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A Value and Ambiguity-Based Ranking of Symmetric hexagonal Intuitionistic Fuzzy Numbers in Decision Making

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Abstract

The aim of this paper is to develop a method for ranking symmetric hexagonal intuitionistic fuzzy numbers in the process of decision making in a intuitionistic fuzzy environment. Arithmetic operations and cut sets over SHIFNs are defined. In the existing real world the circumstances are more fuzzy than specific. Fuzziness comes from a variety of sources such as uncountable information and in many circumstances decision makers have indefinite/vague information about options with respect to attributes. This method has two phases. In the first phase, the decision maker defines the preferred performance for each attribute. In the second phase, an extended MADM method based on the value and Ambiguity based ranking procedure is used to determine the overall performance rating of alternatives.Also, the values and ambiguities of the membership degree and the non- membership degree for SHIFNs is defined and at the same the value-index and ambiguity-index is derived. Here, SHIFNs are introduced as a distinctive type of IFNs, which have advanced applications and can be certainly identified and applied by the decision maker. The concept of the SHIFNs and ranking method as well as applications are discussed in depth. A numerical example is considered to

demonstrate the application process and applicability of the method proposed in this paper.

Keywords and Phrases: Multi attribute decision making (MADM), alpha cut, beta cut Intuitionistic fuzzy number, Symmetric Intuitionistic Hexagonal fuzzy number,

1. Introduction

Multiattribute decision making (MADM) problems are of importance in most kinds of fields such as Engineering, Economics, and Management. In ranking fuzzy numbers many methods have been presented till now, each one is rated based on its special standards proportion and characteristics of fuzzy number. It is observed that the fuzzy set [1] was extended to develop the Intuitionistic fuzzy (IF) set [2, 3] by adding an additional non-membership degree, which may express ample and flexible information as compared with the fuzzy set [4–6]. Fuzzy numbers are a special case of fuzzy sets and are of importance for fuzzy multi attribute decision making problems [7–12]. As a generalization of fuzzy numbers, an IFN seems to suitably describe an unstable quantity, Nehi [13] generalized the concept of characteristic value introduced for the membership and the non-membership functions and proposed a ranking method based on this concept.Wang and Zhang [14] defined the TIFNs and gave a ranking method which transformed the ranking of TIFNs into the ranking of interval numbers. Xiang-tianZeng*et* al., developed a value and ambiguity–based ranking method of trapezoidal Intuitionistic fuzzy numbers [15].

2. PRELIMINARIES

2.1 Intuitionistic Fuzzy number[8]

An Intuitionistic Fuzzy Number (IFN) $\tilde{A}^{I}$ is

i) an intuitionistic fuzzy subset of the real line,

ii) convex for the membership function $\mu_{\tilde{A}^I}(x)$,

(i.e.) $\mu_{\tilde{A}^I}(\lambda x_1+(1-\lambda)x_2)\geq \min\left(\mu_{\tilde{A}^I}(x_1),\mu_{\tilde{A}^I}(x_2)\right)$, for every $x_1, x_2 \in R, \lambda\in[0,1]$.

iii) concave for the membership function $\nu_{\tilde{A}^I}(x)$, that is, $\nu_{\tilde{A}^I}(\lambda x_1+(1-\lambda)x_2)\leq \max\left(\nu_{\tilde{A}^I}(x_1),\nu_{\tilde{A}^I}(x_2)\right)$ for every $x_1, x_2 \in R, \lambda\in[0,1]$.

iv) normal, that is, there is some $x_0\in R$ such that $\mu_{\tilde{A}^I}(x_0)=1, \nu_{\tilde{A}^I}(x_0)=0$.

2.2 Symmetric Hexagonal intuitionistic fuzzy numbers

A symmetric hexagonal intuitionistic fuzzy numbers
$\tilde{A}_H^I = (a_L - s - t,\ a_L - s, a_L, a_U, a_U + s, a_U + s + t, ; a'_L - s' - t', a'_L - s', a_L, a_U, a'_U + s', a'_U + s' + t')$
where $a_L, a_U, a'_L, a'_U, s, t, s', \textit{and } t'$ are real numbers such that

$$(a'_L - s' - t' \le a_L - s - t \le a'_L - s' \le a_L - s \le a_L \le, a_U \le a_U + s \le a'_U + s' \le a_U + s + t \le a'_U + s' + t')$$

and its membership and non-membership functions are given below

$$\mu_{\tilde{A}_H^I}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-(a_L - s - t)}{t}\right) & \textit{for } a_L - s - t \le x \le a_L - s \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-(a_L - s)}{s}\right) & \textit{for } a_L - s \le x \le a_L \\ 1, & \textit{for } a_L \le x \le a_U \\ 1 - \frac{1}{2}\left(\frac{x - a_U}{s}\right), & \textit{for } a_U \le x \le a_U + s \\ \frac{1}{2}\left(\frac{(a_U + s + t) - x}{t}\right), & \textit{for } a_U + s \le x \le a_U + s + t \\ 0, & \textit{otherwise} \end{cases}$$

and

$$v_{\tilde{A}_H^I}(x) = \begin{cases} 1 - \frac{1}{2}\left(\frac{x-(a'_L - s' - t')}{t'}\right) & \textit{for } a'_L - s' - t' \le x \le a'_L - s' \\ \frac{1}{2}\left(\frac{a_L - x}{a_L - (a'_L - s')}\right) & \textit{for } a'_L - s' \le x \le a_L \\ 0, & \textit{for } a_L \le x \le a_U \\ \frac{1}{2}\left(\frac{x - a_U}{a'_U + s' - a_U}\right), & \textit{for } a_U \le x \le a'_U + s' \\ \frac{1}{2} + \frac{1}{2}\left(\frac{(x-(a'_U + s')}{t'}\right), & \textit{for } a'_U + s' \le x \le a'_U + s' + t' \\ 1, & \textit{otherwise} \end{cases}$$

2.3 Arithmetic operations over symmetric hexagonal fuzzy numbers

Let $\tilde{A}_H^I = (a_L - s - t,\ a_L - s, a_L, a_U, a_U + s, a_U + s + t)\,(a'_L - s' - t', a'_L - s', a_L, a_U, a'_U + s', a'_U + s' + t')$
and $\tilde{B}_H^I = (b_L - s_2 - t_2,\ b_L - s_2, b_L, b_U, b_U + s_2, b_U + s_2 + t_2)\,(b_L - s'_2 - t'_2, b_L - s'_2, b_L, b_U, b_U + s'_2, b_U + s'_2 + t'_2)$
be two symmetric hexagonal intuitionistic fuzzy numbers and λ be any positive real number, then the arithmetic operation over the symmetric hexagonal intuitionistic fuzzy numbers is defined as follows.

2.3.1 Addition of two SHIFNs

$$\tilde{A}'_H + \tilde{B}'_H = (a_L + b_L) - t, (a_L + b_L) - s, (a_L + b\ ), (a_U + b_U), (a_U + b_U + s).(a_U + b_U + t);$$
$$(a_L + b_L) - t', (a_L + b_L) - s', (a_L + b_L), (a_U + b_U), (a_U + b_U + s'), (a_U + b_U + t')$$

$$Where \quad s = (s_1 + s_2), \quad t = (s_1 + s_2 + t_1 + t_2) \quad s' = (s'_1 + s'_2), \quad t' = (s_1' + s'_2 + t'_1 + t'_2)$$

2.3.2 Subtraction of two SHIFNs

$$\tilde{A}'_H - \tilde{B}'_H = (a_L - b_U) - t, (a_L - b_U) - s, (a_L - b\ ), (a_U - b_U), (a_U - b_L + s).(a_U - b_L + t);$$
$$(a_L - b_U) - t', (a_L - b_U) - s', (a_L - b_L), (a_U - b_U), (a_U - b_L + s'), (a_U - b_L + t')$$

$$Where \quad s = (s_1 + s_2), \quad t = (s_1 + s_2 + t_1 + t_2) \quad s' = (s'_1 + s'_2), \quad t' = (s_1' + s'_2 + t'_1 + t'_2)$$

2.3.3 Scalar multiplication

$$\lambda \tilde{A}^I_H = \lambda(a_L - s - t),\ \lambda(a_L - s),\ \lambda a_L,\ \lambda a_U,\ \lambda(a_U + s), \lambda(\ a_U + s + t);$$
$$\lambda(a'_L - s' - t'),\ \lambda(a'_L - s'),\ \lambda a_L,\ \lambda a_U,\ \lambda(a'_U + s'),\ \lambda(a'_U + s' + t')$$

$$(\tilde{A}'^{\lambda}_H)^k = \tilde{A}'^{\lambda k}_H$$

2.4 Cut sets of symmetric hexagonal intuitionistic fuzzy numbers

As stated in 2.4 the cut sets of a symmetric hexagonal fuzzy numbers can be defined as follows

Let $\tilde{A}'_H$ be an intuitionistic fuzzy set X. Then $(\alpha.\ ,\beta)$ cut sets of $\tilde{A}'_H$ is a crisp subset R, which is defined as follows.

$$\tilde{A}'_{H(\alpha,\beta)} = \left\{ x \in X : \mu_{\tilde{A}'_H}(x) \geq \alpha, \quad \gamma_{\tilde{A}'_H}(x) \geq \beta \right\} \text{---------- (3)}$$

Where $(\alpha, \beta) \in [0,1]$ where $0 \leq \alpha \leq 1, \quad 0 \leq \beta \leq 1$ and $0 \leq \alpha + \beta \leq 1$

2.5 Degree of membership function of alpha cut

The classical set $\tilde{A}_\alpha$ called alpha cut set of elements whose degree of membership is the set of elements in $\tilde{A}_H = (a_L - s - t,\ a_L - s, a_L,\ a_U,\ a_U + s,\ a_U + s + t\ )$ is not less than α .It is defined as

$$A_\alpha = \left\{ x \in X / \mu_{\tilde{A}_H}(x) \geq \alpha \ \right\} ----------(4)$$

$$= \begin{cases} P_1(\alpha), P_2(\alpha) & for\ \alpha \in [0,0.5) \\ Q_1(\alpha), Q_2(\alpha) & for\ \alpha \in [0.5,1] \end{cases}$$

Using eqn (3) and definition(2.5) we see that
$\tilde{A}_\alpha$ is closed interval denoted by $\tilde{A}_\alpha = [L_{\tilde{A}}(\alpha), R_{\tilde{A}}(\alpha)]$ which can be calculated as follows.

$$[L_{\tilde{A}}(\alpha), R_{\tilde{A}}(\alpha)] = [2\alpha s + a_L - 2s,\ 2\alpha s + a_U + 2s,\ 2\alpha t + (a_l - s - t), -2\alpha t + (a_U + s + t)] \text{ ---- (5)}$$

2.6 Degree of membership function of Beta cut

The classical set $\tilde{A}_\beta$ called Beta cut set of elements whose degree of membership is the set of elements in $\tilde{A}_{H^1} = (a_L - s' - t', a_L - s', a_L,\ a_U,\ a_U + s', a_U + s' + t')$ is not less than β .It is defined as

$$A_\beta = \{x \in X / \gamma_{\tilde{A}_H}(x) \geq \beta \text{----------} (6)$$

$\tilde{A}_\beta$ is closed interval denoted by $\tilde{A}_\beta = [L_{\tilde{A}}(\beta), R_{\tilde{A}}(\beta)]$ which can be calculated as follows.

$$[L_{\tilde{A}}(\beta), R_{\tilde{A}}(\beta)] = [2\beta t' + a_L - s' + t',\ a_L - 2\beta s',\ 2\beta s' + a_U\ , 2\beta t' - t' + a_U + s')]$$

------(7)

3. VALUE AND AMBIGUITY OF SHIFNS.

3.1 Value of SHIFN

Let A_α and A_β be any $\alpha -$cut and $\beta -$cut set of a SHIFN $\tilde{A}'_H$,. The value of the membership function $\mu_{\tilde{A}'_H}(x)$ and non-membership-function $\nu_{\tilde{A}'_H}$ (x) for the SHIFN $\tilde{A}'_H$ is defined as follows:

$$V_\mu = \int_0^1 [L_{\tilde{A}_H}(\alpha) + R_{\tilde{A}'_H}(\alpha)]\, f(\alpha)\, d\alpha \text{------}(8)$$

$$V_\gamma = \int_0^1 [L_{\tilde{A}'_H}(\beta) + R_{\tilde{A}'_H}(\beta)]\, g(\beta)\, d\beta \text{-------}(9)$$

The function f (α) = α gives different weights to elements in different α - cut sets. In fact, α diminishes the contribution of the lower α - cut sets, which is reasonable since these cut sets arising from values of $\mu_{\tilde{A}'_H}(x)$ have a considerable amount of uncertainty. Obviously, $V_\mu = \int_0^1 [L_{\tilde{A}_H}(\alpha) + R_{\tilde{A}'_H}(\alpha)]\, f(\alpha)\, d\alpha$ synthetically reflects the information on every membership degree and may be regarded as central values that represents from the membership function point of view. Similarly, the function g (β) = 1-β has the effect of weighting on the different β- cut sets. g(β) diminishes the contribution of the higher β - cut sets, which is reasonable since these sets arising from values of $V_\gamma = \int_0^1 [L_{\tilde{A}'_H}(\beta) + R_{\tilde{A}'_H}(\beta)]\, g(\beta)\, d\beta$ have a considerable amount of uncertainty. $\nu_{\tilde{A}'_H}$ Synthetically reflects the information on every non-membership degree and may be regarded as a central value that represents from the non-membership function point of view.

According to (8), the value of the membership function of a SHIFN V_μ is calculated as

$$V_\mu = \int_0^1 [R_{\tilde{a}}(\alpha) + L_{\tilde{a}}(\alpha)] f(\alpha) d\alpha, \qquad f(\alpha) = \alpha$$

$$= \int_0^1 2\alpha s + a_L - 2s - 2s\alpha + a_U + 2s + 2\alpha t + a_L - s - t - 2\alpha t + a_U + s + t \text{-------} (10)$$

$$= \int_0^1 (2a_L + 2a_U)\alpha d\alpha \quad Hence$$

$$V_\mu = a_U + a_L \text{-------} (11)$$

In a similar way, according to (9), the value of the non-membership-function of a SHIFN V_ν is calculated as follows:

$$V_\nu = \int_0^1 [R_{\tilde{a}}(\beta) + L_{\tilde{a}}(\beta)]\, g(\beta) d\beta, \qquad g(\beta) = 1 - \beta$$

$$= \int_0^1 (-2\beta\, t' + a_L - s' + t' + a_L - 2\beta s' + 2\beta s' + a_U + 2\beta t' - t' + a_U + s')(1-\beta)\, d\beta \text{---} (12)$$

$$= \int_0^1 (2a_L + 2a_U)(1-\beta) d\beta \quad Hence \qquad V_\nu = a_U + a_L \text{--------} (13)$$

3.2. Ambiguity of SHIFNs.

A_α and A_β be any $\alpha-$cut and $\beta-$cut set of a SHIFN $\widetilde{A}'_H$,. The ambiguity of the membership function $\mu_{\widetilde{A}'_H}(x)$ and non-membership-function $\nu_{\widetilde{A}'_H}$ (x) for the SHIFN $\widetilde{A}'_H$ is defined as follows:

$$A_\mu = \int_0^1 [R_{\widetilde{A}_H}(\alpha) - L_{\widetilde{A}'_H}(\alpha)]\, f(\alpha)\, d\alpha ---(14) \qquad A_\gamma = \int_0^1 [R_{\widetilde{A}'_H}(\beta) - L_{\widetilde{A}'_H}(\beta)]\, g(\beta)\, d\beta -----(15)$$

It is easy to see that $R_{\widetilde{A}'_H}(\alpha) - L_{\widetilde{A}'_H}(\alpha)$ *and* $R_{\widetilde{A}'_H}(\beta) - L_{\widetilde{A}'_H}(\beta)$ are just above the lengths of the interval A_α *and* A_β respectively .Thus, A_μ *and* A_ν can be regarded as the global spreads of the membership function $\mu_{\widetilde{A}'_H}(x)$ and the non=membership function $\nu_{\widetilde{A}'_H}(x)$ .Obviously, A_α *and* A_β basically measures how much there is vagueness in the $\widetilde{A}'_H(x)$

According to (14), the ambiguity of the membership function of a SHIFN is $\widetilde{A}'_H(x)$ is calculated as $[L_{\widetilde{A}'_H}(\alpha), R_{\widetilde{A}'_H}(\alpha)]$, $A_\mu = \int_0^1 [R_{\widetilde{A}_H}(\alpha) - L_{\widetilde{A}'_H}(\alpha)]\, f(\alpha)\, d\alpha$

$$= \int_0^1 (-2\alpha t + a_U + s + t - 2\alpha s + a_U + 2s - 2\alpha t - a_L + s + t - 2\alpha s - a_L + 2s)\alpha d\alpha ---(16)$$

$$= \int_0^1 (-4\alpha t + 2a_U - 2a_L + 6s + 2t - 4\alpha s)\alpha d\alpha \quad Hence \quad A_\mu = -\frac{1}{3}t + a_U - a_L + \frac{5}{3}s ----------(17)$$

Now $A_\gamma = [L_{\widetilde{A}'_H}(\beta), R_{\widetilde{A}'_H}(\beta)]$ $\quad A_r = \int_0^1 [(R_{\widetilde{a}}(\beta) - L_{\widetilde{a}}(\beta)]\,(1-\beta)\, d\beta$

$$= [-2\beta t' + a_L - s' + t', \quad a_L - 2\beta s', \quad 2\beta s' + a_{U,} \quad 2\beta(t^1) - t^1 + a_U + s^1]$$

$$= \int_0^1 \left(2\beta s^1 + a_u + 2\beta t^1 - t^1 + a_u + s^1 + 2\beta t^1 - a_L + s^1 - t^1 - a_L + 2\beta s^1\right)(1-\beta)d\beta -----(18)$$

$$= \int_0^1 \left(4\beta s^1 + 4\beta t^1 + 2a_u - 2a_L + 2s^1 - 2t^1\right)(1-\beta)d\beta \quad Hence \quad A_\gamma = -\frac{1}{3}t' + \frac{5}{3}s' + a_U - a_L --------(19)$$

3.3. The Value and Ambiguity-Based Ranking Method.

Based on the above value and ambiguity of a SHIFN, a new ranking method of SHIFNs is proposed in this sub-section.

$$Let, \widetilde{A}'_H = \begin{pmatrix} a_L - s - t, a_L - s, a_L, a_{U,} a_U + s, a_U + s + t \\ a_L - s' - t', a_L - s', a_L, a_{U,} a_U + s', a_U + s' + t' \end{pmatrix}$$

be a SHIFN, A value index and ambiguity index $\widetilde{A}'_H$ are defined as

$V_\lambda(\widetilde{A}'_H) = \lambda V_\mu(\widetilde{A}'_H) + (1-\lambda) V_\gamma(\widetilde{A}'_H) --(20)$, $A_\lambda(\widetilde{A}'_H) = \lambda A_\mu(\widetilde{A}'_H) + (1-\lambda) A_\gamma(\widetilde{A}'_H) ----(21)$ respectively, where $\lambda \in [0, 1]$ is a weight which represents the decision makers preference formation. $\lambda \in (0.5, 1]$ shows that the decision maker prefers uncertainty or negative feeling; $\lambda \in$ [0, 0.5) shows that the decision maker prefers certainty or positive feeling; λ =0.5 shows that the decision maker is uninterested between positive feeling and negative feeling. Therefore, the value index and the ambiguity index may reflect the decision maker's subjectivity attitude to the SHIFNs. It is easily seen that $V_\lambda(x)$ and $A_\lambda(x)$ have some useful properties, which are summarized in Theorems 3.3.1 and 3.3.2.

Theorem: 3.3.1

$$Let\ \widetilde{A}'_H = (a_L - s_1 - t_1, -a_L - s_1, a_L, a_U, a_U + s_1, a_U + s_1 + t_1\ ; a_L - s'_1 - t'_1, a_L - s'_1, a_L, a_U, a_U + s_1, a_U + s'_1 + t'_1)$$

$$\widetilde{B}_H = (b_L - s_2 - t_2, b_L - s_2, b_L, b_U, b_U + s_2, b_U + s_2 + t_2\ ; b_L - s'_2 - t'_2, b_L - s'_2, b_L, b_U, b_U + s'_2, b_U + s'_2 + t'_2)$$

be two SHIIN, then for $\lambda \in [0,1]\ and\ \gamma \in R^+$
Then the following equation is valid

$$V_\lambda(\widetilde{A}'_H + \widetilde{B}'_H) = V_\lambda(\widetilde{A}'_H) + V_\lambda(\widetilde{B}'_H) \quad \& \quad V_\lambda(\gamma \widetilde{A}'_H) = \gamma V_\lambda(\widetilde{A}'_H) ------------(22)$$

Proof:

According to (2.3.1), we have

$$\widetilde{A}'_H + \widetilde{B}'_H$$
$$= (a_L + b_L) - t, (a_L + b_L) - s, (a_L + b_L), (a_U + b_U), (a_U + b_U + s), (a_U + b_U + t);$$
$$(a_L + b_L) - t', (a_L + b_L) - s', (a_L + b_L), (a_U + b_U), (a_U + b_U + s^1), (a_U + b_U + t')$$
$$where\quad s = (s_1 + s_2), t = (s_1 + s_2 + t_1 + t_2),\ s' = (s'_1 + s'_2),\ t = (s'_1 + s'_2 + t'_1 + t'_2)$$

Using (11) (13) & (20) we get

$$V_\lambda(\widetilde{A}'_H + \widetilde{B}'_H) = \lambda V_\mu(\widetilde{A}'_H + \widetilde{B}'_H) + (1-\lambda) V_\mu(\widetilde{A}'_H + \widetilde{B}'_H)$$
$$= \lambda[a_L + b_L + a_U + b_U] + (1-\lambda)(a_L + b_L + a_U + b_U)$$
$$= \lambda(a_L + a_U) + (1-\lambda)(a_L + a_U)\lambda(b_L + b_u) + (1-\lambda)(b_L + b_u)$$
$$= V_\lambda(\widetilde{A}'_H) + V_\lambda\ (\widetilde{B}'_H) -------(24)$$
$$V_\lambda(\widetilde{A}'_H + \widetilde{B}'_H) = V_\lambda(\widetilde{A}'_H) + V_\lambda(\widetilde{B}'_H)$$

Also: Using (11) (13) & (20) we get

$$\lambda\widetilde{A}'_H = [\lambda(a_L - s_1 - t_1), \quad \lambda(a_L - s_1), \lambda(a_L), \lambda(a_U), \lambda(a_U + s_1)\lambda(a_U + s_1 + t_1);$$
$$\lambda(a_L - s'_1 - t'_1), \lambda(a_L - s'_1), \lambda(a_L - s'_1), \lambda(a_U), \lambda(a_U), \lambda(a_U + s'_1), \lambda(a_U + s'_1 + t'_1)] \quad -- \quad (25)$$

$$V_\lambda\ (\gamma\widetilde{A}'_H) = \lambda V_\mu(\gamma\widetilde{A}'_H) + (1-\lambda)V\gamma(\gamma\widetilde{A}'_H)$$
$$= \lambda(\gamma a_U + \gamma a_U) + (1-\lambda)(\gamma a_L + \gamma a_U)$$
$$= \gamma[(\lambda(a_L + a_U) + (1-\lambda)(a_L - a_U)]$$
$$= \gamma V_\gamma(\widetilde{A}'_H) --- (26)$$

Theorem:3.3.2

Let $\widetilde{A}'_H = (a_L - s_1 - t_1, a_L - s_1, a_L, a_U, a_U + s_1, a_U + s_1 + t_1 \vdots\ a_L - s'_1 - t'_1, a_L - s'_1, a_{L,}, a_U, a_U + s, a_U + s'_1 + t'_1$
$\widetilde{B}'_H = (b_L - s_2 - t_2, b_L - s_2, b_L, b_U, b_U + s_2, b_U + s_2 + t_2 \vdots b_L - s'_2 - t'_2, b_L - s'_2, b_L, b_U, b_U + s'_2, b_U + s'_2 + t'_2)$
be SHIFNs ,Then for any $\lambda \in (0,1]\ \&\ \gamma \in R^+$ *following equation is valid*

$$A_\lambda(\widetilde{A}'_H + \widetilde{B}'_H) = A_\lambda(\widetilde{A}'_H) + A_\lambda(\widetilde{B}'_H) ------(27)$$
$$A_\lambda\ (\gamma\widetilde{A}'_H) \quad = \gamma A_\lambda\ (\widetilde{A}'_H) ------(28)$$
$$A_\mu = -\frac{1}{3}t + a_U - a_L + \frac{5}{3}s \quad \& \quad A_\gamma = \frac{1}{3}t + a_U - a_L + \frac{5}{3}s$$
$$A_\lambda(\widetilde{A}'_H) = \lambda A_\mu(\widetilde{A}'_H) + (1-\lambda)A_\gamma(\widetilde{A}'_H)$$

Now Using (14), (15), and (21), we obtain

$$A_\lambda(\widetilde{A}'_H + \widetilde{B}'_H) = \lambda A_\mu(\widetilde{A}'_H + \widetilde{B}'_H) + (1-\lambda)A_\gamma(\widetilde{A}'_H + \widetilde{B}'_H)$$
$$= \lambda[-\frac{1}{3}(t_1 + t_2) + a_U + b_U - (a_L + b_L) + \frac{5}{3}(s_1 + s_2)] + (1-\lambda)[-\frac{1}{3}(t'_1 + t'_2) + a_U + b_U - (a_L + b_L) + \frac{5}{3}(s'_1 + s'_2)]$$
$$= \lambda[-\frac{1}{3}t_1 + a_U - a_L + \frac{5}{3}s_1] + (1-\lambda)\ [-\frac{1}{3}t'_1 + b_U - b_L + \frac{5}{3}s'_1] + \lambda[-\frac{1}{3}t_2 + a_U - a_L + \frac{5}{3}s_2]$$
$$+ (1-\lambda)[-\frac{1}{3}t'_2 + b_U - b_L + \frac{5}{3}s'_2]$$
$$= [\lambda A_\mu(\widetilde{A}'_H) + (1-\lambda)A_\gamma(\widetilde{A}'_H)] + [\lambda A_\mu(\widetilde{B}'_H) + (1-\lambda)A_\gamma(\widetilde{B}'_H)]$$
$$= A_\lambda(\widetilde{A}'_H) + A_\lambda(\widetilde{B}'_H)$$

Also $A_\lambda(\gamma\widetilde{A}'_H) = \lambda A_\mu(\gamma\widetilde{A}'_H) + (1-\lambda)A_\gamma(\gamma\widetilde{A}'_H)$

$$= \lambda[-\frac{1}{3}\gamma t_1 + \gamma a_L - \gamma a_U + \frac{5}{3}\gamma s_1]\_+ (1-\lambda)[-\frac{1}{3}\gamma t'_1 + \gamma a_L - \gamma a_U + \frac{5}{3}\gamma s'_1]$$
$$= \gamma[\lambda(-\frac{1}{3}t_1 + a_L - a_U + \frac{5}{3}s'_1) + (1-\lambda)(\frac{1}{3}t'_1 + a_L - a_U + \frac{5}{3}s'_1]$$
$$= \gamma A_\lambda(\widetilde{A}'_H)$$

That is $A_\lambda(\widetilde{A}'_H + \widetilde{B}'_H) = A_\lambda(\widetilde{A}'_H) + A_\lambda(\widetilde{B}'_H)$ *and* $A_\lambda(\gamma\widetilde{A}'_H) = \gamma A_\lambda(\widetilde{A}'_H)$

Remark: 3.3.1

Let $\tilde{A}'_H$ *and* $\tilde{B}'_H$ be two SHIFN. A lexicographic ranking procedure based on the value –individual ambiguity index can be summarized as follows.

Rule 1: If $V_\lambda(\tilde{A}'_H) \le V_\lambda(\tilde{B}'_H)$ *than* $\tilde{A}'_H$ *is smaller then* $\tilde{B}'_H$

Rule 2: *If* $V_\lambda(\tilde{A}'_H) \ge V_\lambda(\tilde{B}'_H)$ *than* $\tilde{A}'_H$ *is greater then* $\tilde{B}'_H$

Rule 3: *If* $V_\lambda(\tilde{A}'_H) = V_\lambda(\tilde{B}'_H)$ *and* $A_\lambda(\tilde{A}'_H) \ge A_\lambda(\tilde{B}'_H)$ *than* $\tilde{A}'_H$ *is smaller then* $\tilde{B}'_H$

Rule 4: *If* $V_\lambda(\tilde{A}'_H) = V_\lambda(\tilde{B}'_H)$ *and* $A_\lambda(\tilde{A}'_H) \le A_\lambda(\tilde{B}'_H)$ *then* $\tilde{A}'_H$ *is greatert then* $\tilde{B}'_H$

Rule 5: *If* $V_\lambda(\tilde{A}'_H) = V_\lambda(\tilde{B}'_H)$ *and* $A_\lambda(\tilde{A}'_H) = A_\lambda(\tilde{B}'_H)$ *then* $\tilde{A}'_H = \tilde{B}'_H$

4. AN EXTENDED MADM METHOD BASED ON THE VALUE AND AMBIGUITY- BASED RANKING PROCEDURE

In this section, we will apply the above ranking method of SHIFNs to solve MADM problems in which the ratings of alternatives on attributes are expressed using SHIFNs, Sometimes such MADM problems are called as MADM problems with SHIFNs for short.

4.1 An application to an investment selection problem and comparison analysis of the results obtained.

Let us consider suppose that there is an investment company, which wants to invest a sum of money in best option. There is a panel with four possible sources to invest the money: x_1 is a car company, x_2 is a food company; x_3 is a computer company, and x_4 is an arms company. The investment company must take a decision according to the following three attitudes. a_1 is the risk analysis; a_2 is the growth analysis; and a_3 is the environment impact analysis. The four possible alternatives $x_i\ (j = 1,2,3,4)$ are evaluated using the SHIFNs by decision maker under the above attributes, and the three attributes are benefit attributes; the weighted normalized SHIFNs decision matrix is obtained as shown in Table 4. 1.

Table 4.1: Weighted Normalised Decision Matrix

Attributes	a_1	a_2	a_3
Alternatives x_1	(0.26,0.36,0.46,0.56,0.66,0.76 0.06,0.26,0.46,0.56,0.76,0.96)	(0.24,0.34,0.44,0.54,0.64,0.74 0.04,0.24,0.44.0.54,0.74,0.94)	(0.22,0.32,0.42,0.52,0.62,0.72 0.16,0.29,0.42,0.52,0.65,0.78)
x_2	(0.40,0.50,0.60,0.80,0.90 0.30,0.45,0.60,0.70,85,.1)	(0.35,0.45,0.55,0.65,0.75,0.85 0.25,0.40,0.55,0.65,0.80,0.95)	(0.14,0.24,0.34,0.44,0.54,0.64 0.04,0.19,0.34,0.44,0.59.0.74)
x_3	(0.38,0.48,0.58,0.68,0.78,0.88 0.34,0.46,0.58,0.68,0.80,0.92)	(0.44,0.54,0.64,0.74,0.84,0.94 0.42,0.53,0.64,0.74,0.85,0.96)	(0.36,0.46,0.56,0.66,0.76,0.86 0.32,0.44,0.56,0.66,0.78,0.90)
x_4	(0.30,0.40,0.50,0.60,0.70,0.80 0.20,0.35,0.50,0.60,0.75,0.90)	(0.31,0.41,0.51,0.61,0.71,0.85 0.27,0.39,0.51,0.61,0.73,0.85)	(0.18,0.28,0.38,0.48,0.58,0.68 0.08,0.23,0.38,0.48,0.63,0.78)

$$\left\lfloor \tilde{r}_{ij} = \left(\frac{a_{ij1}}{a_{j6}^{+}}, \frac{a_{ij2}}{a_{j6}}, \frac{a_{ij3}}{a_{j6}}, \frac{a_{ij4}}{a_{j6}}, \frac{a_{ij5}}{a_{j6}}, \frac{a_{ij6}}{a_{j6}}, \frac{b_{ij1}}{a_{j6}}, \frac{b_{ij2}}{a_{j6}}, \frac{b_{ij3}}{a_{j6}}, \frac{b_{ij4}}{a_{j6}}, \frac{b_{ij5}}{a_{j6}}, \frac{b_{ij6}}{a_{j6}} \right\rfloor \right.$$

Using the above formula the weighted comprehensive values of the candidates $x_i\ (i = 1,2,3,4)$ can be obtained as follows:

$\tilde{S}_1 = (0.18,0.26,0.34,0.42,0.56,0.58;\ \ 0.06,0.20,0.34,0.42.0.56,0.70$

$\tilde{S}_2 = (0.19,0.26,0.33,0.40,0.47,0.54;\ \ 0.15,0.24,0.33,0.40,0.49,0.58$

$\tilde{S}_3 = (0.25,0.31,0.37,0.44,0.50,0.56;\ \ 0.23,0.30,0.37,0.44,0.51,0.58$

$\tilde{S}_4 = (0.19,0.26,0.34,0.41,0.49,0.56;\ \ 0.13,0.24,0.34,0.41,0.51,0.62$

According to (11) and (13),(17),(19) and (21) the values and ambiguity of membership functions and non-membership functions of $\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, and\ \tilde{S}_4$ can be calculated as follows:

$V_\mu(\tilde{S}_1) = 0.76,\quad V_\gamma(\tilde{S}_1) = 0.76,$

$V_\mu(\tilde{S}_2) = 0.73,\quad V_\gamma(\tilde{S}_2) = 0.73,$

$V_\mu(\tilde{S}_3) = 0.81,\quad V_\gamma(\tilde{S}_3) = 0.81,$

$V_\mu(\tilde{S}_4) = 0.75,\quad V_\gamma(\tilde{S}_4) = 0.75,$

and

$A_\mu(\tilde{S}_1) = 18.67\quad A_\gamma(\tilde{S}_1) = 26.67,$

$A_\mu(\tilde{S}_2) = 16.33,\quad A_\gamma(\tilde{S}_2) = 17.5,$

$A_\mu(\tilde{S}_3) = 19.00\quad A_\gamma(\tilde{S}_3) = 16.33$

$A_\mu(\tilde{S}_4) = 16.33,\quad A_\gamma(\tilde{S}_4) = 20.0$

It is easy to know that $V_\lambda(\tilde{S}_3) \geq V_\lambda(\tilde{S}_1) \geq V_\lambda(\tilde{S}_4) \geq V_\lambda(\tilde{S}_2)$ for any given weight $\lambda \in [0,1]$ holds. Hence, the ranking order of the four companies is $x_{3,} \geq x_{1,} \geq x_{4,} \geq x_{2,}$ if $\lambda \in [0.1]$ .Best selection is the company x_3 . Obviously, the ranking order of the four companies is related to the attitude parameter $\lambda \in [0, 1]$.

5. CONCLUSION

In this paper, we have studied two characteristics of a SHIFN, that is, the value and ambiguity, which are used to define the value index and ambiguity index of the SHIFN. Then, a ranking method is developed for the ordering of SHIFNs and applied to solve MADM problems with SHIFNs. Due to the fact that a SHIFN is a generalization of a symmetrical hexagonal fuzzy number, the other existing methods of ranking fuzzy numbers may be extended to SHIFNs. More effective ranking methods of SHIFNs can be investigated in the future.

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