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Perfect Vertex Domination in Fuzzy Soft graphs

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Abstract

Let $G_{A,V}$ be a fuzzy soft graph and let x_i and x_j be two vertices of $G_{A,V}$. If $\mu_e(x_i, x_j) \le \rho_e(x_i) \wedge \rho_e(x_j)$ for each parameter $e \in A$ and $\forall i, j = 1,2,3 \cdots n$., then we say that x_i dominates x_j in $G_{A,V}$. A subset S of V is called a ***fuzzy soft dominating set*** if for every $x_j \in V - S$, there exist a vertex $x_i \in S$ such that x_i dominates x_j. In this paper we introduce the concepts of perfect fuzzy soft vertex domination , perfect fuzzy soft vertex domination number, perfect fuzzy soft t-vertex domination.

Keywords: Perfect fuzzy soft vertex domination , perfect fuzzy soft vertex domination number, perfect fuzzy soft t-vertex domination, perfect fuzzy soft t-vertex domination number.

INTRODUCTION

The concept of domination in fuzzy graphs was first introduced by A Somasundaram and S Somasundaram [8]. Perfect vertex (edge) domination in fuzzy graphs was studied by S Ramya and S Lavanya[4]. In 2015,Sumit Mohinta and Samanta[7] introduced the notion of fuzzy soft graphs and some operations in fuzzy soft graphs and later on Muhammed Akram and Saira Nawas[6] introduced different types of fuzzy soft graphs and their properties.

In this paper we introduce the concepts of perfect vertex domination in fuzzy soft graphs, perfect vertex domination number and proved some theorems related to the above concepts.

2. PRELIMINARIES

Definition 2.1 [3] Assume X be a universal set, S be the set of parameters and $P(X)$ denote the power set of X. If there is a mapping $F: S \to P(X),$ then we call the pair (F, S), a ***soft set*** over X.

Definition 2.2 [3] Let X be a universal set, S be the set of parameters and $E \subset S.$ If there is a mapping $F: E \to I^X$, I^X be the set of all fuzzy subsets of S, then we say that (F, E) is a *fuzzy soft set* over X.

Definition 2.3 [7] Let $V = \{x_1, x_2, x_3, \cdots x_n\}$ (non empty set) E (parameters set) and $A \subseteq E.$ Also let

(i) $\rho: A \to F(V),$ collection of all fuzzy subsets in V and each element e of A is mapped to $\rho(e) = \rho_e$ (say) and $\rho_e : V \to [0, 1]$, each element x_i is mapped to $\rho_e(x_i)$ and we call (A, ρ), a fuzzy soft vertex.

(ii) $\mu: A \to F(V \times V),$ collection of all fuzzy subsets in $V \times V,$ which mapped each element e to $\mu(e) = \mu_e$ (say) and $\mu_e : V \times V \to [0,1],$ which mapped each element (x_i, x_j) to $\mu_e(x_i, x_j),$ and we call (A, μ) as a fuzzy soft edge.

Then $((A,\rho),(A,\mu))$, is called fuzzy soft graph if and only if $\mu_e(x_i,x_j) \le \rho_e(x_i) \wedge \rho_e(x_j)\ \forall e \in A$ and $\forall i,j = 1,2,3,\cdots n$, this fuzzy soft graph is denoted by $G_{A,V}$.

Definition2.4[7] The underlying crisp graph of a fuzzy soft graph $G_{A,V} = ((A,\rho),(A,\mu))$ is denoted by $G^* = (\rho^*, \mu^*)$, where

$$\rho^* = \{x_i \in V;\ \rho_e(x_i) > 0\ \textit{for some}\ e \in E\}\ \text{and}$$

$$\mu^* = \{(x_i,x_j) \in V \times V;\ \mu_e(x_i,x_j) > 0\quad \textit{for some}\ e \in E\}.$$

Definition2.5 [6] A fuzzy soft graph $G_{A,V}$ is called a *strong fuzzy soft graph* if $\mu_e(x_i,x_j) = \rho_e(x_i) \wedge \rho_e(x_j)\ \forall (x_i,x_j) \in \mu^*, e \in A.$ and is called a *complete fuzzy soft graph* if $\mu_e(x_i,x_j) = \rho_e(x_i) \wedge \rho_e(x_j)\ \forall x_i, x_j \in \rho^*, e \in A.$

Definition2.6 [6] Let $G_{A,V}$ be a fuzzy soft graph. Then the ***order*** of $G_{A,V}$ is defined as $O(G_{A,V}) = \sum_{e_i \in A}\left(\sum_{x_i \in V} \rho_e(x_i)\right)$ and ***size*** of $G_{A,V}$ is defined as $S(G_{A,V}) = \sum_{e_i \in A}\left(\sum_{x_i,x_j \in V} \mu_e(x_i,x_j)\right)$ and the ***degree*** of a vertex x_i is defined as $d_{(G_{A,V})}(x_i) = \sum_{e_i \in A}\left(\sum_{x_j \in V\ x_i \ne x_j} \mu_e(x_i,x_j)\right)$.

Definition 2.7 Degree of a fuzzy soft graph $G_{A,V}$ is defined as $D_{G_{A,V}} = \max\{d_{G_{A,V}}(x_i); x_i \in V\}$ **Definition 2.8** A fuzzy soft graph $G_{A,V}$ is said to be regular fuzzy soft graph if the fuzzy graph corresponding to each parameter $e \in A$ is a regular fuzzy graph.

3. FUZZY SOFT DOMINATION AND FUZZY SOFT T-DOMINATION

Definition3.1 Let $G_{A,V}$ be a fuzzy soft graph and let x_i and x_j be two vertices of $G_{A,V}$. If $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ for each parameter $e \in A$ and $\forall i, j = 1,2,3 \cdots n$., then we say that x_i dominates x_j in $G_{A,V}$. A subset S of V is called a ***fuzzy soft dominating set*** if for every $x_j \in V - S,$ there exist a vertex $x_i \in S$ such that x_i dominates x_j.

Definition 3.2 A fuzzy soft dominating set S of a fuzzy soft graph $G_{A,V}$ is said to be a ***minimal fuzzy soft dominating set*** if for each parameter $e \in A$, deletion of an element from S is not a fuzzy soft dominating set.

Definition 3.3 The minimum cardinality of all minimal fuzzy soft dominating set is called the ***fuzzy soft domination number*** and is denoted by $\gamma_{fs}(G_{A,V})$.

Definition 3.4 Let $G_{A,V}$ be a fuzzy soft graph and $S \subseteq V$. Then a vertex $x_j \in V - S$ is called a ***fuzzy soft t- dominated*** if it is dominated by at least t-vertices in S for each parameter $e \in A.$ If every vertex in $V - S$ is fuzzy soft t- dominated, then S is called a fuzzy soft t- dominating set.

Definition 3.5 The minimum cardinality of a fuzzy soft t- dominating set is called the ***fuzzy soft t-domination number*** and is denoted by $\gamma_{fs-t}(G_{A,V})$.

4. PERFECT DOMINATION AND PERFECT T-DOMINATION IN FUZZY SOFT GRAPH.

Definition 4.1 Let $G_{A,V}$ be a fuzzy soft graph and S be a dominating set. If for each vertex $x_j \notin S$ is adjacent to exactly one vertex of S for each parameter $e \in A$, then S is called a ***perfect fuzzy soft dominating set (PFSD).***

Definition 4.2 Let S be a perfect dominating set in the fuzzy soft graph $G_{A,V}$. If for each vertex $x_i \in S,$ its deletion from S is not a perfect dominating set, then S is called a ***minimal perfect fuzzy soft dominating set***.

The minimum cardinality of all minimal perfect dominating set is called the ***perfect fuzzy soft domination number*** and is denoted by $\gamma_{pfsd}(G_{A,V})$.

Definition 4.3 Let $G_{A,V}$ be a fuzzy soft graph , $S \subseteq V$ and t be a positive integer. Then S is said to be a ***perfect fuzzy soft t-vertex dominating set*** (PFSD-t)if for each parameter $e \in A$ and for each vertex $x_j \in V - S$ is adjacent to exactly t-vertices of S . The minimum cardinality of a perfect t-vertex dominating set of $G_{A,V}$ is called the ***perfect fuzzy soft t-domination number*** and is denoted by $\gamma_{pfsd-t}(G_{A,V})$.

Theorem 4.4 Every complete fuzzy soft graph with n vertices must have perfect fuzzy soft t-dominating sets, where $1 < t \le n-1$.

Proof: Let $G_{A,V}$ be a complete fuzzy soft graph with n vertices say $\{x_1, x_2, \cdots x_n\}$. If $S \subseteq V$ is a set contains only one element , then each vertex in its complement is adjacent to exactly one vertex in $S.$ therefore S is a PFS-1 vertex dominating set. If S is a two vertex set, then each vertex in its complement is adjacent to exactly 2 vertices in $S.$ so S is a PFS-2 vertex dominating set. Continuing like this up to $n-1$ steps, we get PFS-$(n-1)$ vertex dominating set. Hence there must exist perfect fuzzy soft t-dominating sets with $1 < t \le n-1$and $n > 1$.

Theorem 4.5 Let G^* is an even cycle and $G_{A,V}$ be a regular fuzzy soft graph. Then the set of all alternating vertices in it is always a perfect fuzzy soft 2-vertex dominating set.

Proof: Let $G_{A,V}$ be a regular fuzzy soft graph in an even cycle G^* and S be the set of all alternating vertices. Then any vertex in $V-S$ is adjacent to exactly 2-vertices in S. Hence S is a perfect fuzzy soft 2-dominating set.

Theorem 4.6 Let G^* be a cycle of length n and $G_{A,V}$ be a regular fuzzy soft graph. Then any set of $n-1$ vertices always form a perfect fuzzy soft 2-vertex dominating set.

Proof: Let G^* be a cycle of length n and $G_{A,V}$ be a regular fuzzy soft graph and let S be any set of $n-1$ vertices. Then its complement contains only one vertex and which is adjacent to exactly two vertices in S. There fore S is a perfect fuzzy soft 2-vertex dominating set.

Theorem 4.7 [4] For a fuzzy graph G, $\gamma_{pfd-t}(G) \geq \frac{tn}{\Delta G + t}$, where n is the number of vertices.

Theorem 4.8 If $G_{A,V}$ is any fuzzy soft graph, then $\gamma_{pfs-t}(G_{A,V}) \geq \frac{tn}{(D_{G_{A,V}} + t)}$. Where n is the number of vertices.

Proof: We know that in a fuzzy soft graph, every perfect t-dominating set is a t-dominating set and for each parameter $e \in A$, it is a fuzzy graph. Combining these two results and by above theorem, we get $\gamma_{pfsd-t}(G_{A,V}) \geq \gamma_{fsd-t}(G_{A,V}) \geq \frac{tn}{(D_{G_{A,V}} + t)}$.

Hence $\gamma_{pfsd-t}(G_{A,V}) \geq \frac{tn}{(D_{G_{A,V}} + t)}$.

5. CONCLUSION

In this paper we defined new concepts such as domination in fuzzy soft graphs, perfect fuzzy soft domination, perfect fuzzy soft domination number, perfect t-vertex domination and perfect t-vertex domination number and proved some theorems related to this. The fuzzy soft domination and perfect fuzzy soft domination are very useful for solving wide range of problems.

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