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Ordering of Generalised Trapezoidal Fuzzy Numbers Based on Area Method Using Euler Line of Centroids

A. Hari Ganesh[1] & M. Suresh[2]

[1]Assistant Professor, [2]Research Scholar
Department of Mathematics,
Poompuhar College (Autonomous), Melaiyur – 607 109
Nagapattinam (Dt), Tamil Nadu, India.

Abstract

This paper describes a ranking method based on the area of a generalized trapezoidal fuzzy number. The area used in this method is obtained from the generalized trapezoidal fuzzy number, first by splitting the trapezoidal corresponding to trapezoidal fuzzy number into three plane figures and then calculating the centroids of each plane figure followed by finding the reference point by intersecting the Euler line of the triangle formed by the centroids of these three plane figures and line joining the midpoints of horizontal lines of trapezoid. Finally, the area is found from this reference point to the original point. The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized non – normal trapezoidal fuzzy numbers and also crisp numbers which are the considered to be a special case of fuzzy numbers. Finally, the proposed approach is validated by comparing with different existing approaches.

Keywords: Ranking Function, Centroid, Generalised Trapezoidal Fuzzy Number, Euler Line.

1. INTRODUCTION

Ranking fuzzy numbers play a vital role in decision making; most of the decision making problems that exist in nature are fuzzy, rather than probabilistic or deterministic. In such decision making problems, the fuzzy numbers must be ranked before an action is taken by the decision maker. In order to rank the fuzzy numbers, each fuzzy number has to be converted into a real number by defining a ranking function from the set of fuzzy numbers to a real line R = $(-\infty,+\infty)$ which assigns the real number to each fuzzy number from which a natural order exists.

Various ranking procedure have been developed since the inception of fuzzy sets by Zadeh**[14]** in 1965. Ranking fuzzy numbers was first introduced by Jain **[5]** for decision making in fuzzy environment by representing the ill-defined quantity as a fuzzy set. Subsequently, various ranking procedures were developed by various researchers and few of them were reviewed and compared by Bortolan and Degani**[1]**, Chen & Hwang**[2]**. Among the existing methods of ranking fuzzy numbers, centroid based methods were intensively studied and used in solving numerous decision making problems. Yager **[13]** was the first researcher who contributed the concept of centroid in ranking the fuzzy numbers followed by Murakami et.al **[6]**, who is the first to present the X and Y coordinates of the centroid point as the ranking index in 1983. These methods cannot rank the singleton fuzzy numbers, to overcome this problem cheng **[3]** introduced an approach of ranking fuzzy numbers by distance method which calculates the distance from centroid point to the original point but this technique cannot be used to rank positive and negative fuzzy numbers simultaneously. To overcome this drawback, Chu and Tsao **[4]** proposed a ranking method that calculates the area between the centroid point and the original point. Moreover, Wang et al **[12]** proposed the formula for finding the horizontal and vertical coordinates of the centroid point. In 2007, Shieh **[9]** proposed the accurate formula for finding the horizontal and vertical coordinates of the centroid point. Recently, Phani Bushan Rao and others **[7], [8], [10], [11]** proposed the centroid of formula for finding the center of gravity of generalized trapezoidal fuzzy number using circumcenter of the centroids, orthocentre of centroids, incenter of centroids and centroids of centroids. This paper proposes a new centroid using Euler line of centroids which orders the generalized trapezoidal fuzzy numbers based on area method.

The rest of the paper is organized as follows. Section 2 introduces basic concepts and definitions of fuzzy numbers. Section 3 presents the proposed ranking method based on the proposed centroid formulae of triangle and trapezoidal fuzzy numbers. In section 3, the comparative examples are given to illustrate the advantage of the proposed approach for ranking of fuzzy numbers. Finally, the paper is concluded in Section 4.

2. PRELIMINARIES

In this section some basic definitions of fuzzy set theory are given

Definition 2.1

Let U be a universal set. A fuzzy set $\tilde{A}$ of U is defined by the membership function $f_{\tilde{A}}(x): U \rightarrow [0,1]$ where $f_{\tilde{A}}(x)$ is the degree of $x \ in \ \tilde{A}, \forall x \in U$.

Definition 2.2

A fuzzy set $\tilde{A}$ of universal set U is normal if and only if $Sup_{x \in U} f_{\tilde{A}}(x) = 1$

Definition 2.3

A fuzzy set $\tilde{A}$ of universal set U is convex iff

$$f_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq \min\left(f_{\tilde{A}}(x), f_{\tilde{A}}(y)\right) \forall x, y \in U \ and \ \lambda \in [0,1]$$

Definition 2.4

A fuzzy set $\tilde{A}$ of universal set U is a fuzzy number iff $\tilde{A}$ is normal and convex on U.

Definition 2.5

A real fuzzy number$\tilde{A}$is described as any fuzzy subset of the real Line **R** with membership function$f_{\tilde{A}}(x)$ possessing the following properties:

(a) $f_{\tilde{A}}(x)$is a continuous mapping from **R** to the closed interval $[0, \omega]; 0 < \omega \leq 1$

(b) $f_{\tilde{A}}(x) = 0$ for all$x \epsilon (-\infty, a] \cup [d, \infty)$

(c) $f_{\tilde{A}}(x)$ is strictly increasing on [a,b] and strictly decreasing on [c,d]

(d) $f_{\tilde{A}}(x) = 1$, for all$x \epsilon [b, c]$ where a,b,c and d are real numbers.

Definition 2.6

The membership function of the real fuzzy number$\tilde{A}$ is given by

$$f_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}^{L}(x), & a \leq x \leq b, \\ \omega, & b \leq x \leq c, \\ f_{\tilde{A}}^{R}(x), & c \leq x \leq d, \\ 0, & otherwise \end{cases}$$

Where $0 < \omega \leq 1$ is a constant, a,b c and d are real numbers and $f_{\tilde{A}}^{L}: [a, b] \rightarrow [0, \omega], f_{\tilde{A}}^{R}(x): [c, d] \rightarrow [0, \omega]$ are two strictly monotonic continuous function from R to the closed interval $[0, \omega]$.it is customary to write a fuzzy number as $\tilde{A} = (a, b, c, d: \omega)$. If *w*=1then $\tilde{A} = (a, b, c, d: 1)$ is a normalised fuzzy number, otherwise $\tilde{A}$ is said to be a generalised or non-normal fuzzy number. If the membership function $f_{\tilde{A}}(x)$is piecewise linear, then $\tilde{A}$ is said to be a trapezoidal fuzzy number. The membership function of a trapezoidal fuzzy number is given by

$$f_{\tilde{A}}(x) = \begin{cases} \dfrac{\omega(x-a)}{b-a}, & a \le x \le b, \\ \omega, & b \le x \le c, \\ \dfrac{\omega(x-d)}{c-d}, & c \le x \le d, \\ 0, & otherwise \end{cases}$$

If *w*=1then $\tilde{A} = (a, b, c, d: 1)$ is a normalised trapezoidal fuzzy number and $\tilde{A}$ is said to be a generalised or non-normal trapezoidal fuzzy number if $0 < \omega < 1$. The image of $\tilde{A} = (a, b, c, d: \omega)$ is given by $\widetilde{-A} = (-d, -c, -b, -a: \omega)$. As a particular case if b=c,the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $\tilde{A} = (a, b, d: \omega)$. The value of 'b' corresponds with the mode or core and [a,d] with the support. If *w*=1then $\tilde{A} = (a, b, d: 1)$ is a normalised triangular fuzzy number and $\tilde{A}$ is said to be a generalised or non-normal triangular fuzzy number if $0 < \omega < 1$.

3. PROPOSED RANKING METHOD

The proposed method is based on the area between the proposed centroid point and original point of the trapezoid. The calculation of the proposed centroid is presented from the point of view of analytical geometry.

In a trapezoidal fuzzy number, first the trapezoid is divided into three plane figures. The three plane figures are triangle (APB), rectangle (BPQC) and again a triangle (CQD) respectively. Let the centroids of the three plane figures be G_1, G_2 & G_3 respectively. Subsequently, the centroid of these three plane figures are calculated, followed by this calculation, Euler line of the triangle formed by joining the centroids of these three plane figures and the vertical line joining the centre points of the horizontal lines of the trapezoid are calculated. Finally a ranking function is proposed which is the area between the point of intersection of the Euler line of the centroids and the vertical line trapezoid to the original point.

The reason for selecting the proposed centroid point as a point of reference is that the three centroid points of the plane figures (G1 of triangle APB, G2 of rectangle BPQC, and G3 of triangle CQD) are balancing points of each individual plane figure, and the Euler line is the balancing line of the trapezoidal because the Euler line is passing through the centroid of the centroids of the three plane figures. Furthermore, the line joining the midpoints of the upper and lower sides of the trapezoidal also be the balancing line of the trapezoidal. Therefore, the point of intersection of these two balancing lines would be the better reference point of the trapezoid.

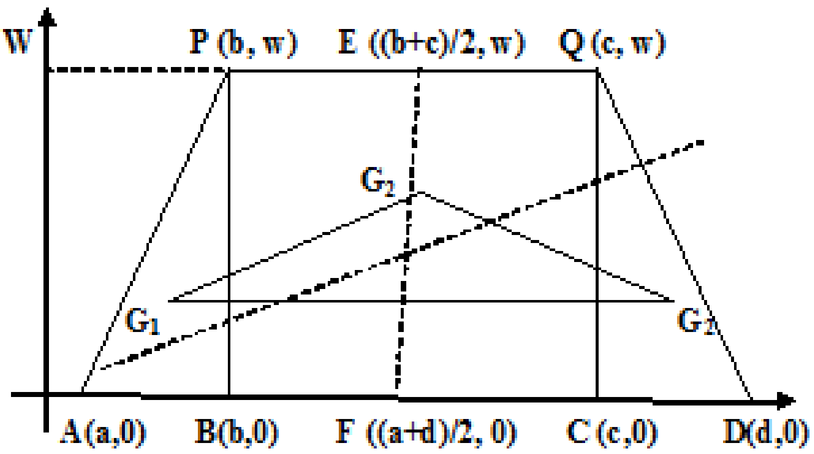

Figure 1: Centroid of Trapezoidal Fuzzy Number based on Euler Line of Centroids

Consider a generalised trapezoid fuzzy number $\tilde{A} = (a, b, c, d: \omega)$ (figure 1). The centroid of the three plane figures are: $G_1 = \left(\frac{a+2b}{3}\right)$; $G_2 = \left(\frac{b+c}{2}\right)$ and $G_3 = \left(\frac{2c+d}{3}\right)$ respectively. Equation of the line joining $\overline{G_1 G_3}$ is $y = \frac{\omega}{3}$ and G_2 does not lie on the line $\overline{G_1 G_3}$. Therefore $G_1 G_2 \& G_3$ are non-collinear and they form a triangle.

We derive the Euler line of the triangle with the vertices G_1, G_2 & G_3 of the generalised trapezoidal fuzzy number $\tilde{A} = (a, b, c, d: \omega)$ as

$$\overline{y_0} + \left\{\frac{[3(2a+b-3c)(c+2d-3b)+\omega^2]}{2\omega(b+c-a-d)}\right\}\overline{x_0} = \frac{(2a+b-3c)(c+2d-3b)(7b+7c+2a+2d)+\omega^2}{12\omega(b+c-a-d)} \quad \text{-------} \quad (1)$$

and the equation of the line joining $E\left(\frac{b+c}{2}, \omega\right)$ and $F\left(\frac{a+d}{2}, 0\right)$ is

$$\overline{y_0} - \frac{2\omega\overline{x_0}}{b+c-a-d} = \frac{-(a+d)\omega}{b+c-a-d} \quad \text{--------} \quad (2)$$

The point of intersection of (1) and (2) is evaluated as

$$S_{\tilde{A}}(\overline{x_0}, \overline{y_0}) = \left\{\frac{(2a+b-3c)(c+2d-3b)(7b+7c+2a+2d)+\omega^2[8a+7b+7c+8d]}{6[3(2a+b-3c)(c+2d-3b)+5\omega^2]}, \frac{7\omega}{3}\left(\frac{(2a+b-3c)(c+2d-3b)+\omega^2}{3(2a+b-3c)(c+2d-3b)+5\omega^2}\right)\right\}$$

-------- (3)

As a special case , for triangular fuzzy number $\tilde{A} = (a, b, d: \omega)$, i.e., c=b in (3) gives

$$S_{\tilde{A}}(\overline{x_0}, \overline{y_0}) = \left\{\frac{4(a-b)(d-b)(a+7b+d)+\omega^2(4a+7b+4d)}{3[12(a-b)(d-b)+5\omega^2]}, \frac{7\omega}{3}\left(\frac{4(a-b)(d-b)+\omega^2}{12(a-b)(d-b)+5\omega^2}\right)\right\} \quad \text{-------} \quad (4)$$

The ranking function of the generalised trapezoidal fuzzy number $\tilde{A} = (a, b, c, d: \omega)$ which maps the set of all fuzzy numbers to a set of all real numbers is defined as :

$$R(\tilde{A}) = r_x^{\tilde{A}} \times r_y^{\tilde{A}}$$

$$= \left\{\frac{(2a+b-3c)(c+2d-3b)(7b+7c+2a+2d)+\omega^2[8a+7b+7c+8d]}{6[3(2a+b-3c)(c+2d-3b)+5\omega^2]}\right\} \times \frac{7\omega}{3}\left(\frac{(2a+b-3c)(c+2d-3b)+\omega^2}{3(2a+b-3c)(c+2d-3b)+5\omega^2}\right) \quad \text{---} \quad (5)$$

This is the area between the original point and the point of intersection of the Euler line and the vertical line joining the midpoints of the horizontal lines of the generalised trapezoidal fuzzy number.

4. NUMERICAL EXAMPLES

Example 4.1

let $\tilde{A} = (0.1,0.3,0.3,0.5; 1)$ and $\tilde{B} = (0.2,0.3,0.3,0.4; 1)$ then

$S_{\tilde{A}}(\overline{x_0}, \overline{y_0}) = (0.3,0.4336)$ and $S_{\tilde{B}}(\overline{x_0}, \overline{y_0}) = (0.3,0.4590)$

Therefore $R(\tilde{A}) = 0.13009$ and $R(\tilde{B}) = 0.1377$

Since $R(\tilde{A}) < R(\tilde{B}) \Rightarrow \tilde{A} < \tilde{B}$

Example 4.2

Let$\tilde{A} = (0.1,0.3,0.3,0.5; 1) \Rightarrow \widetilde{-A} = (-0.5, -0.3, -0.3, -0.1; 1)$ and

$\tilde{B} = (0.2,0.3,0.3,0.4; 1) \Rightarrow -\tilde{B} = (-0.4, -0.3, -0.3, -0.2; 1)$then

$S_{-\tilde{A}}(\overline{x_0}, \overline{y_0}) = (-0.3,0.4336)$ and $S_{-\tilde{B}}(\overline{x_0}, \overline{y_0}) = (-0.3,0.4590)$

Therefore $R(\tilde{A}) = -0.13009$ and $R(\tilde{B}) = -0.1377$

Since $R(-\tilde{A}) > R(-\tilde{B}) \Rightarrow \widetilde{-A} > -\tilde{B}$

Example 4.3

Let $\tilde{A} = (0.1,0.3,0.3,0.5;0.8)$ and $\tilde{B} = (0.1,0.3,0.3,0.5;1)$ then

$S_{\tilde{A}}(\overline{x_0},\overline{y_0}) = (0.3,0.3294)$ and $S_{\tilde{B}}(\overline{x_0},\overline{y_0}) = (0.3,0.4336)$

Therefore $R(\tilde{A}) = 0.0988$ and $R(\tilde{B}) = 0.13009$

Since $R(\tilde{A}) < R(\tilde{B}) \Rightarrow \tilde{A} < \tilde{B}$

Example 4.4

Let $\tilde{A} = (3,5,7;1)$ and $\tilde{B} = (4,5,\frac{51}{8})$ then $S_{\tilde{A}}(\overline{x_0},\overline{y_0}) = (5,0.8682)$ and

$S_{\tilde{B}}(\overline{x_0},\overline{y_0}) = (5.0163,0.913)$

Therefore $R(\tilde{A}) = 4.341$ and $R(\tilde{B}) = 4.6211$

Since $R(\tilde{A}) < R(\tilde{B}) \Rightarrow \tilde{A} < \tilde{B}$

Example 4.5

Consider four fuzzy numbers

A_1 = (0.1,0.2,0.3:1) ; A_2 = (0.2,0.5,0.8;1) ; A_3 = (0.3,0.4,0.9;1) ; A_4 = (0.6,0.7,0.8;1) as shown in the figure.2 which were ranked earlier by Yager,Fortemps and Roubens ,Liou and Wang ,and Chen and Lu as shown in the table 1

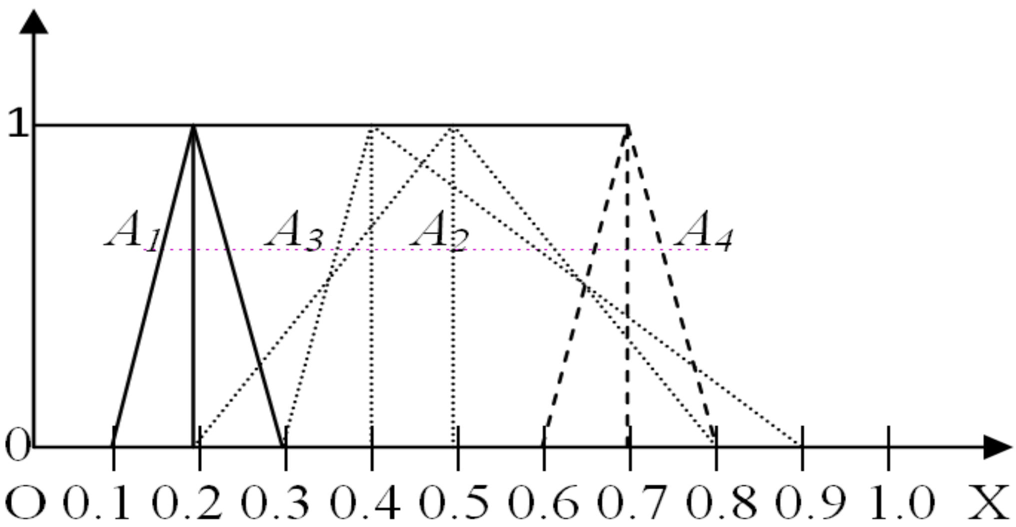

Figure 2 A_1=(0.1,0.2,0.3:1) ; A_2=(0.2,0.5,0.8;1); A_3=(0.3,0.4,0.9;1) ; A_4=(0.6,0.7,0.8;1)

Table 1. Comparison of Various Ranking Methods

Method\fuzzy Number		A_1	A_2	A_3	A_4	Ranking order
Yager		0.20	0.50	0.50	0.70	$A_4>A_3=A_2>A_1$
Fortemps and Roubens		0.20	0.50	0.50	0.70	$A_4>A_3=A_2>A_1$
Liou& Wang	α=1	0.25	0.65	0.65	0.75	$A_4>A_3=A_2>A_1$
	α=0.5	0.20	0.50	0.50	0.70	$A_4>A_3=A_2>A_1$
	α=0	0.15	0.35	0.35	0.65	$A_4>A_3=A_2>A_1$
Chen	β=1	-0.2	0.00	0.00	-0.20	$A_2=A_3>A_1=A_4$
	β=0.5	-0.20	0.00	0.00	-0.20	$A_2=A_3>A_1=A_4$
	β=0	-0.20	0.00	0.00	-0.20	$A_2=A_3>A_1=A_4$
PhaniBushanRao& Ravi Shankar, N		0.4591	0.632	0.6146	0.8129	$A_4>A_2>A_3>A_1$
Proposed Method		0.0918	0.1905	0.2185	0.3267	$A_4>A_3>A_2>A_1$

5. CONCLUSION

This paper proposes a method that ranks fuzzy numbers which is simple and concrete. This method ranks normal and non – normal fuzzy numbers. This method also ranks crisp numbers which are special case of fuzzy numbers. This method which is simple and easier in calculation not only gives satisfactory results to well defined problems, but also gives a correct ranking order to the problems. Moreover, comparative examples are used to illustrate the advantages of the proposed method. Application of this ranking procedure in various decision making problems such as, fuzzy risk analysis and in fuzzy optimization like network analysis is left as future work.

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