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Extension of TOPSIS using L_1 Family of Distance Measures

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Abstract

TOPSIS is an MCDM method based on the technique of obtaining the distance between the shortest distance from positive ideal solution and longest distance from the negative ideal solution that grades the closeness coefficient which allows ranking the alternatives. The traditional method use euclidean norm to find the distance from the ideal solution. In this paper, a modified TOPSIS approach has been proposed using the various distance measures of $\boldsymbol{L_1}$ family. The obtained results from the new proposal have been compared with the traditional TOPSIS method. The comparisons indicate that the proposed distance measures are suitable to solve similar multi criteria decision problems

Keywords: MCDM, TOPSIS, Euclidean Distance, Minkowski's Distance and L_1 family

I. INTRODUCTION

Due to abrupt developments in business sector, the usage of muti criteria decision making has grown to a larger extend in the last few decades. The decision making tools of this kind have become essential for the decision maker to choose the suitable alternative. The various methodologies like AHP, ELECTRE, PROMETHEE, VIKOR, TOPSIS of MCDM [1,2] has been accepted in all fields of optimization. Technique for order performance by similarity to ideal solution (TOPSIS), a classical method is used to such MCDM problems [3-5]. The basic idea is to find the alternative which has the shortest distance from positive ideal solution and a farthest distance from the negative ideal solution. The decision maker shall assume or fix the weights and ratings for the TOPSIS problems.

In the literature, the TOPSIS has been modified in several forms based on normalization technique, distance measure, dependence between alternatives etc., Chen-Tung Chen [6] proposed vertex method to find the distance between PIS and NIS. Jahanshahloo et al. [7,8] extended TOPSIS with interval data and used α-cuts to normalize the fuzzy numbers. Ibrahim A. Baky, Mahmoud A. Abo-Sinna [9] introduced a bi-level programming approach in TOPSIS to MODM problems. Several hybrid approaches integrating AHP and TOPSIS found in literature providing solutions for MCDM problems [10-12].

In this paper the traditional TOPSIS method has been modified by replacing the Euclidean distance by the L_1 family of distance measures to find the distance of each alternative from the PIS and NIS. The optimal solution is identified by the closeness coefficient generated by this distance. Further the six distance measures in the L_1 family are compared with the traditional method.

II. DEFINITIONS

A. Euclidean Distance:

The euclidean distance between a_i and b_i is given by

$$d(a_i, b_i) = \sqrt{\sum_{i=1}^{p} |a_i - b_i|^2}$$

B. L_1 Family of Distance Measures:

The distance between a_i and b_i in L_1 family [13,14] is:

Table 1: L_1 Family of Distance Measures	
Sorensen (or) Bray-Curtis	$d(a_i,b_i) = \frac{\sum_{i=1}^{p}\lvert a_i - b_i\rvert}{\sum_{i=1}^{p}(a_i + b_i)}$
Gower	$d(a_i,b_i) = \frac{1}{p}\sum_{i=1}^{p}\lvert a_i - b_i\rvert$
Soergel	$d(a_i,b_i) = \frac{\sum_{i=1}^{p}\lvert a_i - b_i\rvert}{\sum_{i=1}^{p} max(a_i,b_i)}$
Kulczynski	$d(a_i,b_i) = \frac{\sum_{i=1}^{p}\lvert a_i - b_i\rvert}{\sum_{i=1}^{p} min(a_i,b_i)}$
Canberra	$d(a_i,b_i) = \sum_{i=1}^{p}\frac{\lvert a_i - b_i\rvert}{(a_i + b_i)}$
Lorentzian	$d(a_i,b_i) = \sum_{i=1}^{p}\ln(1 + \lvert a_i - b_i\rvert)$

III. THE PROPOSED METHOD

Step 1- Form a decision matrix.

Step 2- Normalisation: $n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$, i = 1,.,m; j = 1,..,n

Step 3- Weighted normalization: $v_{ij} = w_j n_{ij}$, i = 1,….,m;

j = 1,….,n. and $\sum_{j=1}^{n} w_j$ = 1; (w_j is weight of the j[th] criterion)

Step 4- Find the positive ideal and negative ideal solution by:

$$A^+ = \{v_1^+, \ldots\ldots, v_n^+\} = \left\{\max_i v_{ij},\ j = 1, \ldots\ldots, n\right\},$$

$$A^- = \{v_1^-, \ldots\ldots, v_n^-\} = \left\{\min_i v_{ij},\ j = 1, \ldots\ldots, n\right\},$$

Step 5- Calculate the separation measures:

Table 2: Seperation Measures

Sorensen (or) Bray-Curtis	$d_i^+ = \frac{\sum_{j=1}^{n} \lvert a_{ij} - b_{ij}^+ \rvert}{\sum_{j=1}^{n} (a_{ij} + b_{ij}^+)}$ $d_i^- = \frac{\sum_{j=1}^{n} \lvert a_{ij} - b_{ij}^- \rvert}{\sum_{j=1}^{n} (a_{ij} + b_{ij}^-)}$
Gower	$d_i^+ = \frac{1}{n} \sum_{j=1}^{n} \lvert a_{ij} - b_{ij}^+ \rvert$ $d_i^- = \frac{1}{n} \sum_{j=1}^{n} \lvert a_{ij} - b_{ij}^- \rvert$
Soergel	$d_i^+ = \frac{\sum_{j=1}^{n} \lvert a_{ij} - b_{ij}^+ \rvert}{\sum_{j=1}^{n} max(a_{ij}, b_{ij}^+)}$ $d_i^- = \frac{\sum_{j=1}^{n} \lvert a_{ij} - b_{ij}^- \rvert}{\sum_{j=1}^{n} max(a_{ij}, b_{ij}^-)}$
Kulczynski	$d_i^+ = \frac{\sum_{j=1}^{n} \lvert a_{ij} - b_{ij}^+ \rvert}{\sum_{j=1}^{n} min(a_{ij}, b_{ij}^+)}$ $d_i^- = \frac{\sum_{j=1}^{n} \lvert a_{ij} - b_{ij}^- \rvert}{\sum_{j=1}^{n} min(a_{ij}, b_{ij}^-)}$
Canberra	$d_i^+ = \sum_{j=1}^{n} \frac{\lvert a_{ij} - b_{ij}^+ \rvert}{(a_{ij} + b_{ij}^+)}$ $d_i^- = \sum_{j=1}^{n} \frac{\lvert a_{ij} - b_{ij}^- \rvert}{(a_{ij} + b_{ij}^-)}$
Lorentzian	$d_i^+ = \sum_{j=1}^{n} \ln(1 + \lvert a_{ij} - b_{ij}^+ \rvert)$ $d_i^- = \sum_{j=1}^{n} \ln(1 + \lvert a_{ij} - b_{ij}^- \rvert)$

Step 6- Obtain Relative Closeness: $C_i = \frac{d_i^-}{d_i^+ + d_i^-}$, i = 1,.,m

Step 7- Ranking is given based on the ascending order of the TOPSIS grade C_i

IV. NUMERICAL EXAMPLE

An MCDM problem with 6 alternatives and 5 criteria is taken from the literature. The decision values shown in Table 3 require normalization, since each criteria has different meaning. For the sake of convenience, each criterion is assumed to have equal weights.

Table 3: Decision Matrix

	C1	C2	C3	C4	C5
A1	690	3.1	9	7	4
A2	590	3.9	7	6	10
A3	600	3.6	8	8	7
A4	620	3.8	7	10	6
A5	700	2.8	10	4	6
A6	650	4	6	9	8

Table 4 and 5 shows the normalized and weighted normalized matrix of all alternatives with respect to each criteria respectively. The maximum and the minimum value corresponding to each criteria are taken as positive and negative ideal solution and shown in Table 6.

Table 4: Normalized Matrix

	C1	C2	C3	C4	C5
A1	0.00028	0.04076	0.02375	0.02023	0.01329
A2	0.00024	0.05128	0.01847	0.01734	0.03322
A3	0.00024	0.04733	0.02111	0.02312	0.02326
A4	0.00025	0.04996	0.01847	0.02890	0.01993
A5	0.00028	0.03681	0.02639	0.01156	0.01993
A6	0.00026	0.05259	0.01583	0.02601	0.02658

Table 5: Weighted Normalized Matrix					
	C1	C2	C3	C4	C5
A1	0.000056	0.008151	0.004749	0.004046	0.002658
A2	0.000048	0.010255	0.003694	0.003468	0.006645
A3	0.000048	0.009466	0.004222	0.004624	0.004651
A4	0.000050	0.009992	0.003694	0.005780	0.003987
A5	0.000056	0.007363	0.005277	0.002312	0.003987
A6	0.000052	0.010518	0.003166	0.005202	0.005316

Table 6: Positive and Negative Ideal Solution					
	C1	**C2**	**C3**	**C4**	**C5**
A^+	0.000056	0.010518	0.005277	0.005780	0.006645
A^-	0.000048	0.007363	0.003166	0.002312	0.002658

The separation measures, TOPSIS grade and ranking under each distance measures are calculated using Table 2 and shown in Table 7 to 12. Similarly that of traditional TOPSIS is shown in Table 13. Fig.1 to 6 indicates separation measures between PIS and NIS of L_1 family. Fig.7 compares all the distance measures of L_1 family with traditional method.

Table 7: Sorensen

	d_i^+	d_i^-	**Grade**	**Rank**
A1	0.1521	0.1248	0.4507	5
A2	0.0977	0.1787	0.6466	2
A3	0.1078	0.1688	0.6103	4
A4	0.0996	0.1769	0.6398	3
A5	0.1580	0.1187	0.4290	6
A6	0.0873	0.1889	0.6840	1

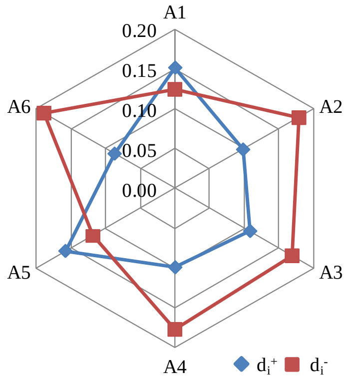

Fig 1: Sorensen

Table 8: Gower

	d_i^+	d_i^-	**Grade**	**Rank**
A1	0.0267	0.0165	0.3822	5
A2	0.0180	0.0252	0.5834	2
A3	0.0197	0.0236	0.5445	4
A4	0.0183	0.0249	0.5761	3
A5	0.0276	0.0156	0.3612	6
A6	0.0162	0.0270	0.6243	1

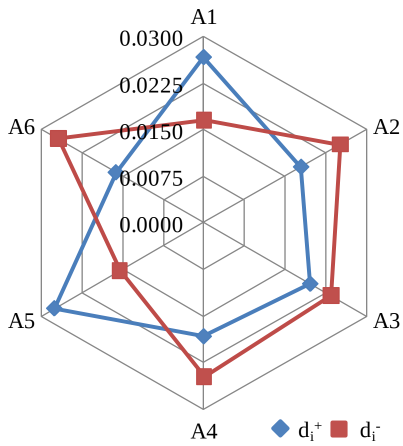

Fig 2: Gower

Table 9: Soergel				
	d_i^+	d_i^-	**Grade**	**Rank**
A1	0.2640	0.2219	0.4567	5
A2	0.1780	0.3033	0.6301	2
A3	0.1946	0.2889	0.5975	4
A4	0.1811	0.3006	0.6240	3
A5	0.2729	0.2123	0.4375	6
A6	0.1605	0.3177	0.6644	1

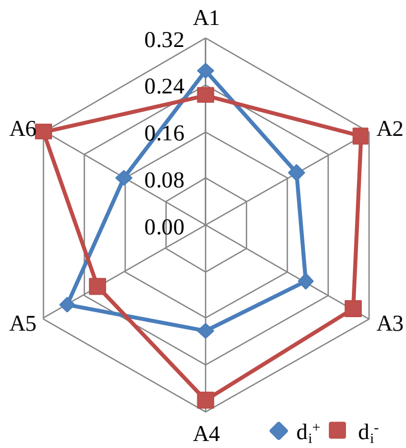

Fig 3: Soergel

Table 10: Kulczynski				
	d_i^+	d_i^-	**Grade**	**Rank**
A1	0.3586	0.2851	0.4429	5
A2	0.2165	0.4352	0.6678	2
A3	0.2416	0.4062	0.6270	4
A4	0.2212	0.4298	0.6602	3
A5	0.3754	0.2695	0.4179	6
A6	0.1912	0.4657	0.7089	1

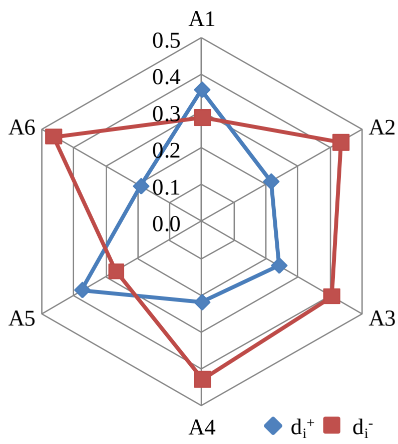

Fig 4: Kulczynski

Table 11: Canberra

	d_i^+	d_i^-	Grade	Rank
A1	0.7916	0.6017	0.4318	5
A2	0.5244	0.8697	0.6238	4
A3	0.5282	0.8823	0.6255	3
A4	0.5127	0.8818	0.6323	2
A5	0.8550	0.5353	0.3850	6
A6	0.4508	0.9428	0.6765	1

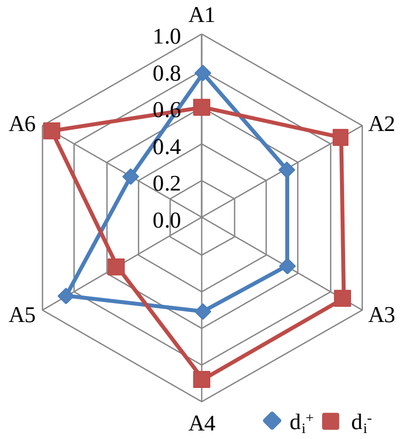

Fig 5: Canberra

Table 12: Lorentzian

	d_i^+	d_i^-	Grade	Rank
A1	0.1305	0.0816	0.3846	5
A2	0.0886	0.1233	0.5818	2
A3	0.0974	0.1159	0.5435	4
A4	0.0901	0.1220	0.5752	3
A5	0.1347	0.0769	0.3635	6
A6	0.0801	0.1322	0.6227	1

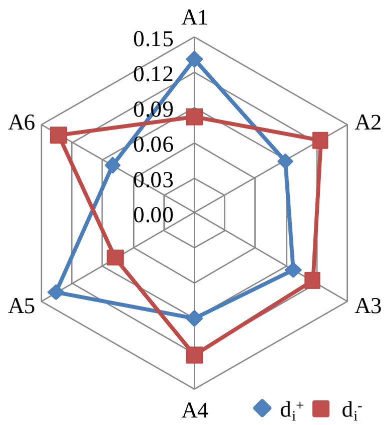

Fig 6: Lorentzian

Table 13: Traditional TOPSIS

	d_i^+	d_i^-	Grade	Rank
A1	0.0797	0.0469	0.3703	5
A2	0.0548	0.0774	0.5856	2
A3	0.0482	0.0617	0.5612	4
A4	0.0566	0.0731	0.5636	3
A5	0.0839	0.0491	0.3693	6
A6	0.0487	0.0764	0.6104	1

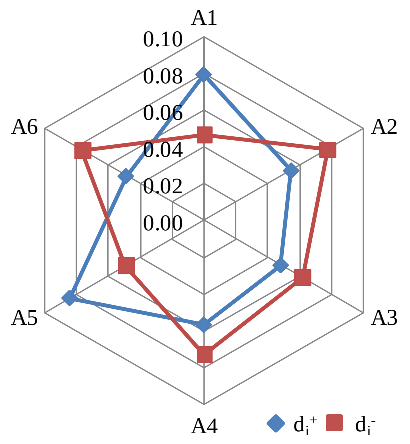

Fig 7: Traditional TOPSIS

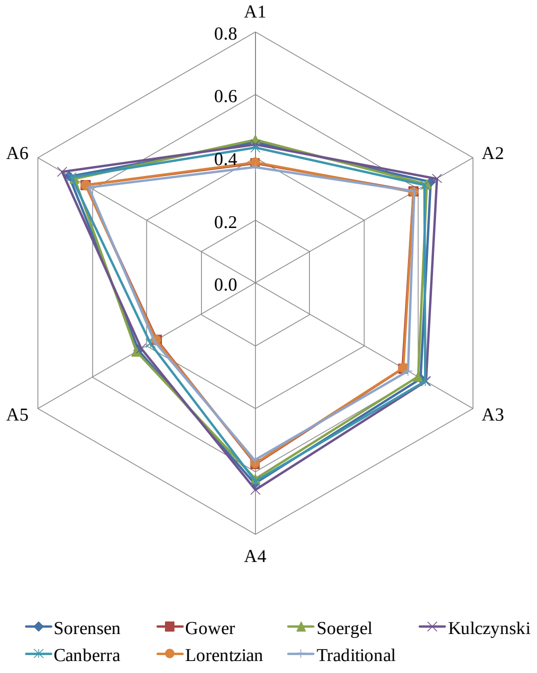

Fig 8: Comparison of all Distances

V. RESUULTS AND DISCUSSION

Various distance measures of L_1 family are classified mainly by the absolute difference. Sorensen or Bray-curtis is used while comparing two pdfs, it is simply the L_1 metric divided by 2. But in Gower distance, the same is divided by p. According to Sung-Hyuk Cha [13], Soergel and Kulczynski distances are non proportional. Canberra and Sorensen look similar, but the absolute difference of the individual level are normalized in Sorensen. Lorentzian distance possess the absolute difference and the natural logarithm is applied. In order to maintain the non-negativity property, 1 is added to logarithmic term.

The difference between the positive and negative ideal solution of each measure seems to be same. But the range of those PIS and NIS of each measure are entirely different. Fig.[1-6] displays the separation measures of each distance in the L_1 family from the PIS and NIS respectively. The alternatives are ranked based on the closeness coefficients obtained from the separation measures. The ranking order on comparison with traditional or any other measures of L_1 family is unchanged. Fig.8 compares all the closeness coefficients of L_1 family with the traditional TOPSIS and display same result, which indicates that the proposed method showcase a better modification to the existing method.

VI. CONCLUSION

In this paper, 1) L_1 family of distance measures has been introduced to modify the traditional TOPSIS method, 2) the euclidean distance is replaced by 6 distance measures namely Sorensen, Gower, Soergel, Kulczynski, Canberra and Lorentzian, 3) the ranking and identification of the best alternative is done for all the distance measures of L_1 family, 4) the comparison of closeness coefficients of the L_1 family and traditional TOPSIS method indicates a close association between them, 5) the method can be extended even in fuzzy environment, 6) the numerical example taken from the literature justify the L_1 family TOPSIS method is effective and suitable tool for MCDM problems under conflicting criteria.

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