \Rightarrow r_i \leq r_{\pi(i)}$

$\Rightarrow r_{\pi(i)} \leq r_{\pi^2(i)}$ and so on.

Thus, $r_i \leq r_{\pi(i)} \leq r_{\pi^2(i)} \leq ...... \leq r_{\pi^{j+1}(i)} = r_i$, where π^{j+1} is the identity map $\Rightarrow r_i = r_{\pi(i)}$

So $r_i = r_{\pi(i)} \wedge r_{\pi(i+1)} \Rightarrow r_i = r_i \wedge r_{\pi(i+1)}$

$\Rightarrow r_i \leq r_{\pi(i+1)} = r_{i+1}$

$\Rightarrow r_i \leq r_{i+1}$

Therefore $r_1 \leq r_2 \leq r_3 \leq ...... \leq r_{2k+1} \leq r_{2k+2} = r_1$

$\Rightarrow r_1 = r_2 = ... = r_{2k+1}$

From (A), $s_i = r_i$

So $r_1 = r_2 = ... = r_{2k+1} = s_1 = s_2 = ... = s_{2k+1}$

This indicates that $\sigma(v_i) = \mu(e_i) = a \in [0, 1]$, $\forall v_i \in$ Supp σ, $e_i \in$ Supp μ, ie; σ and μ are fuzzy sets that take the same constant value.

The converse is immediate.

Proposition 3.10 *Let (σ_W, μ_W) be a partial fuzzy subgraph on the wing graph of a graph G = (V,E). Then it is the fuzzy wing graph of a partial fuzzy subgraph of G iff. $\mu_W(v_i - v_j, v_k - v_l) = \sigma_W(v_i - v_j) \wedge \sigma_W(v_k - v_l)$, for every edge in the wing graph of G.*

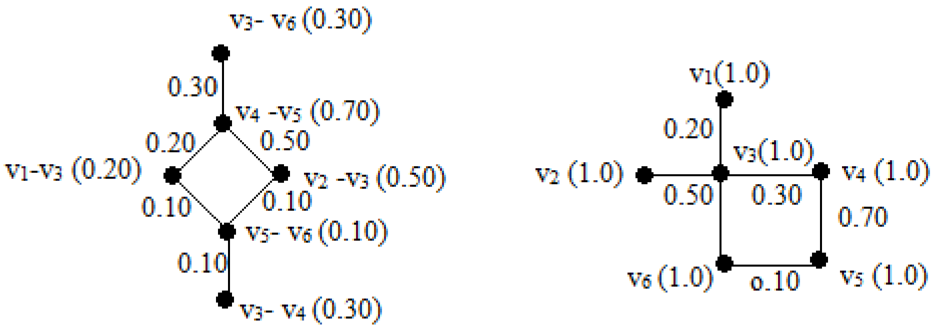
Proof Suppose (σ_W, μ_W) is the fuzzy wing graph of (σ, μ). Then by definition, $\mu_W(v_i - v_j, v_k - v_l) = \sigma_W(v_i - v_j) \wedge \sigma_W(v_k - v_l)$, for every edge in the wing graph of G.

Conversely, let $\mu_W(v_i - v_j, v_k - v_l) = \sigma_W(v_i - v_j) \wedge \sigma_W(v_k - v_l)$, for every edge in the wing graph of G.

We define a partial fuzzy subgraph on G as follows:

$\sigma(v_i) = 1$, $\forall v_i \in V$ and $\mu(v_i - v_j) = \sigma_W(v_i - v_j)$ $\forall v_i - v_j \in E$. Then (σ_W, μ_W) is the fuzzy wing graph of (σ, μ)

Note: The above construction is not unique, '1'may be replaced by any membership grade $\geq$ the supremum of the membership grades of the edges incident on a vertex $v_i \in$ V. Illustrated below is such a construction:



W (G) G

Fig. 3

Proposition 3.11 *(σ, μ) is a fuzzy wing graph iff. (Supp σ, Supp μ) is a wing graph and* $\mu (v_i, v_j) \le \sigma (v_i) \wedge \sigma (v_j), \ \forall v_i - v_j \in Supp\, \mu$.

Proof Let (σ, μ) be the fuzzy wing graph of some graph. Then, by propositions 3. 3 and 3.10,
$\mu (v_i, v_j) \le \sigma (v_i) \wedge \sigma (v_j) \ \ \forall v_i - v_j \in \text{Supp}\ \mu$.
The converse is immediate.

Proposition 3.12 *If a partial fuzzy subgraph on a path is a fuzzy wing graph, then the path must be of length five, six or seven.*

Proof Let (σ, μ) be a fuzzy wing graph on the path P_n of length 'n'. Then by Proposition 3.11, P_n is a wing graph. By Theorem 2.17,'n' is five, six or seven□.

4. RADIUS OF THE FUZZY WING GRAPH - A CONSTRUCTION

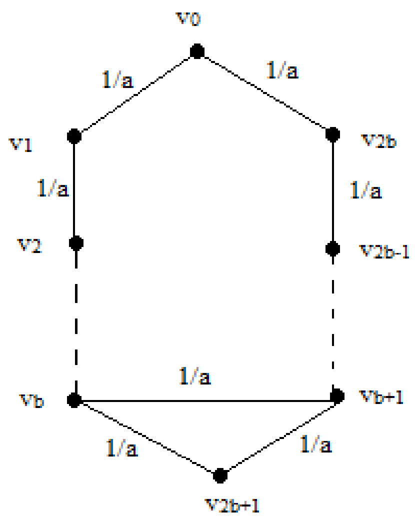
In this section, we give a theorem regarding the fuzzy wing graph whose proof involves the construction of a certain fuzzy graph.

Theorem 4.1 *Given any two integers a, b > 1, there exists a fuzzy graph G = (σ, μ) such that*

$$r (G) = r[W(G)] = ab.$$

Proof Construct a graph G as follows: Let $v_0, v_1, \ldots, v_{2b}$ be the vertices of the cycle C_{2b+1}. Introduce a vertex 'v_{2b+1}' adjacent to two adjacent vertices, say v_b and v_{b+1} of the cycle.

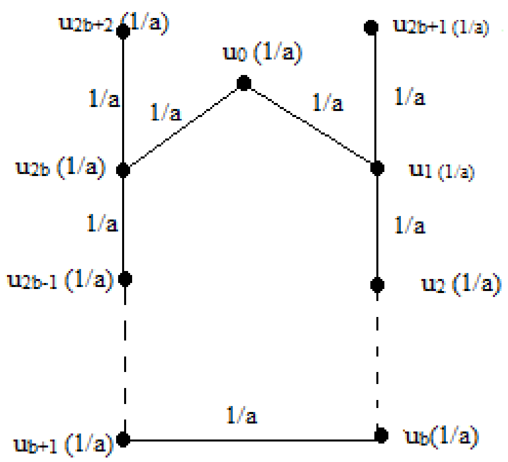
We define the partial fuzzy subgraph (σ, μ) on G as follows:
$\sigma(v_i) = 1, \ \forall \ 0 \leq i \leq 2b+1$ and $\mu(v_i, v_j) = 1/a$, $\forall$ edges $v_i - v_j$ in G. The fuzzy graph hence obtained is illustrated below:



G

Fig. 3

Then $e(v_0) = e(v_{2b+1}) = a(b+1)$ and for all other vertices v_i, $1 \leq i \leq 2b$, $e(v_i) = ab$. Hence
$r(G) = ab$.The fuzzy wing graph of G has 2b+3 vertices, say $u_0, u_1, \ldots, u_{2b+3}$ with all vertices and edges of degree 1/a. It is illustrated below:



W (G)

Fig.4

Then $e(u_i) = ab$, $0 \le i \le .2b+2$ so that r[W (G)] = ab.
Thus we obtain the fuzzy graph G such that r (G) = r [W (G)] = ab.

The following corollary has a constructive proof similar to the construction in the above theorem:

Corollary 4.2 *Given any integer a > 1, there exists a fuzzy graph G = (σ, μ) such that d (G) - d[W(G)] = a.*

Proof Construct a graph G as follows: Let $v_0, v_1, \ldots, v_{2a}$ be the vertices of the cycle C_{2a+1}. Introduce a vertex 'v_{2a+1}' adjacent to two adjacent vertices, say v_a and v_{a+1} of the cycle.

We define the partial fuzzy subgraph (σ, μ) on G as follows:
$\sigma(v_i) = 1$, $\forall\ 0 \le i \le 2a+1$ and $\mu(v_i, v_j) = 1/a$, $\forall$ edges $v_i - v_j$ in G.
Then $e(v_0) = e(v_{2a+1}) = a(a+1)$ and for all other vertices v_i, $1 \le i \le 2a$, $e(v_i) = a^2$.
Hence d (G) = a (a+1).
The fuzzy wing graph of G has 2a+3 vertices, say $u_0, u_1, \ldots, u_{2a+3}$, including two pendant vertices adjacent to a non- adjacent vertices of an induced P_3 in C_{2a+1}. It has all vertices and edges of degree 1/a. The eccentricity of each vertex in it is 'a^2' so that d [W (G)] = a^2.
Thus we obtain the fuzzy graph G such that d (G) - d [W (G)] = a. □

5. CONCLUSION

In this article, we have presented the fuzzy analogues of some classical results regarding wing graphs as well as some independent results. It is possible to explore if the study of fuzzy analogues of the wing graph may help in studies regarding perfect graphs. Further, the construction of a wing graph with the same radius as its fuzzy wing graph may prove to be of use in the study of interconnection networks that minimize faults.

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REFERENCES

1. P. Bhattacharya, Some remarks on fuzzy graphs, Pattern Recognition Letters 6 (1987) 297-302.
2. K. R. Bhutani, A. Rosenfeld, Fuzzy end nodes in fuzzy graphs, Information Sciences 152(2003) 323- 326
3. K. R. Bhutani, A. Rosenfeld, Geodesics in fuzzy graphs, Electronic Notes in Discrete Mathematics 15 (2003) 51- 54
4. K. R. Bhutani, A. Rosenfeld, Strong arcs in fuzzy graphs, Information Sciences 152 (2003) 319 - 322
5. F. Harary, Graph Theory, Narosa Publishing House (1998).
6. C. T. Hoang, On the two edge colorings of perfect graphs, Journal of Graph Theory 19(1995) 271-279
7. S. Hougardy, On wing perfect graphs, Journal of Graph Theory 31(1999) 51-66
8. S. Hougardy, V. B. Le, A. Wagler, Wing triangulated graphs are perfect, Journal of Graph Theory 24 (1997) 25 – 31
9. L. Hsu, C. Lin, Graph Theory and Interconnection Networks, CRC Press (2009)
10. J. N. Mordeson, Fuzzy line graphs, Pattern Recognition Letters 14 (1993) 381 - 384.
11. J. N. Mordeson and P. S. Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Physica Verlag (2000).
12. R. Pramada, On some properties of wing graphs, Pauline Journal of Research & Studies Volume 1, Issue 2 (2014) 247 – 252.
13. O. Ore, Theory of Graphs, American Mathematical Society Colloquium Publications 38 Providence (R.I) (1962) 21.
14. E. Prisner, Graph Dynamics, Longman (1995)
15. A. Rosenfeld, Fuzzy graphs, in: Fuzzy sets and their Applications, L.A. Zadeh, K.S. Fu and M. Shimura (Eds.), Academic Press, New York (1975) pp. 77-95.
16. A. Somasundaram and S. Somasundaram , Domination in fuzzy graphs, Pattern Recognition Letters 19 (1998) 787-791
17. S. Mathew, M. S. Sunitha, Node connectivity and arc connectivity of a fuzzy graph, Information Sciences 180 (2010) 519 – 531
18. S. Mathew, M. S. Sunitha, Menger's Theorem for fuzzy graphs, Information Sciences 222 (2013) 717 - 726
19. J. Xu, Topological Structure and Analysis of Interconnection Networks, Kluver Academic Publishers (2001).