Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 12, Number 1 (2017), pp. 157-166

http://www.ripublication.com

On Generalized Classes of Double Sampling Estimators of Population Mean in Presence of Non Response

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Abstract

In the presence of non response, two generalized double sampling classes of estimators of finite population mean are proposed, their biases and mean square errors are found and their comparative studies have been made with some of the available estimators in the literature. Sub – Classes of optimum estimators in the sense of having minimum mean square error are found and enhancing the practical utility when optimum estimators involve unknown parameters, sub – classes of estimated optimum estimators based on sample observations are also investigated in the presence of non response. Theoretical findings are supported by practical examples also.

Keywords: Generalized class of estimators, two phase sampling, auxiliary information, non response.

1. INTRODUCTION

In most of the sample surveys, the information cannot be obtained from all the units in the survey. An estimator based on such incomplete information is generally biased and the results may be grossly misleading when the respondents differ from the non-respondents. In their seminal paper Hansen and Hurwitz (1946) considered a technique of sub-sampling the non-respondents in order to adjust for the non-response bias in a mail survey.

When the auxiliary information on a variable x is known in the form of its population mean $\bar{X}$, and the non-response is present, then the problem of estimation of population mean $\bar{Y}$ of the study variate y has been dealt by various authors including

Rao (1986, 1987) and Khare and Srivastava (1993, 1997). If the population mean $\bar{X}$ of the auxiliary variate x is not known in presence of non-response then Okafor and Lee (2000) and Tabasum and Khan (2004) have proposed to use double sampling (or two phase sampling) procedure to estimate of population mean $\bar{X}$ on the basis of a large first phase sample of size n' drawn from the finite population of size N by simple random sampling without replacement (SRSWOR). Then a second phase sample of size n $(n < n')$ is drawn from n' by SRSWOR and the information on study variable y under investigation is measured on it. During second phase let n_1 units provide information on y and n_2 units do not respond. Then utilizing Hansen and Hurwitz philosophy, we sub sample from the n_2 non-responding units and a sub sample of size r units is randomly drawn and is recorded by direct interview such that $r = n_2/k;\ k > 1$.

The usual ratio, product and difference estimators along with their alternative counterparts using double sampling in presence of non-response are given by

$$t_r = \frac{\bar{y}^*}{\bar{x}^*}\bar{x}',\ t_{r1} = \frac{\bar{y}^*}{\bar{x}}\bar{x}' \tag{1.1}$$

$$t_p = \frac{\bar{y}^*}{\bar{x}'}\bar{x}^*,\ t_{p1} = \frac{\bar{y}^*}{\bar{x}'}\bar{x} \text{ and} \tag{1.2}$$

$$t_d = \bar{y}^* + k\left(\bar{x}^* - \bar{x}'\right),\ t_{d1} = \bar{y}^* + k\left(\bar{x} - \bar{x}'\right) \tag{1.3}$$

respectively; where $\bar{x}$ and $\bar{x}'$ are the sample means of x based on n and n' units respectively; $\bar{y}^* = \frac{n_1}{n}\bar{y}_1 + \frac{n_2}{n}\bar{y}_{2r}$ and $\bar{x}^* = \frac{n_1}{n}\bar{x}_1 + \frac{n_2}{n}\bar{x}_{2r}$ with sample means $\left(\bar{x}_1, \bar{y}_1\right)$ and $\left(\bar{x}_{2r}, \bar{y}_{2r}\right)$ are the sample means based on n_1 units and sub sample means based on r units of the variables (x, y) respectively. The ratio and product estimators are generally biased although the difference estimator is unbiased. The estimators t_r has been considered by Khare and Srivastava (1993), Okafor and Lee (2000) and Tabasum and Khan (2004); t_{r1} has been considered by Khare and Srivastava (1993) and Tabasum and Khan (2006); t_p has been considered by Khare and Srivastava (1993) etc. The mean square errors (MSE) of these estimators are

$$MSE\left(t_r\right) = \left(\frac{1}{n'} - \frac{1}{N}\right)S_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)S_r^2 + \frac{W_2\left(k-1\right)}{n}S_{r2}^2 \tag{1.4}$$

$$MSE\left(t_{r1}\right) = \left(\frac{1}{n'} - \frac{1}{N}\right)S_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)S_r^2 + \frac{W_2\left(k-1\right)}{n}S_{y2}^2 \tag{1.5}$$

$$MSE\left(t_p\right) = \left(\frac{1}{n'} - \frac{1}{N}\right)S_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)S_p^2 + \frac{W_2\left(k-1\right)}{n}S_{p2}^2 \tag{1.6}$$

$$MSE\left(t_{p1}\right) = \left(\frac{1}{n'} - \frac{1}{N}\right)S_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)S_p^2 + \frac{W_2\left(k-1\right)}{n}S_{y2}^2 \tag{1.7}$$

$$MSE\left(t_d\right) = \left(\frac{1}{n'} - \frac{1}{N}\right)S_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)S_k^2 + \frac{W_2\left(k-1\right)}{n}S_{k2}^2 \tag{1.8}$$

$$MSE\left(t_{d1}\right)=\left(\frac{1}{n'}-\frac{1}{N}\right)S_y^2+\left(\frac{1}{n}-\frac{1}{n'}\right)S_k^2+\frac{W_2\left(k-1\right)}{n}S_{y2}^2 \tag{1.9}$$

where $W_2=N_2/N$, $S_r^2=S_y^2+R^2S_x^2-2R\rho S_xS_y$, $R=\bar{Y}/\bar{X}$, $\rho=S_{xy}/S_xS_y$,
$S_{r2}^2=S_{y2}^2+R^2S_{x2}^2-2R\rho_2S_{x2}S_{y2}$, $\rho_2=S_{xy2}/S_{x2}S_{y2}$, $S_p^2=S_y^2+R^2S_x^2+2R\rho S_xS_y$,
$S_{r2}^2=S_{y2}^2+R^2S_{x2}^2+2R\rho_2S_{x2}S_{y2}$, $S_d^2=S_y^2+k^2S_x^2+2k\rho S_xS_y$, $S_{d2}^2=S_{y2}^2+k^2S_{x2}^2+2k\rho_2S_{x2}S_{y2}$,
$S_{x2}^2=\frac{1}{N_2-1}\sum_{i=1}^{N_2}(x_i-\bar{X}_{(2)})^2$, $S_{y2}^2=\frac{1}{N_2-1}\sum_{i=1}^{N_2}(y_i-\bar{Y}_{(2)})^2$, $\bar{X}_{(2)}=\frac{1}{N_2}\sum_{i=1}^{N_2}x_i$ and $\bar{Y}_{(2)}=\frac{1}{N_2}\sum_{i=1}^{N_2}y_i$.

2. PROPOSED GENERALIZED CLASSES OF ESTIMATORS

In this paper, for estimating the population mean $\bar{Y}$ of study variable, the following classes of estimators are proposed

$$t_f=f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right) \tag{2.1}$$

$$t_g=g\left(\bar{y}^*,\bar{x},\bar{x}'\right) \tag{2.2}$$

where $f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)$ and $g\left(\bar{y}^*,\bar{x},\bar{x}'\right)$ being bounded functions satisfy the following regularity conditions such that

(i) $f\left(\bar{Y},\bar{X},\bar{X}\right)=\bar{Y}$ and $g\left(\bar{Y},\bar{X},\bar{X}\right)=\bar{Y}$ (2.3)

(ii) first order partial derivative with respect to $\bar{y}^*$ at $T=\left(\bar{Y},\bar{X},\bar{X}\right)$ is unity, that is,

$$f_0=\left.\frac{\partial f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)}{\partial\bar{y}^*}\right|_T=1 \text{ and } g_0=\left.\frac{\partial g\left(\bar{y}^*,\bar{x},\bar{x}'\right)}{\partial\bar{y}^*}\right|_T=1 \tag{2.4}$$

(iii) $f_{00}=\frac{\partial^2 f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)}{\partial\bar{y}^{*2}}=0$ and $g_{00}=\frac{\partial^2 g\left(\bar{y}^*,\bar{x},\bar{x}'\right)}{\partial\bar{y}^{*2}}=0$ (2.5)

(iv) first order partial derivative of $f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)$ and $g\left(\bar{y}^*,\bar{x},\bar{x}'\right)$ with respect to $\bar{x}^*$ and $\bar{x}$ respectively at $T=\left(\bar{Y},\bar{X},\bar{X}\right)$ satisfy

$$f_1=\frac{\partial f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)}{\partial\bar{x}^*}=-\frac{\partial f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)}{\partial\bar{x}'}=-f_2 \text{ and } g_1=\frac{\partial g\left(\bar{y}^*,\bar{x},\bar{x}'\right)}{\partial\bar{x}}=-\frac{\partial g\left(\bar{y}^*,\bar{x},\bar{x}'\right)}{\partial\bar{x}'}=-g_2 \tag{2.6}$$

(v) $f_{01}=\left.\frac{\partial^2 f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)}{\partial\bar{y}^*\partial\bar{x}^*}\right|_T=-\left.\frac{\partial^2 f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)}{\partial\bar{y}^*\partial\bar{x}'}\right|_T=-f_{02}$ and

$$g_{01}=\left.\frac{\partial^2 g\left(\bar{y}^*,\bar{x},\bar{x}'\right)}{\partial\bar{y}^*\partial\bar{x}}\right|_T=-\left.\frac{\partial^2 g\left(\bar{y}^*,\bar{x},\bar{x}'\right)}{\partial\bar{y}^*\partial\bar{x}'}\right|_T=-g_{02} \tag{2.7}$$

Some of the members belonging to the following to the generalized double sampling class of estimators $t_f=f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)$ are

$t_0=\bar{y}^*$, $t_1=\frac{\bar{y}^*}{\bar{x}^*}\bar{x}'=t_r$, $t_2=\frac{\bar{y}^*}{\bar{x}'}\bar{x}^*=t_p$, $t_3=\bar{y}^*+k\left(\bar{x}^*-\bar{x}'\right)=t_d$, $t_4=\bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)^k$,

$$t_5 = \bar{y}^* \left(\frac{\bar{x}^*}{\bar{x}'} \right)^{k1} + k_2 \left(\bar{x}^* - \bar{x}' \right), \; t_6 = \bar{y}^* \left(\frac{\bar{x}^*}{\bar{x}'} \right)^{k1} + k_2 \left(\bar{x}^{*k_3} - \bar{x}'^{k_3} \right), \; t_7 = \bar{y}^* \left\{ 1 + k \frac{\left(\bar{x}^* - \bar{x}' \right)}{\bar{x}'} \right\} \text{etc.} \quad (2.8)$$

where k, k_1, k_2 and k_3 are suitably chosen characterizing scalars.

It may be easily verified that the regularity conditions (2.3) to (2.7) are satisfied by t_6 and its particular cases t_i $\left(i = 0,1,2,3,4,5,6\right)$ and that the conditions (2.3) to (2.7) are satisfied by t_7 along with some other estimators available in literature (eg Reddy (1973, 74), Ray and Singh (1979), Srivenkataramana, Tracy (1980, 81) etc).

Also, some of the members belonging to the following to the generalized double sampling class of estimators $t_g = g\left(\bar{y}^*, \bar{x}, \bar{x}'\right)$ are

$$t_8 = \frac{\bar{y}^*}{\bar{x}} \bar{x}' = t_{r1}, \; t_9 = \frac{\bar{y}^*}{\bar{x}'} \bar{x} = t_{p1}, \; t_{10} = \bar{y}^* + k\left(\bar{x} - \bar{x}'\right) = t_{d1}, \; t_{11} = \bar{y}^* \left(\frac{\bar{x}}{\bar{x}'} \right)^k,$$

$$t_{12} = \bar{y}^* \left(\frac{\bar{x}}{\bar{x}'} \right)^{k1} + k_2 \left(\bar{x} - \bar{x}' \right), \; t_{13} = \bar{y}^* \left(\frac{\bar{x}}{\bar{x}'} \right)^{k1} + k_2 \left(\bar{x}^{k_3} - \bar{x}'^{k_3} \right), \; t_{14} = \bar{y}^* \left\{ 1 + k \frac{\left(\bar{x} - \bar{x}' \right)}{\bar{x}'} \right\} \text{ etc} \quad (2.9)$$

where k, k_1, k_2 and k_3 are suitably chosen characterizing scalars.

Similarly, it may be easily verified that the regularity conditions (2.3) to (2.7) are satisfied by t_{13} and its particular cases t_i $\left(i = 0,8,9,10,11,12,13,14\right)$ and that the conditions (2.3) to (2.7) are satisfied by t_{14} along with some other estimators available in literature.

3. BIAS AND MSE OF THE PROPOSED CLASSES

Expanding $t_f = f\left(\bar{y}^*, \bar{x}^*, \bar{x}'\right)$ in Taylor's series expansion about the point $T = \left(\bar{Y}, \bar{X}, \bar{X}\right)$ and using the regularity conditions (2.3) to (2.7), we have

$$t_f = \bar{Y} + \left(\bar{y}^* - \bar{Y}\right) + \left(\bar{x}^* - \bar{X}\right) f_1 - \left(\bar{x}' - \bar{X}\right) f_1 + \frac{1}{2} \left\{ \left(\bar{x}^* - \bar{X}\right)^2 f_{11} + \left(\bar{x}' - \bar{X}\right)^2 f_{22} \right.$$

$$+ 2\left(\bar{y}^* - \bar{Y}\right)\left(\bar{x}^* - \bar{X}\right) f_{01} - 2\left(\bar{y}^* - \bar{Y}\right)\left(\bar{x}' - \bar{X}\right) f_{01} + 2\left(\bar{x}^* - \bar{X}\right)\left(\bar{x}' - \bar{X}\right) f_{12}$$

$$+ \frac{1}{3!} \left\{ \left(\bar{y}^* - \bar{Y}\right) \frac{\partial}{\partial \bar{y}^*} + \left(\bar{x}^* - \bar{X}\right) \frac{\partial}{\partial \bar{x}^*} + \left(\bar{x}' - \bar{X}\right) \frac{\partial}{\partial \bar{x}'} \right\}^3 f\left(\bar{y}_0^*, \bar{x}_0^*, \bar{x}_0'\right) \quad (3.1)$$

where f_0, f_{00}, f_1, f_2, f_{01} and f_{02} are already defined in (2.3) to (2.7), second order partial derivatives f_{11}, f_{22} and f_{12} are given by

$$f_{11}=\left.\frac{\partial^2 f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)}{\partial\bar{x}^{*2}}\right|_T,\quad f_{22}=\left.\frac{\partial^2 f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)}{\partial\bar{x}'^2}\right|_T \text{ and } f_{12}=\left.\frac{\partial^2 f\left(\bar{y}^*,\bar{x}^*,\bar{x}'\right)}{\partial\bar{x}^*\partial\bar{x}'}\right|_T;$$

$$\bar{y}_0^*=\bar{Y}+\theta\left(\bar{y}^*-\bar{Y}\right),\ \bar{x}_0^*=\bar{X}+\theta\left(\bar{x}^*-\bar{X}\right),\ \bar{x}_0'=\bar{X}+\theta\left(\bar{x}'-\bar{X}\right);\ 0<\theta<1.$$

In order to obtain the bias and MSE, let us consider

$\bar{y}^*=\bar{Y}+e_0^*,\ \bar{x}^*=\bar{X}+e_1^*,\ \bar{x}=\bar{X}+e_1,\ \bar{x}'=\bar{X}+e_1'$ such that

$$E\left(e_0^*\right)=E\left(e_1^*\right)=E\left(e_1\right)=E\left(e_1'\right)=0 \text{ and } E\left(e_0^{*2}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)S_y^2+\frac{W_2\left(k-1\right)}{n}S_{y2}^2$$

$$E\left(e_1^{*2}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)S_x^2+\frac{W_2\left(k-1\right)}{n}S_{x2}^2\ ;\ E\left(e_0^*e_1^*\right)=\left(\frac{1}{n}-\frac{1}{N}\right)S_{xy}+\frac{W_2\left(k-1\right)}{n}S_{xy2}$$

$$E\left(e_1^2\right)=\left(\frac{1}{n}-\frac{1}{N}\right)S_x^2\ ;\ E\left(e_1'^2\right)=\left(\frac{1}{n'}-\frac{1}{N}\right)S_x^2\ \ E\left(e_0^*e_1\right)=\left(\frac{1}{n}-\frac{1}{N}\right)S_{xy}\ E\left(e_0^*e_1'\right)=\left(\frac{1}{n'}-\frac{1}{N}\right)S_{xy}$$

$$E\left(e_1^*e_1\right)=\left(\frac{1}{n}-\frac{1}{N}\right)S_x^2\ ;\ E\left(e_1^*e_1'\right)=\left(\frac{1}{n'}-\frac{1}{N}\right)S_x^2 \tag{3.2}$$

Now, putting these values in (3.1) and neglecting terms of powers of $e's$ greater than two, to the first degree of approximation, we have

$$t_f-\bar{Y}=e_0^*+\left(e_1^*-e_1'\right)f_1+\frac{1}{2}\left\{e_1^{*2}f_{11}+e_1'^2 f_{22}+2e_0^*e_1^*f_{01}-2e_0^*e_1'f_{01}+2e_1^*e_1'f_{12}\right\} \tag{3.3}$$

Taking expectation on both sides, we get

$$Bias\left(t_f\right)=\frac{1}{2}\left[\left\{\left(\frac{1}{n}-\frac{1}{N}\right)S_x^2+\frac{W_2\left(k-1\right)}{n}S_{x2}^2\right\}f_{11}+\left(\frac{1}{n'}-\frac{1}{N}\right)S_x^2 f_{22}\right.$$
$$\left.+2\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)S_{xy}+\frac{W_2\left(k-1\right)}{n}S_{xy2}\right\}f_{01}+2\left(\frac{1}{n'}-\frac{1}{N}\right)S_x^2 f_{12}\right] \tag{3.4}$$

Squaring (3.3) and taking expectation upto the first order of approximation, we have

$$MSE\left(t_f\right)=\left(\frac{1}{n'}-\frac{1}{N}\right)S_y^2+\left(\frac{1}{n}-\frac{1}{n'}\right)\left(S_y^2+f_1^2S_x^2-2f_1\rho S_xS_y\right)+\frac{W_2\left(k-1\right)}{n}\left(S_{y2}^2+f_1^2S_{x2}^2-2f_1\rho_2S_{x2}S_{y2}\right)$$

$$= \left(\frac{1}{n'} - \frac{1}{N}\right) S_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) S_f^2 + \frac{W_2 (k-1)}{n} S_{f2}^2 \tag{3.5}$$

where $S_f^2 = S_y^2 + f_1^2 S_x^2 - 2 f_1 \rho S_x S_y$ and $S_{f2}^2 = S_{y2}^2 + f_1^2 S_{x2}^2 - 2 f_1 \rho_2 S_{x2} S_{y2}$.

Proceeding similarly, we obtain the bias and MSE of t_g given by

$$Bias(t_g) = \frac{1}{2}\left[\left(\frac{1}{n} - \frac{1}{N}\right) S_x^2 g_{11} + \left(\frac{1}{n'} - \frac{1}{N}\right) S_x^2 g_{22} + 2\left(\frac{1}{n} - \frac{1}{n'}\right) S_{xy} g_{01} + 2\left(\frac{1}{n'} - \frac{1}{N}\right) S_x^2 g_{12}\right] \tag{3.6}$$

$$MSE(t_g) = \left(\frac{1}{n'} - \frac{1}{N}\right) S_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)\left(S_y^2 + g_1^2 S_x^2 - 2 g_1 \rho S_x S_y\right) + \frac{W_2 (k-1)}{n} S_{y2}^2 \tag{3.7}$$

4. ESTIMATORS BASED ON OPTIMUM AND ESTIMATED OPTIMUM VALUES

It can be easily verified that the optimum value of f_1 minimizing the $MSE(t_f)$ is

$$f_{1opt} = \frac{\left(\frac{1}{n} - \frac{1}{n'}\right) \rho S_x S_y + \frac{W_2 (k-1)}{n} \rho_2 S_{x2} S_{y2}}{\left(\frac{1}{n} - \frac{1}{n'}\right) S_x^2 + \frac{W_2 (k-1)}{n} S_{x2}^2} \tag{4.1}$$

and the minimum $MSE(t_f)$ is given by

$$MSE(t_f)_{\min} = \left(\frac{1}{n} - \frac{1}{N}\right) S_y^2 + \frac{W_2 (k-1)}{n} S_{y2}^2 - \frac{\left\{\left(\frac{1}{n} - \frac{1}{n'}\right) \rho S_x S_y + \frac{W_2 (k-1)}{n} \rho_2 S_{x2} S_{y2}\right\}^2}{\left(\frac{1}{n} - \frac{1}{n'}\right) S_x^2 + \frac{W_2 (k-1)}{n} S_{x2}^2} \tag{4.2}$$

Similarly, it can be easily verified that the optimum value of g_1 minimizing the $MSE(t_g)$ is

$$g_{1opt} = \frac{\rho S_x S_y}{S_x^2} \tag{4.3}$$

and the minimum $MSE(t_g)$ is given by

$$MSE(t_g)_{\min} = \left(\frac{1}{n'} - \frac{1}{N}\right) S_y^2 + \frac{W_2 (k-1)}{n} S_{y2}^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)\left(1 - \rho^2\right) S_y^2 \tag{4.4}$$

It has been observed that such optimum values f_{1opt} and g_{1opt} depends on the unknown population parameters which may not always be known in practice. Hence, we propose to estimate the unknown parameters by their consistent estimators based on the sample data

$$\hat{f}_{1opt} = \frac{\left(\frac{1}{n}-\frac{1}{n'}\right)s_{xy}^{*} + \frac{W_2\left(k-1\right)}{n}s_{xy2}}{\left(\frac{1}{n}-\frac{1}{n'}\right)s_{x}^{*2} + \frac{W_2\left(k-1\right)}{n}s_{x2}^{2}} \tag{4.5}$$

$$\hat{g}_{1opt} = \frac{s_{xy}^{**}}{s_x^2} \tag{4.6}$$

where $s_{xy}^{*} = \frac{1}{n-1}\left(\sum_{i=1}^{n_1} x_i y_i + r\sum_{j=1}^{r} x_j y_j - n\bar{x}^{*}\bar{y}^{*}\right)$, $s_x^{2*} = \frac{1}{n-1}\left(\sum_{i=1}^{n_1} x_i^2 + r\sum_{j=1}^{r} x_j^2 - n\bar{x}^{*2}\right)$,

$s_{xy}^{**} = \frac{1}{n-1}\left(\sum_{i=1}^{n_1} x_i y_i + r\sum_{j=1}^{r} x_j y_j - n\bar{x}\,\bar{y}^{*}\right)$, $s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$,

$s_{x2}^2 = \frac{1}{r-1}\sum_{j=1}^{r}\left(x_j - \bar{x}_{2r}\right)^2$ and $s_{xy2} = \frac{1}{r-1}\sum_{j=1}^{r}\left(x_j - \bar{x}_{2r}\right)\left(y_j - \bar{y}_{2r}\right)$.

It can be shown to the first order of approximation that

$$MSE\left(t_{\hat{f}}\right) = \left(\frac{1}{n}-\frac{1}{N}\right)S_y^2 + \frac{W_2\left(k-1\right)}{n}S_{y2}^2 - \frac{\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)\rho S_x S_y + \frac{W_2\left(k-1\right)}{n}\rho_2 S_{x2} S_{y2}\right\}^2}{\left(\frac{1}{n}-\frac{1}{n'}\right)S_x^2 + \frac{W_2\left(k-1\right)}{n}S_{x2}^2} \tag{4.7}$$

and $$MSE\left(t_{\hat{g}}\right) = \left(\frac{1}{n'}-\frac{1}{N}\right)S_y^2 + \frac{W_2\left(k-1\right)}{n}S_{y2}^2 + \left(\frac{1}{n}-\frac{1}{n'}\right)\left(1-\rho^2\right)S_y^2 \tag{4.8}$$

5. CONCLUDING REMARKS

1. Some estimators belonging to the class t_f are given in (2.8) satisfying the regularity conditions (2.3) to (2.7). It has been shown in (4.1) that the sub – class of these estimators attaining the minimum MSE satisfies

$$f_{1opt} = \frac{\left(\frac{1}{n} - \frac{1}{n'}\right)\rho S_x S_y + \frac{W_2(k-1)}{n}\rho_2 S_{x2} S_{y2}}{\left(\frac{1}{n} - \frac{1}{n'}\right)S_x^2 + \frac{W_2(k-1)}{n}S_{x2}^2} \tag{5.1}$$

and each member of this sub – class attains minimum MSE given by

$$MSE(t_f)_{\min} = \left(\frac{1}{n} - \frac{1}{N}\right)S_y^2 + \frac{W_2(k-1)}{n}S_{y2}^2 - \frac{\left\{\left(\frac{1}{n} - \frac{1}{n'}\right)\rho S_x S_y + \frac{W_2(k-1)}{n}\rho_2 S_{x2} S_{y2}\right\}^2}{\left(\frac{1}{n} - \frac{1}{n'}\right)S_x^2 + \frac{W_2(k-1)}{n}S_{x2}^2} \tag{5.2}$$

2. Since the optimum value f_{1opt} involves unknown parameter, therefore, in order to enhance the practical utility of proposed class of estimator, a class of estimators $t_{\hat{f}}$ depending upon the estimated optimum value $\hat{f}_{1opt}$ is proposed whose each member attains the minimum MSE given by (5.2) within the class t_f.

3. Some examples of estimators belonging to the class t_g are given in (2.9) satisfying the regularity conditions (2.3) to (2.7). It has been shown in (4.3) that the sub – class of these estimators attaining the minimum MSE satisfies

$$g_{1opt} = \frac{\rho S_x S_y}{S_x^2} \tag{5.3}$$

and each member of this sub – class attains minimum MSE given by

$$MSE(t_g)_{\min} = \left(\frac{1}{n'} - \frac{1}{N}\right)S_y^2 + \frac{W_2(k-1)}{n}S_{y2}^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)(1-\rho^2)S_y^2 \tag{5.4}$$

4. Since the optimum value g_{1opt} involves unknown parameter, therefore, in order to enhance the practical utility of proposed class of estimator, a class of estimators $t_{\hat{g}}$ depending upon the estimated optimum value $\hat{g}_{1opt}$ is proposed whose each member attains the minimum MSE given by (5.4) within the class t_g.

5. It can be easily verified that the proposed sub classes of estimators are more efficient than the Hansen Hurwitz strategy.

6. Further, it can be easily verified that the results of single phase can be obtained as special cases of these results by replacing n' by N.

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