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Decomposition of Locally Closed Sets in Topological Spaces

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Abstract

The purpose of this paper is to introduce $g^*\omega\alpha$-lc sets, $g^*\omega\alpha^*$-lc sets and $g^*\omega\alpha^{**}$-lc sets and different notions of generalizations of continuous functions in topological spaces and study their properties.

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1. Introduction

The notion of locally closed sets in the literature was first introduced and studied by Kuratowaski and Sierpienski [4]. Bourbaki [2] defined, a subset A of a topological space X is locally closed if it is the intersection of an open set and a closed set. Further, Stone [8] has used the term FG for a locally closed subset. Using the concept of a locally closed set, in 1989 Ganster and Reilly [3] continued their work and introduced the concept of LC-continuous and LC-irresolute maps to find a decomposition of continuous functions.

In 1996, Balachandran et al. [1] introduced and investigated the concepts of generalized locally closed sets and obtained different notions of continuity called GLC-continuity and GLC-irresolue maps. Various authors contributed to the development of generalizations of locally closed sets and locally closed continuous functions in topological spaces.

In this paper, we introduce new classes of sets called g$^*\omega\alpha$-lc sets, g$^*\omega\alpha^*$-lc sets and g$^*\omega\alpha^{**}$-lc sets by using the notion of g$^*\omega\alpha$-closed and g$^*\omega\alpha$-open sets. We study some of their properties and the relationship among these classes and the other existing classes of sets. Finally, we also introduce and study different classes of weaker forms of continuity and irresoluteness and some of their properties in topological spaces.

2. Preliminaries

Throughout this paper (X, τ), (Y, μ) and (Z, σ) (or simply X, Y and Z) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X the closure and interior of A with respect to τ are denoted by cl(A) and int(A) respectively.

Definition 2.1. **[5]** Let A be a subset of X. Then A is said to be a generalized star $\omega\alpha$-closed (briefly $g^*\omega\alpha$-closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\omega\alpha$-open in X.

Definition 2.2. A subset A of a space X is called a

(i) locally closed [3] if $A = G \cap F$ where G is open and F is closed in X.

(ii) generalized locally closed (briefly glc-closed) [1] if $A = G \cap F$ where G is g-open and F is g-closed in X.

Theorem 2.3. **[6]** In a topological space X, if every $g^*\omega\alpha$-closed set is closed then X is said to be $T_{g^*\omega\alpha}$-space.

Definition 2.4. **[7]** A function f : X $\rightarrow$ Y is called a

(i) $g^*\omega\alpha$-continuous if f^{-1}(V) is $g^*\omega\alpha$-closed in X for every closed set V in Y.

(ii) $g^*\omega\alpha$-irresolute if f^{-1}(V) is $g^*\omega\alpha$-closed in X for every $g^*\omega\alpha$-closed set V in Y.

Proposition 2.5. **[5]** Following statements are true for a topological space X:

(i) A is $g^*\omega\alpha$-closed in X if and only if $g^*\omega\alpha$-cl(A) = A.

(ii) $g^*\omega\alpha$-cl(A) is $g^*\omega\alpha$-closed in X.

(iii) $x \in g^*\omega\alpha - cl(A)$ if and only if $A \cap U \neq \phi$ for every $U \in G^*\omega\alpha O(X, x)$.

3. $g^*\omega\alpha$-Locally Closed Sets in Topological Spaces

Definition 3.1. Let $A \subseteq X$. Then A is said to be a

(i) $g^*\omega\alpha$-locally closed (briefly $g^*\omega\alpha$-lc) if A = G $\cap$ F where G is $g^*\omega\alpha$-open and F is $g^*\omega\alpha$-closed in X.

(ii) $g^*\omega\alpha^*$-locally closed (briefly $g^*\omega\alpha^*$-lc) if A = G $\cap$ F where G is $g^*\omega\alpha$-open and F is closed in X.

(iii) $g^*\omega\alpha^{**}$-locally closed (briefly $g^*\omega\alpha^{**}$-lc) if A = G $\cap$ F where G is open and F is $g^*\omega\alpha$-closed in X.

Remark 3.2. The family of all $g^*\omega\alpha$-lc sets (resp. $g^*\omega\alpha^*$-lc sets, $g^*\omega\alpha^{**}$-lc sets) of (X, τ) will be denoted by G$^*\omega\alpha$-LC(X, τ)(resp. G$^*\omega\alpha^*$-LC(X, τ), G$^*\omega\alpha^{**}$-LC(X, τ)).

Remark 3.3. It is obvious that every $g^*\omega\alpha$-closed (resp. $g^*\omega\alpha$-open) set is $g^*\omega\alpha$-locally closed.

Remark 3.4. Every locally closed set is $g^*\omega\alpha$-locally closed but not conversely.

Example 3.5. $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. A subset $\{a, b\}$ of the space X is $g^*\omega\alpha$-locally closed but not locally closed.

Remark 3.6.

(i) Every locally closed set is $g^*\omega\alpha^*$-lc and $g^*\omega\alpha^{**}$-lc set.

(ii) Every $g^*\omega\alpha$-lc set is $g^*\omega\alpha^*$-lc set and $g^*\omega\alpha^{**}$-lc set. However, converses of the above remark need not be true in general as seen from the following example.

Example 3.7. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$. A subset $\{a, c\}$ of X is $g^*\omega\alpha$-locally closed but not $g^*\omega\alpha^*$-locally closed and locally closed. The set $\{c\}$ is $g^*\omega\alpha$-locally closed but not $g^*\omega\alpha^{**}$-locally closed.

Remark 3.8. The class of $g^*\omega\alpha^*$-locally closed and $g^*\omega\alpha^{**}$-locally closed sets are independent of each other as seen from the following example.

Example 3.9. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, c\}\}$. In this topological space X, the set $A = \{b\}$ is $g^*\omega\alpha^*$-locally closed but not $g^*\omega\alpha^{**}$-locally closed.

Example 3.10. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$. In this topological space X, the set $A = \{a\}$ is $g^*\omega\alpha^{**}$-locally closed but not $g^*\omega\alpha^*$-locally closed.

We have the following characterizations:

Proposition 3.11. The following properties holds for a $T_{g^*\omega\alpha}$-space:

(i) $G^*\omega\alpha - LC(X, \tau) = LC(X, \tau)$.

(ii) $G^*\omega\alpha - LC(X, \tau) \subseteq G\omega\alpha - LC(X, \tau)$.

Proof.

(i) Follows from the fact that every closed set is $g^*\omega\alpha$-closed by [5] and from the definition of $T_{g^*\omega\alpha}$-space.

(ii) In any space (X, τ), $LC(X, \tau) \subseteq G\omega\alpha - LC(X, \tau)$ and from (i), we have $G^*\omega\alpha - LC(X, \tau) \subseteq G\omega\alpha - LC(X, \tau)$. ■

Theorem 3.12. The following properties are equivalent for any subset A of a space X:

(i) $A \in G^*\omega\alpha$-LC(X, τ)

(ii) $A = G \cap g^*\omega\alpha - cl(A)$ for some $g^*\omega\alpha$-open set G

(iii) $g^*\omega\alpha - cl(A) \setminus$ A is $g^*\omega\alpha$-closed

(iv) $A \cup (X \setminus g^*\omega\alpha - cl(A))$ is $g^*\omega\alpha$-open.

Proof.

(i) → (ii): Let A $\in G^*\omega\alpha$-LC(X, τ). Then there exist $g^*\omega\alpha$-open set G and $g^*\omega\alpha$-closed set F such that A = $G \cap F$. Since $A \subseteq G$ and $A \subseteq g^*\omega\alpha - cl(A)$ then $A \subseteq G \cap g^*\omega\alpha - cl(A)$.

Conversely, from Proposition 2.1(ii), we have $g^*\omega\alpha - cl(A) \subseteq$ F and hence $G \cap g^*\omega\alpha - cl(A) \subseteq G \cap F = A$.

(ii) → (i): From hypothesis and Proposition 2.1(ii), $g^*\omega\alpha - cl(A)$ is $g^*\omega\alpha$-closed and hence A = $G \cap g^*\omega\alpha - cl(A) \in G^*\omega\alpha$-LC(X, τ).

(ii) → (iii): Suppose (ii) holds then $g^*\omega\alpha - cl(A) \setminus$ A = $g^*\omega\alpha - cl(A) \cap (X \setminus G)$ which is $g^*\omega\alpha$-closed. Hence $g^*\omega\alpha - cl(A) \setminus$ A is $g^*\omega\alpha$-closed.

(iii) → (ii): Let G = $X \setminus (g^*\omega\alpha - cl(A) \setminus A)$. Then from (iii), G is $g^*\omega\alpha$-open in X and hence A = $G \cap g^*\omega\alpha - cl(A)$ holds.

(iii) → (iv): Let F = $g^*\omega\alpha - cl(A) \setminus A$. Then $X \setminus F = A \cup (X \setminus g^*\omega\alpha - cl(A))$ holds and $X \setminus F$ is $g^*\omega\alpha$-open. Hence $A \cup (X \setminus g^*\omega\alpha - cl(A))$ is g$^*\omega\alpha$-open.

(iv) → (iii): Let G = $A \cup (X \setminus g^*\omega\alpha - cl(A))$. Since $X \setminus G$ is $g^*\omega\alpha$-closed and $X \setminus G = g^*\omega\alpha - cl(A) \setminus A$ holds. Hence $g^*\omega\alpha - cl(A) \setminus A$ is $g^*\omega\alpha$-closed. ■

Theorem 3.13. Let A be any subset of a space (X, τ). Then the following properties are equivalent:

(i) $A \in G^*\omega\alpha^*$-LC(X, τ)

(ii) $A = U \cap cl(A)$ for some $g^*\omega\alpha$-open set G

(iii) $cl(A) \setminus A$ is $g^*\omega\alpha$-closed

(iv) $A \cup (X \setminus cl(A))$ is $g^*\omega\alpha$-open.

Proposition 3.14. Let A be a subset of (X, τ). If $A \in G^*\omega\alpha^{**}$-LC(X, τ) if and only if $A = U \cap g^*\omega\alpha - cl(A)$ for some open set U.

Proof. Necessity: Let $A \in G^*\omega\alpha^{**}$-LC(X, τ). Then there exist open set G and $g^*\omega\alpha$-closed set F such that $A = G \cap F$. Then from Proposition 2.1 (ii), $A \subseteq F$ implies

$g^*\omega\alpha$-cl(A) $\subseteq$ F. Now A = $A \cap g^*\omega\alpha$-cl(A) = $G \cap F \cap g^*\omega\alpha$-cl(A) = $G \cap g^*\omega\alpha$-cl(A).

Sufficiency: Let $A = G \cap g^*\omega\alpha$-cl(A) for some open set G. Then from Proposition 2.1 (ii), $g^*\omega\alpha$-cl(A) is $g^*\omega\alpha$-closed and hence $A = G \cap g^*\omega\alpha$-cl(A) $\in G^*\omega\alpha^{**}$-LC(X, τ). ■

Theorem 3.15. Let A be a subset of (X, τ). If A $\in G^*\omega\alpha^{**}$-LC(X, τ) then

(i) $g^*\omega\alpha$-cl(A)$\backslash A$ is $g^*\omega\alpha$-closed.

(ii) $A \cup (X \setminus g^*\omega\alpha$-cl(A)) is $g^*\omega\alpha$-open.

Proof. (i) Let $A \in G^*\omega\alpha^{**}$-LC(X, τ) then A = $G \cap F$ where G is open and F is $g^*\omega\alpha$-closed. Since $A \subseteq G$ and $A \subseteq g^*\omega\alpha$-cl(A) so $A \subseteq G \cap g^*\omega\alpha$-cl(A).

Conversely, from Proposition 2.1 (ii), we have $g^*\omega\alpha$-cl(A) $\subseteq F$ and hence $G \cap g^*\omega\alpha$-cl(A) $\subseteq G \cap F$ = A. Therefore A = $G \cap g^*\omega\alpha$-cl(A). Then it follows from the assumption that $g^*\omega\alpha$-cl(A) $\setminus$ A = $g^*\omega\alpha$-cl(A) $\cap (X \setminus G)$ is $g^*\omega\alpha$-closed in X.

(ii) From (i), $g^*\omega\alpha$-cl(A) $\setminus$ A is $g^*\omega\alpha$-closed in X and let F = $g^*\omega\alpha$-cl(A)$\backslash A$. Since $X \setminus F = A \cup (X \setminus g^*\omega\alpha$-cl(A)) holds so $X \setminus F$ is $g^*\omega\alpha$-open. Therefore $A \cup (X \setminus g^*\omega\alpha$-cl(A)) is $g^*\omega\alpha$-open. ■

Definition 3.16. Let A and B be any two subsets of a space X. Then A and B are said to be separated if $A \cap cl(B) = \phi$ and $B \cap cl(A) = \phi$.

Theorem 3.17. Let $A, B \in G^*\omega\alpha^*$-LC(X, τ). Suppose that the collection of all $g^*\omega\alpha$-open sets of (X, τ) are closed under finite unions. If A and B are separated in (X, τ) then $A \cup B \in g^*\omega\alpha^*$-LC(X, τ).

Proof. Since A, B $\in g^*\omega\alpha^*$-LC(X, τ). Then from Theorem 3.2 (ii), there exist $g^*\omega\alpha$-open sets G and F in X such that $A = G \cap cl(A)$ and $B = F \cap cl(B)$. Put $U = G \cap (X - cl(B))$ and $V = F \cap (X - cl(A))$. Then U and V are $g^*\omega\alpha$-open subsets of X implies that $A = U \cap cl(A)$, $B = U \cap cl(B)$, $U \cap cl(B)) = \phi$ and $U \cap cl(A) = \phi$. Therefore $A \cup B = (U \cup V) \cap (cl(A \cup B))$, that is $A \cup B \in g^*\omega\alpha^*$-LC(X, τ). ■

Remark 3.18. From the following example we can observe that assumption A and B are separated cannot be removed.

Example 3.19. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a, b\}\}$. The set $\{b, c\} \in g^*\omega\alpha^*$-lc and $\{c\} \in g^*\omega\alpha^*$-lc but $\{b, c\} \notin g^*\omega\alpha^*$-lc.

Remark 3.20. Union of two $g^*\omega\alpha$-lc (resp. $g^*\omega\alpha^*$-lc, $g^*\omega\alpha^{**}$-lc) sets need not be $g^*\omega\alpha$-lc (resp. $g^*\omega\alpha^*$-lc, $g^*\omega\alpha^{**}$-lc) sets as seen from the following example.

Example 3.21. Let $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\}$.Then $\{a\}$ and $\{c\}$ are $g^*\omega\alpha$-lc sets but $\{a\} \cup \{c\} = \{a, c\}$ is not $g^*\omega\alpha$-lc set in X.

Example 3.22. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$.Then $\{b\}$ and $\{c\}$ are $g^*\omega\alpha^*$-lc and $g^*\omega\alpha^{**}$-lc sets but $\{b, c\}$ is not $g^*\omega\alpha$-lc and $g^*\omega\alpha^{**}$-lc in X.

Theorem 3.23. Let $A, B \subseteq X$. Suppose that the collection of $g^*\omega\alpha$-closed set of X is closed under finite intersection then the following properties holds:

(i) if $A \in g^*\omega\alpha$-LC(X, τ) and B is $g^*\omega\alpha$-open and $g^*\omega\alpha$-closed then $A \cap B \in g^*\omega\alpha$-LC(X, τ).

(ii) if $A, B \in g^*\omega\alpha^*$-LC(X, τ) then $A \cap B \in g^*\omega\alpha^*$-LC(X, τ).

Proof. (i) Let $A \in g^*\omega\alpha$-LC(X, τ). Then there exist $g^*\omega\alpha$-open set G and $g^*\omega\alpha$-closed set F such that $A = G \cap F$, so $A \cap B = (G \cap F) \cap B$.

If B is $g^*\omega\alpha$-open then $A \cap B = (G \cap B) \cap F \in g^*\omega\alpha$-LC(X, τ).

If B is $g^*\omega\alpha$-closed then $A \cap B = G \cap (F \cap B) \in g^*\omega\alpha$-LC(X, τ). Hence $A \cap B$ is $g^*\omega\alpha$-closed. Thus $A \cap B \in g^*\omega\alpha$-LC(X, τ).

(ii) Let $A, B \in g^*\omega\alpha^*$-LC(X, τ). Then from Theorem 3.2 (ii), there exist $g^*\omega\alpha$-open sets P and Q such that $A = P \cap cl(A))$ and $B = Q \cap cl(B))$. Then $A \cap B = (P \cap cl(A)) \cap (Q \cap cl(B)) = (P \cap Q) \cap (cl(A) \cap cl(B)) \in g^*\omega\alpha^*$-LC(X, τ). ■

Proposition 3.24. Let A and B be any two subsets of a space X such that $A \subseteq B$. Suppose the collection of $g^*\omega\alpha$-open sets of X are closed under finite intersection. If B is $g^*\omega\alpha$-open in X and $A \in g^*\omega\alpha^* - LC(B, \tau/B)$ then $A \in g^*\omega\alpha^* - LC(X, \tau)$.

Proof. Let $A \in g^*\omega\alpha^* - LC(B, \tau/B)$ then there exists $g^*\omega\alpha$-open set G in $(X, \tau/B)$ such that $A = G \cap cl(A)_B$ where $cl(A)_B = B \cap cl(A)$. Since G and B are $g^*\omega\alpha$-open then $G \cap B$ be is also $g^*\omega\alpha$-open [5]. This implies that $A = (G \cap B) \cap cl(A) \in g^*\omega\alpha^* - LC(X, \tau)$. ■

Definition 3.25. Let X be a topological space. Then X is said to be $g^*\omega\alpha$-submaximal if every desne subset is $g^*\omega\alpha$-open.

Remark 3.26. Every submaximal space is $g^*\omega\alpha$-submaximal. However the converse need not be true as seen from the following example.

Example 3.27. $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. Let $A = \{a, b\}$ then A is dense in X such that A is $g^*\omega\alpha$-open but not open in X.

Theorem 3.28. A topological space X is $g^*\omega\alpha$-submaximal if and only if P(X) = $g^*\omega\alpha - LC(X, \tau)$.

Proof. Let X be a $g^*\omega\alpha$-submaximal and A ∈ P(X). Let $U = A \cup (X \setminus cl(A))$. Then cl(U) = X and hence U is dense in X. Since X is $g^*\omega\alpha$ submaximal, so U is $g^*\omega\alpha$-open in X. Then from Theorem 3.2, $A \in g^*\omega\alpha^* - LC(X, \tau)$. This implies that P(X) = $g^*\omega\alpha^* - LC(X, \tau)$.

Conversely, let A be dense in X. Then $A \cup (X \setminus cl(A)) = A \cup \phi = A$. Since $A \in g^*\omega\alpha^* - LC(X, \tau)$, $A = A \cup (X \setminus cl(A))$ is $g^*\omega\alpha$-open from Theorem 3.2 (iv). Hence X is $g^*\omega\alpha$-submaximal. ■

Proposition 3.29. Let $\{Z_i : i \in \tau\}$ be a cover of X, where τ is finite set and A be a subset of X. Suppose $\{Z_i : i \in \tau\}$ is $g^*\omega\alpha$-closed in X and the collection of $g^*\omega\alpha$-closed sets is closed under finite unions. If $A \cap Z_i \in g^*\omega\alpha^{**} - LC(Z_i, \tau/Z_i)$ for each $i \in \tau$ then $A \in g^*\omega\alpha^{**} - LC(X, \tau)$.

Proof. Let $i \in \tau$. Since $A \cap Z_i \in g^*\omega\alpha^{**} - LC(Z_i, \tau/Z_i)$ there exist an open set U_i of (X, τ) and $g^*\omega\alpha$-closed set F_i of $(Z_i, \tau/Z_i)$ such that $A \cap Z_i = (U_i \cap Z_i) \cap F_i = U_i \cap (Z_i \cap F_i)$. Then $A = \cup\{A \cap Z_i : i \in \tau\} = \cup\{U_i : i \in \tau\} \cap (\cup\{Z_i \cap F_i : i \in \tau\})$. Hence $A \in g^*\omega\alpha^{**} - LC(X, \tau)$. ■

4. $g^*\omega\alpha$-LC Continuous and $g^*\omega\alpha$-LC Irresolute Functions in Topological Spaces

Definition 4.1. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then f is called a

(a) $g^*\omega\alpha - LC$ (resp. $g^*\omega\alpha^* - LC$ and $g^*\omega\alpha^{**} - LC$) continuous if for each $V \in (Y, \sigma)$, $f^{-1}(V) \in g^*\omega\alpha - LC(X, \tau)$ (resp. $g^*\omega\alpha^* - LC(X, \tau)$ and $g^*\omega\alpha^{**} - LC(X, \tau)$).

(b) $g^*\omega\alpha - LC$ (resp. $g^*\omega\alpha^* - LC$ and $g^*\omega\alpha^{**} - LC$) irresolute if for each $V \in g^*\omega\alpha - LC(Y, \sigma)$, $f^{-1}(V) \in g^*\omega\alpha - LC(X, \tau)$ (resp. $g^*\omega\alpha^* - LC(X, \tau)$ and $g^*\omega\alpha^{**} - LC(X, \tau)$).

Theorem 4.2. The following properties holds for a function $f : (X, \tau) \to (Y, \sigma)$:

(a) if f is LC-continuous then f is $g^*\omega\alpha - LC$ continuous, $g^*\omega\alpha^* - LC$ continuous and $g^*\omega\alpha^{**} - LC$ continuous.

(b) if f is $g^*\omega\alpha^* - LC$ continuous or $g^*\omega\alpha^{**} - LC$ continuous then f is $g^*\omega\alpha - LC$ continuous.

(c) if f is $g^*\omega\alpha^* - LC$ (resp. $g^*\omega\alpha^* - LC$ and $g^*\omega\alpha^{**} - LC$) irresolte then it is $g^*\omega\alpha - LC$ (resp. $g^*\omega\alpha^* - LC$ and $g^*\omega\alpha^{**} - LC$) continuous.

Proof.

(a) It follows from the Remark 3.4.

(b) Since every $g^*\omega\alpha^*$-lc and $g^*\omega\alpha^{**}$-lc set is $g^*\omega\alpha$-lc set and hence the proof follows.

(c) Since every open set is $g^*\omega\alpha$-open, $g^*\omega\alpha^*$-open and $g^*\omega\alpha^{**}$-open sets. However the converse of the above statements need not be true as seen from the following examples. ■

Example 4.3. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a, b\}\}$. Then the identity function $f : X \to Y$ is $g^*\omega\alpha^* - lc$ continuous but not lc-continuous, since for the set $A = \{a, b\}$ in Y, $f^{-1}(\{a, b\}) = \{a, c\}$ is not locally closed in X.

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a, b\}\}$. Then the identity function f is $g^*\omega\alpha^* - lc$-continuous but not $g^*\omega\alpha - lc$-irresolute, $g^*\omega\alpha^* - lc$-continuous and $g^*\omega\alpha^{**} - lc$-continuous. Consider the set $\{a, c\}$ in Y, $f^{-1}(\{a, c\}) = \{a, c\}$ is not $g^*\omega\alpha$-locally closed in X.

Proposition 4.4. Let $f : (X, \tau) \to (Y, \sigma)$ be $g^*\omega\alpha$-irresolute injective map. Then

(a) if $B \in g^*\omega\alpha - LC(Y, \sigma)$ then $f^{-1}(B) \in g^*\omega\alpha - LC(X, \tau)$.

(b) if X is $T_{g^*\omega\alpha}$-space and $B \in g^*\omega\alpha - LC(Y, \sigma)$ then $f^{-1}(B) \in LC(X, \tau)$.

Proof.

(a) Let $B \in g^*\omega\alpha - LC(X, \tau)$. Then there exist $g^*\omega\alpha$-open set G and $g^*\omega\alpha$-closed set F such that $B = G \cap F$, $f^{-1}(B) = f^{-1}(G) \cap f^{-1}(F)$. Since f is $g^*\omega\alpha$-irresolute, $f^{-1}(G)$ and $f^{-1}(F)$ are $g^*\omega\alpha$-open and $g^*\omega\alpha$-closed sets in X respectively. Hence $f^{-1}(B) \in g^*\omega\alpha - LC(X, \tau)$.

(b) Let $B \in g^*\omega\alpha - LC(Y, \sigma)$. There exist $g^*\omega\alpha$-open set G and $g^*\omega\alpha$-closed set F such that $B = G \cap F$, $f^{-1}(B) = f^{-1}(G) \cap f^{-1}(F)$. Since f is $g^*\omega\alpha$-irresolute map, $f^{-1}(G)$ and $f^{-1}(F)$ are $g^*\omega\alpha$-open and $g^*\omega\alpha$-closed sets in (X, τ) respectively. From hypothesis $f^{-1}(G)$ and $f^{-1}(F)$ are open and closed sets in X. Hence $f^{-1}(B) \in LC(X, \tau)$. ■

Theorem 4.5. Any map defined on a door space is $g^*\omega\alpha - LC$ continuous (resp. $g^*\omega\alpha - LC$ irresolute).

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a function where (X, τ) is a door space. Let $A \in (Y, \sigma)$ (resp. $A \in g^*\omega\alpha - LC(Y, \sigma)$) then $f^{-1}(A)$ is either open or closed. Since every open or closed set is $g^*\omega\alpha$-open or $g^*\omega\alpha$-closed [5] respectively and hence $f^{-1}(A) \in g^*\omega\alpha - LC(X, \tau)$. Hence f is $g^*\omega\alpha - LC$ continuous (resp. $g^*\omega\alpha - LC$ irresolute). ■

Proposition 4.6. $g^*\omega\alpha - LC$ continuous and contra-continuous maps defined on a $T_{g^*\omega\alpha}$-space is $g^*\omega\alpha - LC$ irresolute.

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be $g^*\omega\alpha - LC$-continuous and contra-continuous maps and (Y, σ) be $T_{g^*\omega\alpha}$-space. Let $G \in g^*\omega\alpha - LC(Y, \sigma)$ then $G = U \cap F$ where U is $g^*\omega\alpha$-open and F is $g^*\omega\alpha$-closed then U is open and F is closed in (Y, σ). Consider $f^{-1}(G) = f^{-1}(U) \cap f^{-1}(F)$, where $f^{-1}(U)$ is $g^*\omega\alpha$-locally closed and $f^{-1}(F)$ is open. Therefore $f^{-1}(G)$ is $g^*\omega\alpha$-locally closed in (X, τ) by Theorem 3.5. ■

Proposition 4.7. A topological space (X, τ) is $g^*\omega\alpha$-submaximal if and only if every function having (X, τ) as a domain is $g^*\omega\alpha - LC$-continuous.

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then from Theorem 3.6, P(X) = $g^*\omega\alpha - LC(X, \tau)$. Let U be an open set in (Y, σ) then $f^{-1}(U) \in P(X) = g^*\omega\alpha - LC(X, \tau)$, so f is $g^*\omega\alpha - LC$ continuous.

Conversely, let us consider the Sierpinski space $Y = \{0, 1\}$ with $\sigma = \{\phi, Y, \{0\}\}$. Let V be a subset of X and define a function $f : (X, \tau) \to (Y, \sigma)$ as f(x) = 0 if $x \in V$ and f(x) = 1 if $x \notin V$. Then it follows from the assumption that $f^{-1}\{(0)\}$ = V ∈ $g^*\omega\alpha - LC(X, \tau)$. Therefore, we have P(X) = $g^*\omega\alpha - LC(X, \tau)$ and so X is $g^*\omega\alpha$-submaximal. ■

Now we will recall the definition of combination of two functions.
Let $X = A \cup B$ and $f : A \to Y$ and $h : B \to Y$ be any two functions. We say that f and h are compatible if $f \mid A \cap B = h \mid A \cap B$, we define a function $f \bigtriangledown h : X \to Y$ as follows:
$(f \bigtriangledown h)(x) = f(x)$ for every $x \in A$
$(f \bigtriangledown h)(x) = h(x)$ for every $x \in B$.
Then the function $f \bigtriangledown h : X \to Y$ is called the combination of f and h.

Proposition 4.8. Let $X = A \cup B$ where A and B are $g^*\omega\alpha$-closed sets of X and $f : (A, \tau \mid A) \to (Y, \sigma)$ and $h : (B, \tau \mid B) \to (Y, \sigma)$ be compatible functions. If f and h are $g^*\omega\alpha^{**} - LC$ continuous (resp. $g^*\omega\alpha^{**} - LC$ irresolute) then $f \bigtriangledown h : (X, \tau) \to (Y, \sigma)$ is $g^*\omega\alpha^{**} - LC$ continuous (resp. $g^*\omega\alpha^{**} - LC$ irresolute).

Proof. Let $V \in (Y, \sigma)$ (resp. $g^*\omega\alpha^{**} - LC(Y, \sigma)$) then $(f \bigtriangledown h)^{-1}(V) \cap A = f^{-1}(V)$ and $(f \bigtriangledown h)^{-1}(V) \cap B = h^{-1}(V)$ holds. By assumption, we have $(f \bigtriangledown h)^{-1}(V) \cap A \in g^*\omega\alpha^{**} - LC(A, \tau/A)$ and $(f \bigtriangledown h)^{-1}(V) \cap B \in g^*\omega\alpha^{**} - LC(B, \tau/B)$. Then it follows that $(f \bigtriangledown h)^{-1}(V) \in g^*\omega\alpha^{**} - LC(X, \tau)$ and hence $f \bigtriangledown h$ is $g^*\omega\alpha^{**} - LC$ continuous (resp. $g^*\omega\alpha^{**} - LC$ irresolute). ■

Now we have the theorems concerning to composition of maps:

Theorem 4.9. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ are any two functions. Then

(a) if f and g are $g^*\omega\alpha - LC$ irresolute (resp. $g^*\omega\alpha^* - LC$ irresolute and $g^*\omega\alpha^{**} - LC$ irresolute) then $g \circ f$ is $g^*\omega\alpha - LC$ irresolute (resp. $g^*\omega\alpha^* - LC$ irresolute and $g^*\omega\alpha^{**} - LC$ irresolute).

(b) if f is $g^*\omega\alpha - LC$ irresolute and g is $g^*\omega\alpha - LC$ continuous then $g \circ f$ is $g^*\omega\alpha - LC$ continuous.

Proof.

(a) Let $V \in g^*\omega\alpha - LC(Z)$ (resp. $g^*\omega\alpha^* - LC(Z)$ and $g^*\omega\alpha^{**} - LC(Z)$) then $g^{-1}(V) \in g^*\omega\alpha - LC(Y)$ (resp. $g^*\omega\alpha^* - LC(Y)$ and $g^*\omega\alpha^{**} - LC(Y)$) and since f is $g^*\omega\alpha - LC$ irresolute (resp. $g^*\omega\alpha^* - LC$ irresolute and $g^*\omega\alpha^{**} - LC$ irresolute) , $(f^{-1})^{-1}(V)) = (g \circ f)^{-1}(V) \in g^*\omega\alpha - LC(X)$ (resp. $g^*\omega\alpha^* - LC(X)$ and $g^*\omega\alpha^{**} - LC(X)$). Therefore $(g \circ f)$ is $g^*\omega\alpha - LC$ irresolute (resp. $g^*\omega\alpha^* - LC$ irresolute and $g^*\omega\alpha^{**} - LC$ irresolute).

(b) Let $V \in Z$ then $g^{-1}(V) \in g^*\omega\alpha - LC(Y)$ and $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) \in g^*\omega\alpha - LC(X)$ as f is $g^*\omega\alpha - LC$ irresolute. Therefore $(g \circ f)^{-1}(V) \in g^*\omega\alpha - LC(X)$. Hence $g \circ f$ is $g^*\omega\alpha - LC$ continuous. ■

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