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Fuzzy Transportation by Using Monte Carlo method

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Abstract

In this article we consider fuzzy balance transportation problem by using triangular fuzzy number. For finding the initial solution we convert given transportation problem into linear programming problem and find optimum solution by using Simplex Method. The main objective of this paper is to find optimal solution to given transportation problem using Monte Carlo Method i.e. by using fuzzy random number.

Keywords: Monte Carlo Method, Simplex Method Random number, fuzzy transportation problem, central triangular fuzzy number.

1. INTRODUCTION

The Monte Carlo technique is used to solve many numerical problems in engineering, finance statistics and science by using random number. Transportation problem generally studied in operation research field which has been used to simulate different type of real life problems. The basic application of Transportation problem is shipping commodity from source to destination. Further transportation model can be extended to other areas of operation such as employment scheduling, personnel assignment and inventory control. Fuzzy mathematics is used in many areas such as engineering, business, mathematics, psychology, management, medicine and image processing and pattern recognition. To obtain optimum solution of transportation model many times we are faced with the problem of incompleteness uncertain data. This is due to by a lack of knowledge about the consider system or changing nature of the world.

Random number is heart of Monte Carlo Method. Many numerical problems in applied science and technology can be solved by using this method. Monte Carlo Method gives approximate optimum solution close to crisp solution. Also Monte Carlo Method is very useful to obtain approximate solution to fuzzy optimization problems (Jowers, 2008). Fuzzy Set Theory gives the formalization of approximate reasoning, and preserves the original information contents of imprecision. Hitchcock (L, 1941) first time developed the basic transportation problem. Appa (M, 1973) discussed different method of the transportation problem. In general, transportation problems are solved with the assumptions that unit cost of transportation from each source to each destination, supply of the product at each source and demand at each destination are specified in a exact way i.e., in crisp environment. But in practice, the parameters of the transportation problem are not always exactly known and stable. This imprecision may follow from the lack of exact information; uncertainty in judgment etc. Therefore, Zadeh (A Z. L., 1965) introduced the concept of fuzzy numbers. Saad& Abbas (A S. O., 2003) discussed an algorithm for solving the transportation problems in fuzzy environment. Das & Baruah (K, 2007) proposed vogel's approximation method to find the fuzzy initial basic feasible solution of fuzzy transportation problems in which all the parameters are represented by triangular fuzzy numbers. Basirzadeh (H, 2011) used the classical algorithms to find the fuzzy optimal solution of fully fuzzy transportation problems by transforming the fuzzy parameters into crisp parameters. Kaur& Kumar (A K. A., 2011) proposed a new method for the fuzzy transportation problems using ranking function. Deepika Rani, T R Gulati&Amit Kumar (Deepika Rani, 2014) developed method for unbalanced transportation problems in fuzzy environment.

This paper is organized as follows. In section 2, some preliminaries and definitions are given, also the triangular membership function is defined. In the next section, the general transportation problem with fuzzy triangular numbers is discussed. This is followed by the solution of transportation problem using Monte Carlo Method in section 4. Section 5 illustrates the solution of transportation problem through a numerical example. The computational complexity of the problem is given in this section. Finally, in section 6 conclusions are given.

2. PRELIMINARIES AND DEFINITIONS:

Fuzzy set: A fuzzy set is defined by $\{(x, \mu_A(x)): x \in A, \mu_A(x) \in [0, 1]\}$. In the pair $(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$, belong to the interval [0, 1], called Membership function.

Normality: A fuzzy set is called **normal** if its core is nonempty. In other words, there is at least one point $x \in X$ with $\mu_A(x) = 1$.

***α*–cut:** α-cut of a fuzzy set A is denoted by $A\alpha$ and is defined as $A\alpha = \{x \in X : \mu_A(x) \geq \alpha\}$

Fuzzy Number: A fuzzy set A on R must possess at least the following three properties to qualify as a fuzzy number,

(i) Set A must be a normal fuzzy set;

(ii) α –cut must be closed interval for every $\alpha \in [0, 1]$

(iii) The support of, α, must be bounded

Triangular Fuzzy Number: A triangular fuzzy number A or simply triangular number represented with three points as follows (a1, aM, a2) holds the following conditions

$(i)\ a1\ toaMisincreasingfunction$

$(ii) aMtoa2\ isdecreasingfunctio\ n$

$$(iii)\ a1 \leq aM \leq a2.$$

This representation is interpreted as membership functions

$$\mu_A(x) = \begin{cases} \frac{x-a1}{aM-a1} & a1 \leq x \leq aM \\ \frac{x-a2}{aM-a2} & aM \leq x \leq a2 \\ 0 & otherwise \end{cases}$$

Where [a1; a2] is the supporting interval and the point (aM; 1) is the peak

3. TRANSPORTATION PROBLEM

The transportation problem is to transport various amount of a single homogeneous commodity that are initially stored at various origins to different destinations in such a way that total transportation cost is a minimum.

Tabular Representation: Suppose there are m factories and n warehouses then transportation problem is usually represented in tabular form

Table 1 Tabular Representation of Crisp Transportation Problem

Destination / Origin	D1	D2	D3	------	Dn	Supply
O1	C_{11}	C_{12}	C_{13}	------	C_{1n}	A1
O2	C_{21}	C_{22}	C_{23}	------	C_{2n}	A2
⋮	⋮	⋮	⋮		⋮	⋮
Om	C_{m1}	C_{m2}	C_{m3}	------	C_{mn}	Am
Demand	B1	B2	B3	------	Bn	$\sum_{i=1}^{n} Bi = \sum_{j=1}^{m} Aj$

Feasible Solution: a set of non-negative individual allocation which simultaneously removes deficiencies is called feasible solution.

Basic feasible solution: A feasible solution to a m-origin and n- destination problem is said to be basic if the number of positive allocation are m + n -1.

Theorem 1.The transportation problem has triangular basis.

Theorem 2.There always exist an optimal solution to a balanced transportation problem.

Theorem 3.The number of basic variables in a transportation problem is at most m + n -1.

Transportation Problem into crisp Linear Programming Problem:

Let there be m origins, ith origin possessing Ai units (see table 1) of a certain product, whereas there are n destinations with destination on j requiring Bj units. Let Cij be the cost of shipping one unit product from ith origin to jth destination and 'Xij' be the amount to be shipped from ith origin to jth destination. Here we assume that $\sum Ai = \sum Bj$ i= 1,2,..,m and j= 1,2,...,n.

LPP formulation of above transportation problem is given below

$$\text{Min } z = \sum_{i=1}^{m} \sum_{j=1}^{n} Xij\ Cij \qquad \text{(objective function)}$$

Subject to

$$\sum_{j=1}^{n} xij = Ai \qquad \text{for } i= 1,2,.....,m$$

$$\sum_{i=1}^{m} xij = Bj \qquad \text{for } j = 1,2,.....,n$$

The problem is to determine non negative values of Xij satisfying both availability constraints.

Transportation Problem into crisp Linear Programming Problem:

Let there be m origins, ith origin possessing $\overline{Ai}$ units (see table 2) of a certain product, whereas there are n destinations with destination on j requiring $\overline{Bj}$ (see table 2) units. Let $\overline{Cij}$ be the cost of shipping one unit product from ith origin to jth destination and '$\overline{Xij}$' be the amount to be shipped from ith origin to jth destination. Here we assume that $\sum \overline{Ai} = \sum \overline{Bj}$ i= 1,2,..,m and j= 1,2,...,n. (see table 2)

LPP formulation of above transportation problem is given below

$$\text{Min } z = \sum_{i=1}^{m} \sum_{j=1}^{n} \overline{Xij\ Cij} \qquad \text{(objective function)}$$

Subject to

$$\sum_{j=1}^{n} \overline{Xij} = \overline{Ai} \qquad \text{for } i= 1, 2,....., m$$

$$\sum_{i=1}^{m} \overline{Xij} = \overline{Bj} \qquad \text{for } j = 1, 2,....., n$$

Where $\overline{Ai}$, $\overline{Bj}$, $\overline{Cij}$, $\overline{Xij}$ are all fuzzy triangular number.

The problem is to determine non negative fuzzy values of '$\overline{X_{ij}}$' satisfying both availability constraints.

4. MONTE CARLO METHOD:

Monte Carlo Method gives approximate solution to fuzzy optimization problem. It is a numerical method that makes use of random number to solve mathematical problem for which an analytical solution is not known; that is trough random number experiment on computer. To compare two random triangular fuzzy number say $\bar{X}$ =(x1/x2/x3) and $\bar{Y}$= (y1/y2/y3) we find here α cut say Xα and Yα. If each α cut Xα is less than or equal to each α-cut of Yα then we can say that fuzzy number $\bar{X} \leq \bar{Y}$.

Random Number:

Monte Carlo Method is deals with use of random number. We use Matlab function r = rand() to generate random number in the interval [0,1].then by using function (b-a)*r + a we can generate random number in any interval [a,b]. By using sort and reshape function of Matlab we can convert these random numbers into fuzzy triangular numbers.

Interval Containing Solution:

Range of interval is very important because exact selection of this interval will make Monte Carlo Method more efficient. If interval is to large then too many of random number rejected and if it is very small then we can miss optimum solution. Suppose there are m equation in n(x1,x2,....xn) variable then put n-1 variable equal to zero find value of x1 similarly find the values of x2,x3,...,xn by equating all variable equal to zero. Finally to obtain upper bound take maximum of x1, x2 ...xn.

Defuzzification: If $\bar{x}$ = (a, b, c) given central triangular fuzzy number then we use mean method to Defuzzify. i.e. x = $\frac{a+b+c}{3}$ or centre method i.e. x = b.

METHODOLOGY

In this section, first the algorithm and methodology are explained and then the system functions and testing are illustrated.

Algorithm

Step I: convert a given transportation problem into crisp linear programming problem.

Step II: Find optimum solution to given Linear programming problem by using Simplex method.

Step III: convert a given transportation problem into fuzzy linear programming problem by using triangular fuzzy number.

Step IV: apply Monte Carlo Method to find an optimum solution to given fuzzy linear programming problem.

5. NUMERICAL EXAMPLE

Consider the following balance transportation problem having four destinations and three origins.

Table 2 Crisp Transportation Problem

Destination / Origin	D1	D2	Supply
O1	10	15	20
O2	2	13	25
Demand	15	30	45

Transportation problem into crisp linear programming problem:

Min z = 10x1 + 15x2 + 2x3 + 13x4

Subject to

x1 + x2 = 20

x3 + x4 = 25

x1 + x3 = 15

x2 + x4 = 30

x1,x2,x3,x4 ≥ 0

Solution by using Simplex Method:

Min z = 10x1 + 15x2 + 2x3 + 13x4 +MR1 + MR2 + MR3 + MR4

Subject to

x1 + x2 + R1 = 20

x3 + x4 + R2 = 25

x1 + x3 + R3 = 15

x2 + x4 + R4 = 30

Where x1,x2,x3,x4 ,R1,R2,R3,R4 ≥ 0.

Table 3 Optimum Solution of Transportation problem by Using Simplex Method

Z	z	10	15	21	13	M	M	M	M	Sol.
	basic	X1	X2	X3	X4	R1	R2	R3	R4	
M	R1	1	1	0	0	1	0	0	0	20
M	R2	0	0	1	1	0	1	0	0	25
M	R3	1	0	1	0	0	0	1	0	15
M	R4	0	1	0	1	0	0	0	1	30
	Cj-zj	2M-10	2M-15	2M-2	2M-13	0	0	0	0	
M	R1	1	1	0	0	1	0	0	0	20

M	R2	-1	0	0	1	0	1	-1	0	10
2	X3	1	0	1	0	0	0	1	0	15
M	R4	0	1	0	1	0	0	0	1	30
	Cj-zj	-8	2M-15	0	2M-13	0	0	2M-2	0	
M	R1	1	1	0	0	1	0	0	0	20
13	X4	-1	0	0	1	0	1	-1	0	10
2	X3	1	0	1	0	0	0	1	0	15
M	R4	1	1	0	0	0	-1	1	1	20
	Cj -zj	2M-21	2M-15	0	0	0	-2M -13	-11	0	
10	X1	1	1	0	0	1	0	0	0	20
13	X4	0	1	0	1	1	1	-1	0	30
2	X3	0	-1	1	0	-1	0	1	0	-5
M	R4	0	0	0	0	-1	-1	1	1	0

	Cj-zj	0	6	0	0	-2M +21	-2M +13	-11	-2M	
15	X2	1	1	0	0	1	0	0	0	20
13	X4	-1	0	0	1	0	1	-1	0	10
2	X3	1	0	1	0	0	0	1	0	15
M	R4	0	0	0	0	-1	-1	1	1	0
	Cj-zj	-6	0	0	0	-2M +15	-2M + 13	-2M -11	0	

All cj-zj ≤ 0 we get optimum solution with x1 = 0, x2 = 20, x3 = 15, x4 = 10 and minimum value of z = 460.

Fuzzy Monte Carlo Method:

Step I - Fuzzy Linear Programming Problem:

Min z = (9/10/11)x1 + (14/15/16)x2 +(1/2/3)x3 +(12/ 13/14)x4

Subject to

x1 + x2 = (19/20/21)

x3 + x4 = (24/25/26)

x1 + x3 = (14/15/16)

x2 + x4 = (29/30/31)

Where x1,x2,x3,x4 ≥ 0

Step II: Matlab Program

```
%Matlab program for solution of fuzzy transportation problem by Using Monte Carlo Method;
clc
a1=rand(99999,1);
a1=sort(a1);
a1=reshape(a1,3,33333);
z01=10000;
z02=10000;
z03=10000;
z0=[z01,z02,z03];
'Enter the Interval a & b'
a=input('');
b=input('');
x2=(b-a)*a1+a;
b1=rand(99999,1);
b1=sort(b1);
b1=reshape(b1,3,33333);
y2=(b-a)*b1+a;
c1=rand(99999,1);
c1=sort(c1);
c1=reshape(c1,3,33333);
z2=(b-a)*c1+a;
d1=rand(99999,1);
d1=sort(d1);
d1=reshape(d1,3,33333);
w2=(b-a)*b1+a;
for i1=1:33333
```

```
for i2=1:33333
    for i3=1:33333
        for i4=1:33333
count=0;
p=[x2(1,i1),x2(2,i1),x2(3,i1)]+[y2(1,i2),y2(2,i2),y2(3,i2)];
q=[z2(1,i3),z2(2,i3),z2(3,i3)]+[w2(1,i4),w2(2,i4),w2(3,i4)];
r=[x2(1,i1),x2(2,i1),x2(3,i1)]+[z2(1,i3),z2(2,i3),z2(3,i3)];
s=[y2(1,i2),y2(2,i2),y2(3,i2)]+[w2(1,i4),w2(2,i4),w2(3,i4)];
m11=p(1,1);
m21=p(1,2);
m31=p(1,3);
m1=[m11,m21,m31];
n11=19;
n21=20;
n31=21;
n1=[n11,n21,n31];
m12=q(1,1);
m22=q(1,2);
m32=q(1,3);
m2=[m12,m22,m32];
n12=24;
n22=25;
n32=26;
n2=[n12,n22,n32];
m13=r(1,1);
m23=r(1,2);
m33=r(1,3);
m3=[m13,m23,m33];
```

```
   n13=14;
   n23=15;
   n33=16;
   n3=[n13,n23,n33];
   m14=s(1,1);
   m24=s(1,2);
   m34=s(1,3);
   m4=[m14,m24,m34];
   n14=29;
   n24=30;
   n34=31;
   n4=[n14,n24,n34];
  l=rand(100,1);
for j=1:100
m11alpha=m11+l(j,1)*(m21-m11);
m21alpha=m21+l(j,1)*(m21-m31);
m12alpha=m12+l(j,1)*(m22-m12);
m22alpha=m22+l(j,1)*(m22-m32);
m13alpha=m13+l(j,1)*(m23-m13);
m23alpha=m23+l(j,1)*(m23-m33);
m14alpha=m14+l(j,1)*(m24-m14);
m24alpha=m24+l(j,1)*(m24-m34);
n11alpha=n11+l(j,1)*(n21-n11);
n21alpha=n21+l(j,1)*(n21-n31);
n12alpha=n12+l(j,1)*(n22-n12);
n22alpha=n22+l(j,1)*(n22-n32);
n13alpha=n13+l(j,1)*(n23-m13);
n23alpha=n23+l(j,1)*(n23-m33);
```

```
n14alpha=n14+l(j,1)*(n24-n14);
n24alpha=n24+l(j,1)*(n24-n34);
ab1=abs(m11alpha-n11alpha);
ab2=abs(m21alpha-n21alpha);
ab3=abs(m12alpha-n12alpha);
ab4=abs(m22alpha-n22alpha);
ab5=abs(m13alpha-n13alpha);
ab6=abs(m23alpha-n23alpha);
ab7=abs(m14alpha-n14alpha);
ab8=abs(m24alpha-n24alpha);
if(ab1<=0.01 && ab2<=0.01 && ab3<=0.01 && ab4<=0.01 && ab5<=0.01 && ab6<=0.01 && ab7<=0.01 && ab8<=0.01)
   count=count+1;
end
if(count==100)
 z1=[9*x2(1,i1),10*x2(2,i1),11*x2(3,i1)]+[14*y2(1,i2),15*y2(2,i2),16*y2(3,i2)]+[1*z2(1,i3),2*z2(2,i3),3*z2(3,i3)]+[12*w2(1,i4),13*w2(2,i4),14*w2(3,i4)]
 x11=[x2(1,i1),x2(2,i1),x2(3,i1)];
 y11=[y2(1,i2),y2(2,i2),y2(3,i2)];
 z11=[z2(1,i3),z2(2,i3),z2(3,i3)];
 w11=[w2(1,i4),w2(2,i4),w2(3,i4)];
 if(z1(1,1)<z0(1,1) && z1(1,2)<z0(1,2) && z1(1,3)<z0(1,3))
    z0=z1;
  f1=i1;
  f2=i2;
  f3=i3;
  f4=i4;
end
```

```
        end
                end
                        end

                                end
                                        end
                                                end
z0
 x =[x2(1,f1),x2(2,f1),x2(3,f1)]
 y =[y2(1,f2),y2(2,f2),y2(3,f2)]
 z =[z2(1,f3),z2(2,f3),z2(3,f3)]
 w =[w2(1,f4),w2(2,f4),w2(3,f4)]
```

Step III: Some Solution obtained by Matlab Program

```
Enter the Interval a & b =   0     20

z1 = 398.2346        460.2340        521.2398

z1 =   401.7551      462.3979        523.7904

z1 =   405.6125      465.6430        517.4126

z1 = 411.0819        470.0149        522.2230

z1 =   411.9123      468.9648        519.7082

z1 = 417.9564        474.5760        527.2952
```

z1 = 413.7564	470.4091	517.3545
z1 = 408.1158	465.9482	515.0163
z1 = 415.3522	472.9012	520.9474
z1 = 420.5435	467.4171	520.2579
z1 = 426.2693	474.2749	528.8053

Optimum Solution:

z0 = 398.2346	**460.2340**	**521.2398**
x = 0.7122	**0.0922**	**1.3010**
y = 18.2939	**19.8881**	**20.4244**
z = 13.5339	**15.5829**	**16.1986**
w = 10.4748	**10.9991**	**11.1495**

If we Defuzzify this triangular fuzzy number obtained from Matlab program we get min value of z = 460.2340 with x = 0.0922, y= 19.8881, z = 15.5829 and w = 10.9991 which are close to crisp solution obtain by Simplex method x = 0, y = 20, z = 15, w = 10 and minimum value of z = 460.

6. **CONCLUSION AND FUTURE WORK**

In this article we discussed a method of finding minimum fuzzy transportation cost by using Fuzzy Monte Carlo Method i.e. by using random triangular fuzzy number. In this method, through a numerical example, we conclude that Monte Carlo Method produced a solution closest to crisp solution. In future, we want to extend our work doing more research by using random trapezoidal fuzzy number.

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