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Application of Fuzzy Assignment Problem

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Abstract

Transportation and assignment models play significant role in logistics and supply chain management for reducing cost and time, for better service. In this paper a fuzzy Multi-objective Assignment Model is proposed and for defuzzification, the fuzzy ranking method, i.e., centroid of centroids from its' λ-cuts for linear and nonlinear reference functions is used. The proposed Multi-objective Assignment model is tested by considering an example with three parameters as fuzzy cost, fuzzy time and fuzzy quality with generalized LR trapezoidal fuzzy numbers and a practical data is tested with two parameters fuzzy cost and fuzzy time as generalized LR trapezoidal fuzzy numbers for a civil construction process by various cases. This method helps in assigning and implementing the persons to works successfully on one-one basis so that it increases the number of potential bidders and it minimize the total fuzzy assignment cost and total fuzzy time.

Key words: Multi objective assignment; centroid - Centroid ranking index; LR fuzzy numbers; linear programming, λ-cut.

1 Introduction

Assignment problem is used worldwide in solving real world problems. It plays an important role in the industry and is used very often in solving problems of engineering, management science and it has many other applications. Project management is designed to control organization resources on a given set of activities, within time, cost and quality. Therefore, the limited resources must be utilized efficiently such that the optimal available resources can be assigned to the most needed tasks so as to maximize and minimize the profit and cost respectively. Taha (1992), Hiller and Libermann (2001), Swarup et al. (2003) and Murthy (2007) discussed a single objective function in a crisp environment for different types of assignment problems. Dhingra et al. (1991) formulated a multiple objective nonlinear programming problem with the design of high speed planar mechanisms. Geetha et al. (1993) expressed the cost-time minimizing assignment as the multicriteria problem. Tsai et al. (1999) proposed a thorough formulation of the deployment of manpower to solve a balanced multi-objective decision making problem which is related to cost, time and quality in fuzzy environment. Belacela and Boulasselb (2001) used multi-criterion fuzzy assignment problem for solving medical classification problems by a fuzzy classification method called PROAFTN. Haddad et al. (2002) presented two models for solving the general assignment problem in which the assignment cost and capacity of agents are considered as fuzzy numbers. In order to obtain near optimal solution a novel hybrid algorithm is presented using simulated annealing (SA) method and max-min fuzzy. The multi-objective assignment problem was studied by Bao et al. (2007) and used 0-1 programming method to convert multi-objective assignment problem to single objective assignment problem. Kagade and Bajaj (2009) used linear and non-linear membership functions to solve the multi-objective assignment problem as a vector minimum problem. Kagade and Bajaj (2010) obtained the solution procedure to the multi-objective assignment problem where the cost coefficients of the objective functions are interval values and the equivalent transformed problem explained using fuzzy programming techniques. Mukherjee and Basu (2010) resolved an assignment problem with fuzzy cost, by Yager's ranking index (1981) which transforms the fuzzy assignment problem into a crisp assignment problem. Kumar and Gupta (2011) by using different membership functions and Yager (1981) ranking index proposed methods for solving fuzzy assignment problems and fuzzy travelling salesman problems. Pramanik and Biswas (2011) considered a multi-objective assignment problem with cost, time and quality as trapezoidal fuzzy numbers in the case of military affairs; they have taken weights of the objectives according to their priorities and used Yager (1981) ranking method to form a single objective problem of a multi-objective fuzzy assignment problem. Emrouznejad (2011) developed a procedure based on the data envelopment analysis method to solve the assignment problems with fuzzy costs or fuzzy profits and to rank the fuzzy numbers a discrete approach is presented. Pramanik and Biswas (2012) developed a priority based fuzzy goal programming method for generalized trapezoidal fuzzy numbers and applied it for multi-objective assignment problem in which cost and time are considered as a generalized trapezoidal fuzzy number. Tapkan et al. (2013) by using different ranking methods in bees algorithm the direct solution approach for solving the fuzzy multiple objective generalized assignment problem is proposed in which the coefficients and right hand side values of the constraints and the objective function coefficients are defined as fuzzy numbers.

In this paper, the fuzzy Multi-objective Assignment Model is proposed and a fuzzy Assignment Problem with three parameters fuzzy cost, fuzzy time and fuzzy quality are proposed to find the minimum total fuzzy assignment cost, time and quality. For defuzzification, the fuzzy ranking method centroid of centroids from its' λ-cuts for linear and nonlinear reference functions is used. The proposed Multi-objective Assignment model with three parameters as fuzzy cost, fuzzy time and fuzzy quality with generalized LR trapezoidal fuzzy numbers is tested by considering an example. The Multi-objective Assignment model with two parameters fuzzy cost and fuzzy time as generalized LR trapezoidal fuzzy numbers is tested with a practical data for civil construction process which is collected from an Organization GITAM University (GU), Visakhapatnam (VSP), Andhra Pradesh (AP), India. This method helps in assigning and implementing the persons to works successfully on one-one basis so that it increase the number of potential bidders and it minimize the total fuzzy assignment cost and total fuzzy time and it will be benefited to the management.

The rest of the paper is organised as follows: In section 2, Preliminaries of LR fuzzy numbers, λ-cut of LR fuzzy number, reference functions and arithmetic operations are reviewed. In section 3, a Ranking Method using Centroid of Centroids from its $\lambda-$cut is reviewed. In section 4, Fuzzy Multi-objective Assignment Model with fuzzy cost, fuzzy time, fuzzy quality and so on and mathematical formulation of fuzzy Multi-objective Assignment Model and working rule to find the optimal solution is given. In section 5, the proposed Multi-objective Assignment Model with three parameters fuzzy cost, fuzzy time and fuzzy quality tests with numerical example. In Section 6, the proposed Multi-objective Assignment Model with two parameters fuzzy cost, and fuzzy time is tested with a practical data which is collected from GITAM University (GU), Visakhapatnam (VSP), Andhra Pradesh, India. Finally, the conclusion of the work is given in section 7.

2. Preliminaries

In this section, LR fuzzy number, λ-cut of LR fuzzy number, and reference functions are reviewed from Dubois and Prade (1980)

LR fuzzy number

A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be an LR fuzzy number if

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \le m,\ \alpha > 0, \\ R\left(\frac{x-n}{\beta}\right), & x \ge n,\ \beta > 0, \\ 1, & \text{otherwise.} \end{cases}$$

If m = n then $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ will be converted into $\tilde{A} = (m, \alpha, \beta)_{LR}$ and is said to be an triangular LR fuzzy number. L and R are called reference functions, which are continuous, non-increasing functions that define the left and right shapes of $\mu_{\tilde{A}}(x)$ respectively and L(0) = R(0) =1. Two special cases are triangular and trapezoidal fuzzy number, for which $L(x) = R(x) = \{\max 0, 1-|x|\}$ are linear functions. Commonly used nonlinear reference functions with parameters p, are denoted as $RF_p(x)$. Linear and Non-linear reference functions with their inverses are presented in Table 1

Table 1: Reference functions and their inverses

Function Name	**Reference Functions** $RF_p(x)$	**Inverse of Reference function** $\alpha \in (0,1]$
Linear	$RF_p(x) = \max\{0, 1-\|x\|\}$	$RF_p^{-1}(x) = (1-\alpha)$
Exponential	$RF_p(x) = e^{-px},\ p \ge 1$	$RF_p^{-1}(x) = -(In\alpha)/p$
Power	$RF_p(x) = \max(0, 1-x^p),\ p \ge 1$	$RF_p^{-1}(x) = \sqrt[p]{1-\alpha}$
Exponential power	$RF_p(x) = e^{-x^p},\ p \ge 1$	$RF_p^{-1}(x) = \sqrt[p]{-In\alpha}$
Rational	$RF_p(x) = 1/(1+x^p),\ p \ge 1$	$RF_p^{-1}(x) = \sqrt[p]{(1-\alpha)/\alpha}$

λ-cut of L-R fuzzy number

Let $\tilde{A} = (m, n, \alpha, \beta)_{L-R}$ be an L-R fuzzy number and λ be a real number in the interval [0, 1]. Then the crisp set $A_\lambda = \{x \in X : \mu_{\tilde{A}}(x) \ge \lambda\} = [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$ is said to be λ-cut of $\tilde{A}$.

Arithmetic Operations on LR fuzzy number

(i) LR Fuzzy numbers addition:

Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be two LR fuzzy numbers. Then $\tilde{A}_1 \oplus \tilde{A}_2 = (m_1 + m_2,\ n_1 + n_2,\ \alpha_1 + \alpha_2,\ \beta_1 + \beta_2)_{LR}$

3. Ranking Method

In this section, a Ranking Method using Centroid of Centroids from its' $\lambda-$cut is reviewed from Thorani and Ravi Shankar (2014).

Generally, the Centroid is considered as the balancing point of a trapezoid. In this new ranking method the trapezoid (APQD) is divided into the triangle (APC), the triangle (QCD) and the triangle (PQC) as shown in Fig.1. and the centroids of these three triangles APC, QCD and PQC be G_1, G_2 and G_3 respectively as shown in Fig.1.

The centroid of the triangle (APC), where A = (m- α, 0), P = (m, w), C = (n, 0) is,

$$G_1 = \left(\frac{2m + n - \alpha}{3}, \frac{w}{3} \right)$$

The centroid of the triangle (QCD), where Q = (n, w), C = (n, 0), D = (n+β, 0) is,

$$G_2 = \left(\frac{3n + \beta}{3}, \frac{w}{3} \right)$$

The centroid of the triangle (PQC), where P = (m, w), Q = (n, w), C = (n, 0) is,

$$G_3 = \left(\frac{m + 2n}{3}, \frac{2w}{3} \right)$$

Equation of the line G_1G_2 is $y = \frac{w}{3}$ and G_3 does not lie on the line G_1G_2. Therefore G_1, G_2 and G_3 are non-collinear and they form a triangle.

The Centroid of the triangle $(G_1G_2G_3)$ is the intersection of the three centroid points G_1, G_2 and G_3 as shown in Fig.1. Since each Centroid point is a balancing point of each individual triangle, therefore Centroid of the triangle $(G_1G_2G_3)$ is taken as the better point of reference than the centroid pont of the trapezoid, as it is much more balancing point, it is used to define the ranking of generalized LR fuzzy numbers $\tilde{A} = (m, n, \alpha, \beta;\ w)_{LR}$.

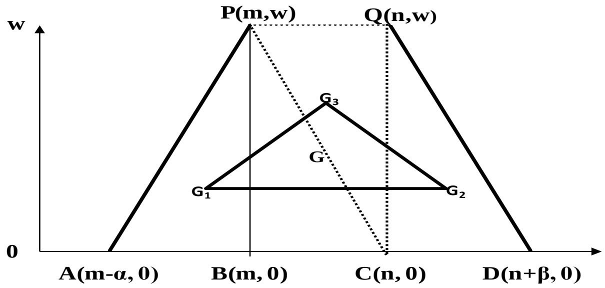

Fig. 1: Generalized LR fuzzy number

The Centroid $G_{\tilde{A}}(x,y)$ of the triangle $(G_1G_2G_3)$ with vertices G_1, G_2 and G_3 for the generalized trapezoidal LR fuzzy number $\tilde{A} = (m,n,\alpha,\beta;\, w)_{LR}$ is

$$G_{\tilde{A}}(x,y) = \left(\frac{3m + 6n - \alpha + \beta}{9}, \frac{4w}{9} \right) \tag{1}$$

The Centroid $G_{\tilde{A}}(x,y)$ of the triangle $(G_1G_2G_3)$ with vertices G_1, G_2 and G_3 for the generalized triangular LR fuzzy number $\tilde{A} = (m,n,\alpha,\beta;\, w)_{LR}$ i.e., m = n is

$$G_{\tilde{A}}(x,y) = \left(\frac{9m - \alpha + \beta}{9}, \frac{4w}{9} \right) \tag{2}$$

The ranking function of the generalized trapezoidal LR fuzzy number $\tilde{A} = (m,n,\alpha,\beta;\, w)_{LR}$ which maps the set of all fuzzy numbers to a set of real numbers is defined as:

$$R(\tilde{A}) = (x \times y) = \left(\frac{3m + 6n - \alpha + \beta}{9} \times \frac{4w}{9} \right) \tag{3}$$

This the Area between the Centroid of the Centroids $G_{\tilde{A}}(x,y)$ as defined in Eq. (3.1) and the original point.

The ranking function of the generalized trapezoidal LR fuzzy number $\tilde{A} = (m,n,\alpha,\beta;w)_{LR}$ of Eq. (1) from its $\lambda-$cut is defined as

$$R(\tilde{A}) = 2\left(\frac{4w}{81} \int_0^1 \left(3m - \alpha L^{-1}(\lambda)\right) d\lambda + \int_0^1 \left(6n + \beta R^{-1}(\lambda)\right) d\lambda \right) \tag{4}$$

Since $R(\tilde{A})$ is calculated from the extreme values of λ-cut of $\tilde{A}$, rather than its membership function, it is not required knowing the explicit form of the membership functions of the fuzzy numbers to be ranked. That is unlike most of the ranking methods that require the knowledge of the membership functions of all fuzzy numbers to be ranked. This centroid of centroid index is still applicable even if the explicit form the membership function of the fuzzy numbers is unknown.

The ranking indexes for Eq. (4) for linear and nonlinear reference functions are:

Case (i) $L(x) = R(x) = \max\{0, 1-|x|\}$

$$R(\tilde{A}) = \frac{8w}{81}\left[3m + 6n - \frac{\alpha}{2} + \frac{\beta}{2}\right]$$

Case (ii) $L(x) = R(x) = e^{-x}$

$$R(\tilde{A}) = \frac{8w}{81}\left[3m + 6n - \alpha + \beta\right]$$

Case (iii) $L(x) = \max\{0, 1-|x|\}$ **and** $R(x) = e^{-x}$

$$R(\tilde{A}) = \frac{8w}{81}\left[3m + 6n - \frac{\alpha}{2} + \beta\right]$$

Case (iv) $L(x) = e^{-x}$ **and** $R(x) = \max\{0, 1-|x|\}$

$$R(\tilde{A}) = \frac{8w}{81}\left[3m + 6n + \alpha + \frac{\beta}{2}\right]$$

Case (v) $L(x) = e^{-px}$ **and** $R(x) = \max\{0, 1-x^p\}$

$$R(\tilde{A}) = \frac{8w}{81}\left[3m + 6n - \frac{\alpha}{p} + \frac{\beta p}{p+1}\right]$$

Case (vi) $L(x) = \max\{0, 1-|x^p|\}$ **and** $R(x) = e^{-px}$

$$R(\tilde{A}) = \frac{8w}{81}\left[3m + 6n + \frac{\alpha p}{p+1} + \frac{\beta}{p}\right]$$

The Mode (M) of the generalized LR fuzzy number $\tilde{A} = (m, n, \alpha, \beta;\ w)_{LR}$ is defined as:

$$M(\tilde{A}) = \frac{1}{2}\int_0^w (m+n)\, dx = \frac{w}{2}(m+n) \quad (5)$$

The Spread (S) of the generalized LR fuzzy number $\tilde{A} = (m, n, \alpha, \beta; w)_{LR}$ is defined as:

$$S(\tilde{A}) = \int_0^w (\alpha + \beta + n - m)\, dx = w(\alpha + \beta + n - m) \quad (6)$$

The Left spread (LS) of the generalized LR fuzzy number $\tilde{A} = (m, n, \alpha, \beta; w)_{LR}$ is defined as:

$$LS(\tilde{A}) = \int_0^w \alpha\, dx = w\,\alpha \quad (7)$$

The Right spread (RS) of the generalized LR fuzzy number $\tilde{A} = (m, n, \alpha, \beta; w)_{LR}$ is defined as:

$$RS(\tilde{A}) = \int_0^w \beta\, dx = w\,\beta \quad (8)$$

Working rule to find the ranks of two generalized LR fuzzy numbers $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1; w_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2; w_2)_{LR}$, by using the ranking indexes for Eq. (4) linear and nonlinear reference functions, and using the equations from Eq. (5) to Eq. (8), is defined as follows:

Step 1: Calculate $R(\tilde{A}_1)$ and $R(\tilde{A}_2)$

(i) If $R(\tilde{A}_1) > R(\tilde{A}_2)$ then $\tilde{A}_1 > \tilde{A}_2$

(ii) If $R(\tilde{A}_1) < R(\tilde{A}_2)$ then $\tilde{A}_1 < \tilde{A}_2$

(iii) If $R(\tilde{A}_1) = R(\tilde{A}_2)$, then go to step 2.

Step 2: Calculate $M(\tilde{A}_1)$ and $M(\tilde{A}_2)$

(i) If $M(\tilde{A}_1) > M(\tilde{A}_2)$ then $\tilde{A}_1 > \tilde{A}_2$

(ii) If $M(\tilde{A}_1) < M(\tilde{A}_2)$ then $\tilde{A}_1 < \tilde{A}_2$

(iii) If $M(\tilde{A}_1) = M(\tilde{A}_2)$, then go to step 3.

Step 3: Calculate $S(\tilde{A}_1)$ and $S(\tilde{A}_2)$

(i) If $S(\tilde{A}_1) > S(\tilde{A}_2)$ then $\tilde{A}_1 < \tilde{A}_2$

(ii) If $S(\tilde{A}_1) < S(\tilde{A}_2)$ then $\tilde{A}_1 > \tilde{A}_2$

(iii) If $S(\tilde{A}_1) = S(\tilde{A}_2)$, then go to step 4.

Step 4: Calculate $LS(\tilde{A}_1)$ and $LS(\tilde{A}_2)$

(i) If $LS(\tilde{A}_1) > LS(\tilde{A}_2)$ then $\tilde{A}_1 > \tilde{A}_2$

(ii) If $LS(\tilde{A}_1) < LS(\tilde{A}_2)$ then $\tilde{A}_1 < \tilde{A}_2$

(iii) If $LS(\tilde{A}_1) = LS(\tilde{A}_2)$, then go to step 5.

Step 5: Examine w_1 and w_2

(i) If $w_1 > w_2$ then $\tilde{A}_1 > \tilde{A}_2$

(ii) If $w_1 < w_2$ then $\tilde{A}_1 < \tilde{A}_2$

(iii) If $w_1 = w_2$ then $\tilde{A}_1 \approx \tilde{A}_2$

4. Fuzzy Multi-objective Assignment Model Uing Linear Programming

In this Section, Fuzzy Multi-objective Assignment Model with fuzzy cost, fuzzy time, fuzzy quality and so on and mathematical formulation of fuzzy Multi-objective Assignment Model is given.

In a general Assignment Problem, 'n' works to be performed by 'n' persons depending on their efficiency to do the job in one-one basis such that the assignment cost is minimum or maximum. If the objective of an assignment problem is to minimize fuzzy cost, fuzzy time fuzzy quality, and so on, then this type of fuzzy Assignment problem is treated as a fuzzy Multi-objective Assignment problem. Here, a fuzzy assignment problem with several fuzzy parameters in the following form of $n \times n$ fuzzy matrix where each cell having a fuzzy cost $(\tilde{c}_{ij})$, fuzzy time $(\tilde{t}_{ij})$, fuzzy quality $(\tilde{q}_{ij})$ and so on is shown in Table 2.

Table 2: General Fuzzy Multi-objective Assignment Model

PERSONS \ WORKS	1	2	…	j	…	N
1	$\tilde{c}_{11},\tilde{t}_{11},\tilde{q}_{11},...$	$\tilde{c}_{12},\tilde{t}_{12},\tilde{q}_{12},...$	…	$\tilde{c}_{1j},\tilde{t}_{1j},\tilde{q}_{1j},...$	…	$\tilde{c}_{1n},\tilde{t}_{1n},\tilde{q}_{1n},...$
2	$\tilde{c}_{21},\tilde{t}_{21},\tilde{q}_{21},...$	$\tilde{c}_{22},\tilde{t}_{22},\tilde{q}_{22},...$	…	$\tilde{c}_{2j},\tilde{t}_{2j},\tilde{q}_{2j},...$	…	$\tilde{c}_{2n},\tilde{t}_{2n},\tilde{q}_{2n},...$
…	…	…	…	…	…	…
I	$\tilde{c}_{i1},\tilde{t}_{i1},\tilde{q}_{i1},...$	$\tilde{c}_{i2},\tilde{t}_{i2},\tilde{q}_{i2},...$	…	$\tilde{c}_{ij},\tilde{t}_{ij},\tilde{q}_{ij},...$	…	$\tilde{c}_{in},\tilde{t}_{in},\tilde{q}_{in},...$
…	…	…	…	…	…	…
N	$\tilde{c}_{n1},\tilde{t}_{n1},\tilde{q}_{n1},...$	$\tilde{c}_{n2},\tilde{t}_{n2},\tilde{q}_{n2},...$	…	$\tilde{c}_{nj},\tilde{t}_{nj},\tilde{q}_{nj},....$	…	$\tilde{c}_{nn},\tilde{t}_{nn},\tilde{q}_{nn},...$

4.1 Mathematical Formulation of Fuzzy Multi-objective Assignment Problem

Mathematically, the fuzzy Multi-objective Assignment Problem in Table 1 can be stated as:

$$\text{Minimize } \tilde{z}_k = \sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{p}_{ij}^{k})x_{ij},\ k = 1,2,...,n$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1,2,...,n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1,2,...,n$$

$x_{ij} = 0$ or 1, for all i =1,2,…,n and j=1,2,…,n.

$$x_{ij} = \begin{cases} 1 \text{ if } i^{th} \text{ person is assigned } j^{th} \text{ work} \\ 0 \text{ otherwise} \end{cases}$$

where $\tilde{z}_k = \{\tilde{z}_1, \tilde{z}_2, \tilde{z}_3, \ldots \tilde{z}_k\}$ is a vector of k-objective functions.

The fuzzy Multi-objective Assignment Problem is converted into a single objective Assignment Problem by considering the weights w_1, w_2,… based on the priorities of the objective in such a way that $w_1 + w_2 + \ldots = 1$.

$$\text{Minimize} \quad \tilde{z} = w_1 \sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{c}_{ij})x_{ij} + w_2 \sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{t}_{ij})x_{ij} + w_3 \sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{q}_{ij})x_{ij} + \ldots$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,\ldots,n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,\ldots,n$$

and $w_1 + w_2 + w_3 + \ldots = 1$.

$$x_{ij} = 0 \text{ or } 1, \text{ for all } i=1,2,\ldots,n \text{ and } j=1,2,\ldots,n.$$

$$x_{ij} = \begin{cases} 1 \text{ if } i^{th} \text{ person is assigned } j^{th} \text{ work} \\ 0 \text{ otherwise} \end{cases}$$

If the objective function $\tilde{z}_1$ denotes the fuzzy cost function, objective function is defined as: Minimize $\tilde{z}_1 = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$,

If the objective function $\tilde{z}_2$ denotes the fuzzy time function, objective function is defined as: Minimize $\tilde{z}_2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{t}_{ij} x_{ij}$,

If the objective function $\tilde{z}_3$ denotes the fuzzy quality function, objective function is defined as: Minimize $\tilde{z}_3 = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{q}_{ij} x_{ij}$,

Then it is called as a three objective fuzzy assignment problem.

Here, a fuzzy Assignment Problem with three parameters in the following form of $n \times n$ fuzzy matrix where each cell having a fuzzy cost $(\tilde{c}_{ij})$, fuzzy time $(\tilde{t}_{ij})$, and fuzzy quality $(\tilde{q}_{ij})$ is considered as shown in Table 3.

Table 3: Three objective fuzzy Assignment Problem with cost, time, and quality

WORKS

PERSONS	1	2	…	j	…	n
1	$\tilde{c}_{11};\tilde{t}_{11};\tilde{q}_{11}$	$\tilde{c}_{12};\tilde{t}_{12};\tilde{q}_{12}$	…	$\tilde{c}_{1j};\tilde{t}_{1j};\tilde{q}_{1j}$	…	$\tilde{c}_{1n};\tilde{t}_{1n};\tilde{q}_{1n}$
2	$\tilde{c}_{21};\tilde{t}_{21};\tilde{q}_{21}$	$\tilde{c}_{22};\tilde{t}_{22};\tilde{q}_{22}$	…	$\tilde{c}_{2j};\tilde{t}_{2j};\tilde{q}_{2j}$	…	$\tilde{c}_{2n};\tilde{t}_{2n};\tilde{q}_{2n}$
…	…	…	…	…	…	…
i	$\tilde{c}_{i1};\tilde{t}_{i1};\tilde{q}_{i1}$	$\tilde{c}_{i2};\tilde{t}_{i2};\tilde{q}_{i2}$	…	$\tilde{c}_{ij};\tilde{t}_{ij};\tilde{q}_{ij}$	…	$\tilde{c}_{in};\tilde{t}_{in};\tilde{q}_{in}$
…	…	…	…	…	…	…
N	$\tilde{c}_{n1};\tilde{t}_{n1};\tilde{q}_{n1}$	$\tilde{c}_{n2};\tilde{t}_{n2};\tilde{q}_{n2}$	…	$\tilde{c}_{nj};\tilde{t}_{nj};\tilde{q}_{nj}$	…	$\tilde{c}_{nn};\tilde{t}_{nn};\tilde{q}_{nn}$

The three objective fuzzy Assignment Problem is converted into a single objective fuzzy Assignment Problem, by normalizing the fuzzy cost $(\tilde{c}_{ij})$, fuzzy time $(\tilde{t}_{ij})$ and fuzzy quality $(\tilde{q}_{ij})$ and by considering the weights w_1, w_2 and w_3 based on the priorities of the objective. The assignment problem is not affected by normalized data.

$$\text{Minimize } \tilde{z} = w_1\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{c}_{ij})x_{ij} + w_2\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{t}_{ij})x_{ij} + w_3\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{q}_{ij})x_{ij}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,\dots,n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,\dots,n$$

$$x_{ij} = 0 \text{ or } 1, \text{ for all } i=1,2,\dots,n \text{ and } j=1,2,\dots,n.$$

$$x_{ij} = \begin{cases} 1 \text{ if } i^{th} \text{ person is assigned } j^{th} \text{ work} \\ 0 \text{ otherwise} \end{cases}$$

and $w_1 + w_2 + w_3 = 1$.

where $\tilde{c}_{ij}$: Fuzzy cost to i^{th} person for doing j^{th} work.

$\tilde{t}_{ij}$: Fuzzy time for i^{th} person for doing j^{th} work.

$\tilde{q}_{ij}$: Fuzzy quality of the i^{th} person for doing j^{th} work.

$\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{c}_{ij})x_{ij}$: Total fuzzy cost for performing all the works

$\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{t}_{ij})x_{ij}$: Total fuzzy time for performing all the works

$\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{q}_{ij})x_{ij}$: Total fuzzy quality for performing all the works

4.2 Optimal Solution to the Fuzzy Multi-objective Assignment Problem

In this Section, a working rule to find the optimal solution, the total fuzzy cost, total fuzzy time and total fuzzy quality to the fuzzy Multi-objective Assignment Problem and its technique is explained with an example where fuzzy cost $(\tilde{c}_{ij})$, fuzzy time $(\tilde{t}_{ij})$, and fuzzy quality $(\tilde{q}_{ij})$ as generalized LR fuzzy numbers.

Working Rule to find the optimal solution

Step 1: First test whether the given fuzzy Multi-objective assignment matrix is a balanced one or not. If it is a balanced one (i.e., number of persons are equal to the number of works) then go to step 3. If it is an unbalanced one (i.e., number of persons is not equal to the number of works) then go to step 2.

Step 2: Introduce dummy rows and/or columns with zero fuzzy costs, time and quality so as to form a balanced one.

Step 3: Consider the fuzzy Linear Programming Model as mentioned in Section 4.1.

Step 4: Convert the fuzzy Multi-objective Assignment Problem into the following crisp Linear Programming Problem

$$\text{Minimize} \quad z = w_1\sum_{i=1}^{n}\sum_{j=1}^{n}R(\tilde{c}_{ij})x_{ij} + w_2\sum_{i=1}^{n}\sum_{j=1}^{n}R(\tilde{t}_{ij})x_{ij} + w_3\sum_{i=1}^{n}\sum_{j=1}^{n}R(\tilde{q}_{ij})x_{ij}$$

subject to

$$\sum_{i=1}^{n}x_{ij} = 1, \quad j = 1,2,\ldots,n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,...,n$$

$$x_{ij} = 0 \text{ or } 1, \text{ for all } i=1,2,\ldots,n \text{ and } j=1,2,\ldots,n.$$

$$x_{ij} = \begin{cases} 1 \text{ if } i^{th} \text{ person is assigned } j^{th} \text{ work} \\ 0 \text{ otherwise} \end{cases}$$

and $w_1 + w_2 + w_3 = 1$.

Step 5: Calculate the values of $R(\tilde{c}_{ij}), R(\tilde{t}_{ij})$ and $R(\tilde{q}_{ij})$, $\forall i, j$ by using the ranking procedure as mentioned in Section 3.

Step 6: For the values obtained in step 5, choose the maximum cost, time and quality. To normalize operation cost, time and quality, let the maximum cost = $1/k_1$ (say), the maximum time = $1/k_2$ (say), the maximum quality = $1/k_3$ (say).The objective of the assignment problem is to minimize cost, time, quality. Let $a = w_1k_1$, $b = w_2k_2$ and $c = w_3k_3$.

Step 7: Using the values of a, b and c obtained in step 6, and using the values of $R(\tilde{c}_{ij}), R(\tilde{t}_{ij})$ and $R(\tilde{q}_{ij})$ obtained in step 5, the fuzzy Multi-objective Assignment Problem in step 4 is converted into the following crisp linear programming problem

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} a\, R(\tilde{c}_{ij})\, x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} b\, R(\tilde{t}_{ij})\, x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} c\, R(\tilde{q}_{ij})\, x_{ij}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,...,n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,...,n$$

$$x_{ij} = 0 \text{ or } 1, \text{ for all } i=1,2,\ldots,n \text{ and } j=1,2,\ldots,n.$$

$$x_{ij} = \begin{cases} 1 \text{ if } i^{th} \text{ person is assigned } j^{th} \text{ work} \\ 0 \text{ otherwise} \end{cases}$$

Step 8: The model obtained in step 7 is converted into a single objective crisp Assignment problem as follows:

$$\text{Minimize} \quad z = \sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij} x_{ij}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,...,n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,...,n$$

$$x_{ij} = 0 \text{ or } 1, \text{ for all } i=1,2,\ldots,n \text{ and } j=1,2,\ldots,n.$$

$$x_{ij} = \begin{cases} 1 \text{ if } j^{th} \text{ work is assigned } i^{th} \text{ person} \\ 0 \text{ otherwise} \end{cases}$$

where Q_{ij} is a constant.

Step 9: For the crisp linear programming problem obtained in step 8, the optimal solution $\{x_{ij} : i = 1,2,\ldots,n; j = 1,2,\ldots,n\}$ is obtained by using the software applications like TORA or LINGO.

Step 10: By using step 9, the number of persons is assigned to the number of works are obtained. The total fuzzy assignment cost, time and quality of the original problem is obtained by adding the respective units with the fuzzy assignment cost, time and quality per unit of the original problem by using the fuzzy arithmetic operators as mentioned in Section 2.

5. Numerical Example for Fuzzy Multi-objective Assignment Problem

To illustrate the proposed model, six persons A, B, C, D, E, F and six works I, II, III, IV, V, and VI respectively are considered as a fuzzy Multi-objective Assignment Problem to minimize the fuzzy cost, fuzzy time and fuzzy quality. The fuzzy cost coefficients, the fuzzy time and the fuzzy quality in fuzzy Multi- objective Assignment problem are considered as LR fuzzy numbers for allocating each person. The fuzzy Multi-objective Assignment Problem with fuzzy cost, fuzzy time and the fuzzy quality is shown in Table 4 and it is solved for linear and nonlinear reference functions as mentioned in Section 3.

Table 4: Fuzzy assignment problem with fuzzy cost, fuzzy time and fuzzy quality

PERSONS \ WORKS	I	II	III	IV	V	VI
A	(6,7,2,2) (9,11,2,2) (0.16,0.19,0.01,0.02)	(5,7,2,3) (9,10,3,2) (0.11,0.13,0.01,0.01)	(8,10,3,2) (4,5,2,2) (0.04,0.06,0.03,0.03)	(5,8,1,2) (12,13,4,2) (0.18,0.20,0.02,0.1)	(7,10,1,2) (10,11,1,1) (0.16,0.18,0.02,0.02)	(4,6,1,3) (11,13,3,2) (0.07,0.09,0.02,0.02)
B	(3,5,1,2) (7,10,1,2) (0.12,0.15,0.03,0.03)	(7,8,2,3) (12,14,3,3) (0.16,0.18,0.02,0.02)	(8,7,2,4) (5,8,2,1) (0.08,0.09,0.03,0.01)	(5,7,2,2) (9,12,1,1) (0.09,0.15,0.03,0.03)	(6,7,1,3) (8,10,1,1) (0.21,0.23,0.01,0.02)	(7,9,3,2) (8,12,2,1) (0.18,0.22,0.03,0.03)
C	(8,10,2,2) (4,5,1,2) (0.20,0.22,0.02,0.02)	(7,12,2,2) (5,7,1,2) (0.15,0.17,0.02,0.02)	(8,9,1,2) (7,9,1,2) (0.11,0.14,0.01,0.02)	(6,8,2,2) (6,8,1,4) (0.22,0.23,0.02,0.02)	(7,9,1,1) (7,8,1,3) (0.22,0.24,0.02,0.03)	(5,7,1,2) (4,6,1,1) (0.16,0.18,0.01,0.02)
D	(7,10,4,2) (6,8,2,2) (0.18,0.20,0.03,0.02)	(7,10,1,2) (7,8,2,2) (0.21,0.23,0.02,0.02)	(11,13,3,2) (7,10,1,2) (0.17,0.20,0.02,0.02)	(9,10,3,2) (4,6,2,2) (0.10,0.11,0.01,0.04)	(10,11,3,2) (5,7,1,1) (0.13,0.14,0.01,0.01)	(7,10,2,4) (9,11,4,4) (0.14,0.16,0.04,0.02)
E	(6,8,2,2) (11,13,2,2) (0.09,0.11,0.01,0.01)	(24,26,4,2) (2,4,2,2) (0.14,0.18,0.02,0.02)	(12,14,2,2) (12,13,1,2) (0.03,0.04,0.01,0.02)	(9,10,1,2) (5,6,1,1) (0.03,0.05,0.02,0.02)	(16,17,2,3) (8,10,2,2) (0.14,0.15,0.01,0.01)	(8,11,2,2) (16,18,2,2) (0.12,0.14,0.02,0.01)
F	(16,19,2,1) (18,20,2,2) (0.07,0.08,0.01,0.03)	(24,26,2,2) (6,8,2,2) (0.32,0.35,0.02,0.02)	(21,23,1,1) (20,21,2,2) (0.06,0.08,0.02,0.02)	(20,22,2,2) (18,20,2,2) (0.22,0.24,0.01,0.02)	(22,24,2,2) (10,11,2,2) (0.06,0.12,0.02,0.03)	(8,10,2,2) (11,13,2,1) (0.10,0.15,0.02,0.03)

Note: " (grey) " indicates time and " (red) " indicates quality.

Case (i) $L(x) = R(x) = \max\{0, 1-|x|\}$

Step 1: The fuzzy Multi-objective Assignment Problem in Table 4 is a balanced one.

Step 2: Using step 3 of Section 4.2, the fuzzy Multi-objective Assignment Problem is converted into a single fuzzy objective problem as follows:

$$\text{Minimize } \tilde{z} = 0.5\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{c}_{ij})x_{ij} + 0.4\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{t}_{ij})x_{ij} + 0.1\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{q}_{ij})x_{ij}$$

subject to

$$\sum_{i=1}^{6} x_{ij} = 1, \quad j = 1,2,\ldots,6$$

$$\sum_{j=1}^{6} x_{ij} = 1, \quad i = 1,2,...,6$$

$$x_{ij} = 0 \text{ or } 1, \text{ for all i=1,2,3,4,5,6 and j=1,2,3,4,5,6.}$$

$$x_{ij} = \begin{cases} 1 \text{ if } i^{th} \text{ person is assigned } j^{th} \text{ work} \\ 0 \quad \text{otherwise} \end{cases}$$

Here, $w_1 = 0.5$, $w_2 = 0.4$, $w_3 = 0.1$.

Step 3: The fuzzy Multi-objective Assignment problem is converted into the following crisp linear programming problem as:

$$\text{Minimize } z = 0.5\sum_{i=1}^{6}\sum_{j=1}^{6} R(\tilde{c}_{ij})x_{ij} + 0.4\sum_{i=1}^{6}\sum_{j=1}^{6} R(\tilde{t}_{ij})x_{ij} + 0.1\sum_{i=1}^{6}\sum_{j=1}^{6} R(\tilde{q}_{ij})x_{ij}$$

subject to

$$\sum_{i=1}^{6} x_{ij} = 1, \quad j = 1,2,...,6$$

$$\sum_{j=1}^{6} x_{ij} = 1, \quad i = 1,2,...,6$$

$$x_{ij} = 0 \text{ or } 1, \text{ for all i=1,2,3,4,5,6 and j=1,2,3,4,5,6.}$$

Step 4: Using Section 3, the values $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$ and $R(\tilde{q}_{ij})$, $\forall i, j$ of Table 4 are calculated and given in Table 5.

Table 5: Ranks of Fuzzy Cost, Fuzzy Time and Fuzzy Quality for case (i)

PERSONS	WORKS I	II	III	IV	V	VI
A	5.92 9.18 0.16	5.67 8.54 0.10	8.24 4.14 0.04	6.27 11.16 0.17	8.04 9.48 0.15	4.83 10.91 0.07
B	3.90 8.04 0.12	6.86 11.85 0.15	6.61 6.17 0.07	5.62 9.77 0.11	6.02 8.29 0.199	7.35 9.43 0.18
C	8.29 4.19 0.18	9.18 5.67 0.14	7.75 7.45 0.11	6.51 6.66 0.20	7.40 6.91 0.20	5.67 4.74 0.15
D	7.90 6.51 0.17	8.04 6.81 0.19	10.91 8.04 0.16	8.54 4.74 0.09	9.43 5.62 0.12	8.09 9.18 0.13
E	6.51 10.91 0.09	22.41 2.96 0.14	11.85 11.30 0.03	8.64 5.03 0.03	14.86 8.29 0.13	8.88 15.40 0.11
F	15.95 17.18 0.06	22.51 6.51 0.30	19.85 18.37 0.06	18.96 17.18 0.20	20.74 9.48 0.08	8.29 10.91 0.11

Step 5: Using step 6 of Section 4.2, highest cost, highest time and highest quality from Table 5 respectively are 22.51, 18.37 and 0.30. For w_1= 0.5, w_2 = 0.4 and w_3 = 0.1, the values of a, b, and c are: a = 0.02, b = 0.01 and c = 0.333.

Step 6: Using step 7 of Section 4.2, the fuzzy Multi-objective Assignment Problem is converted into the following crisp linear programming as:

$$\text{Minimize } z = (0.35)x_{11} + (0.31)x_{12} + (0.26)x_{13} + (0.40)x_{14} + (0.39)x_{15} + (0.33)x_{16} +$$
$$(0.27)x_{21} + (0.42)x_{22} + (0.27)x_{23} + (0.34)x_{24} + (0.34)x_{25} + (0.39)x_{26} +$$
$$(0.30)x_{31} + (0.34)x_{32} + (0.34)x_{33} + (0.32)x_{34} + (0.35)x_{35} + (0.25)x_{36} +$$
$$(0.34)x_{41} + (0.35)x_{42} + (0.43)x_{43} + (0.29)x_{44} + (0.34)x_{45} + (0.38)x_{46} +$$
$$(0.37)x_{51} + (0.55)x_{52} + (0.47)x_{53} + (0.28)x_{54} + (0.50)x_{55} + (0.52)x_{56} +$$

$$(0.68)x_{61} + (0.67)x_{62} + (0.78)x_{63} + (0.78)x_{64} + (0.63)x_{65} + (0.42)x_{66}$$

subject to

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 1 \ , \quad x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 1,$$
$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 1 \ , \quad x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 1,$$
$$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 1 \ , \quad x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} = 1,$$
$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 1 \ , \quad x_{11} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 1,$$
$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 1 \ , \quad x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{65} = 1,$$
$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 1 \ , \quad x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 1.$$
$$x_{ij} = 0 \text{ or } 1, \text{ for all } i=1,2,3,4,5,6 \text{ and } j=1,2,3,4,5,6.$$

Step 7: The optimal solution is $x_{13} = 1$, $x_{21} = 1$, $x_{32} = 1$, $x_{45} = 1$, $x_{54} = 1$, $x_{66} = 1$.

Step 8: Calculations for minimum total fuzzy assignment cost, the fuzzy assignment time and the fuzzy assignment quality are

Persons	Works	Fuzzy Cost	Fuzzy Time	Fuzzy Quality
A	III	(8,10,3,2)	(4,5,2,2)	(0.04,0.06,0.03,0.03)
B	I	(3,5,1,2)	(7,10,1,2)	(0.12,0.15,0.03,0.03)
C	II	(7,12,2,2)	(5,7,1,2)	(0.15,0.17,0.02,0.02)
D	V	(10,11,3,2)	(5,7,1,1)	(0.13,0.14,0.01,0.01)
E	IV	(9,10,1,2)	(5,6,1,1)	(0.03,0.05,0.02,0.02)
F	VI	(8,10,2,2)	(11,13,2,1)	(0.10,0.15,0.02,0.03)
Total fuzzy Assignment cost = (45,58,12,12) Total fuzzy Assignment time = (37,48,8,9) Total fuzzy Assignment quality = (0.57,0.72,0.13,0.14)				

Similarly for other cases of Section 3, the optimal total fuzzy Assignment cost, time and quality for linear and nonlinear reference functions are given in Table 6.

Table 6 : Total Optimal Fuzzy Assignment Cost, Time and Quality

Linear and nonlinear reference functions	Optimal Assignment	Total Optimal Fuzzy Assignment Cost, Time and Quality
Case(i)	A-III, B-I, C-II, D-V, E-IV, F-VI.	(45, 58, 12, 12) (37, 48, 8, 9) (0.57, 0.72, 0.13, 0.14)
Case (ii)	A-III, B-I, C-II, D-V, E-IV, F-VI.	(45, 58, 12, 12) (37, 48, 8, 9) (0.57, 0.72, 0.13, 0.14)
Case (iii)	A-III, B-I, C-II, D-V, E-IV, F-VI.	(45, 58, 12, 12) (37, 48, 8, 9) (0.57, 0.72, 0.13, 0.14)
Case (iv)	A-III, B-I, C-II, D-V, E-IV, F-VI.	(45, 58, 12, 12) (37, 48, 8, 9) (0.57, 0.72, 0.13, 0.14)
Case (v)	A-III, B-I, C-II, D-V, E-IV, F-VI.	(45, 58, 12, 12) (37, 48, 8, 9) (0.57, 0.72, 0.13, 0.14)
Case (vi)	A-III, B-I, C-II, D-V, E-IV, F-VI.	(45, 58, 12, 12) (37, 48, 8, 9) (0.57, 0.72, 0.13, 0.14)

6. APPLICATION TO CIVIL CONSTRUCTION PROCESS

In this Section, the proposed Multi-objective Assignment Model with two parameters fuzzy cost, and fuzzy time is tested with a practical data which is collected from GITAM University (GU), Visakhapatnam (VSP), Andhra Pradesh, India.

In GU various civil construction process is going on, for the civil construction purpose the Organization invites various contractors for bidding. The various contractors like SBEC Projects Pvt. Ltd., M/s Venkateswara Infra, M/s Navya constructions and M/s Hemanth sai constructions have bid for doing the main civil works such as Earth work, Concrete work, Brick masonry, finishing, plumbing and sanitary works for the building with seven floors named Vinay Sadhan Bavan (Boys Hostel) in GU, VSP, AP, India. In GU, the higher authority providing the whole contract to the contractor who is quoting with minimum cost, minimum time and with good quality. The contractors are quoting the cost, time and quality in the exact quantity data, but in the real life they may arise certain uncertainty cases during the civil construction process such as production and availability of materials, project-wide safety measures, utility construction disagreement and postponement, design management assessment and omission, maintenance of traffic, incident management, work zone safety, asserts, environmental obligations, project schedule, errors and blunders, irregularity of workers, contractors may drop suddenly from the work, due to lack of finance, and so on. And the general uncertainty cases arising in the GU civil construction process are taken into consideration and to meet this uncertainty cases the crisp data collected from the GU Superintending Engineers Office is converted into fuzzy data by interacting with experts. The proposed fuzzy Multi-objective Assignment Model with fuzzy cost $\left(\tilde{c}_{ij}\right)$ and fuzzy time $\left(\tilde{t}_{ij}\right)$ are considered as generalized LR fuzzy numbers and is applied to test the validity of the model. The main advantage of the proposed model is to assign one work to one contractor, i.e., the assignment is done on one-one basis. By assigning one work to one contractor the total cost as well as the total time can be reduced and the organization can benefited instead of giving the whole contract to a single contractor.

6.1 Data collection in civil construction process

In this Section, the crisp data collected from GU, Superintending Engineer's Office is given and it is converted into fuzzy environment by interacting with the experts. The cost in units and time in months for doing the work by the contractor in crisp and fuzzy environment is given in Table 7 and 7a respectively. Here the time in crisp and fuzzy is indicated by “ ”.

The civil construction works

1. Earth Work in excavations in foundations of structure, pipe trenches, upto required depth in Cum.
2. Concrete Work
3. Brick Masonry
4. Finishing work includes
    (i) Plasting
    (ii) Flooring Vitrified
    (iii) Painting

5. Plumbing and Sanitary works include

(i) Water pipe line (ii) Drainage out line
(iii) Washbasin (iv)Commodes
(v) Taps and other fittings

Various contractors for doing the above work

1. SBEC Projects Pvt. Ltd., **2.** M/s Venkateswara Infra,
3. M/s Navya constructions and **4.** M/s Hemanth sai constructions

Table 7: Crisp Assignment cost and time for construction process

	Earth Work	Concrete Work	Brick Masonry	Finishing	Plumbing & Sanitary
SBEC Projects Pvt Ltd	84 0.8	11,060 9	4,100 9	1,372 3	14,430 2
M/s Venkateswara Infra	86 0.86	11,750 10	4,050 8	1,387 3	14,640 2
M/s Navya Constructions	85 0.8	10,855 9	4,025 9	1,386 2	14,885 3
M/s Hemanthi sai constructions	100 0.76	10,925 9	4,000 10	1,400 4	14,770 2

Table 7a: Fuzzy Assignment cost and time for construction process

	Earth Work	Concrete Work	Brick Masonry	Finishing	Plumbing & Sanitary
SBEC Projects Pvt Ltd	(0.08,0.09,0.01,0.01) (0.7,0.8,0.1,0.2)	(11,11.05,0.2,0.2) (8,9,1,1)	(4,4.2,0.1,0.2) (8,9,1,1)	(1.25,1.38,0.01,0.02) (2,3,1,1)	(14,15.50,0.2,0.1) (1,2,0.5,0.5)
M/s Venkateswara Infra	(0.07,0.09,0.01,0.01) (0.8,0.9,0.1,0.2)	(11.25,11.90,0.2,0.1) (9,10,1,1)	(3.9,4.15,0.1,0.2) (7,8,0.5,0.5)	(1.35,1.4,0.01,0.02) (2,3,1,1)	(13.9,14.65,0.1,0.1) (1,2,0.5,0.5)
M/s Navya Constructions	(0.06,0.09,0.02,0.01) (0.7,0.8,0.1,0.2)	(10.50,10.90,0.1,0.2) (8,9,1,1)	(3.5,4,0.1,0.1) (8,9,1,1)	(1.34,1.39,0.01,0.02) (1,2,0.5,0.5)	(14,14.9,0.1,0.1) (2,3,1,1)
M/s Hemanthi sai constructions	(0.09,0.12,0.01,0.01) (0.6,0.7,0.1,0.2)	(10.60,11,0.1,0.2) (8,9,1,1)	(3.9,4.2,0.1,0.2) (9,10,1,1)	(1.39,1.42,0.01,0.02) (3,4,0.5,0.5)	(14,14.8,0.1,0.1) (1,2,0.5,0.5)

6.2 Calculation of the Data in Fuzzy environment

In this Section, the proposed Assignment model mentioned in Section 4.2, is applied to calculate the minimum total fuzzy cost and minimum fuzzy time by using ranking procedure as mentioned in Section 3, for linear and nonlinear reference functions.

Case (i) $\mathbf{L(x) = R(x) = \max\{0, 1-|x|\}}$

Step 1: The fuzzy Multi-objective Assignment Problem in Table 7a is an unbalanced one i.e., 4×5 matrix.

	Earth Work	**Concrete Work**	**Brick Masonry**	**Finishing**	**Plumbing & Sanitary**
SBEC Projects Pvt Ltd	(0.08,0.09,0.01,0.01) (0.7,0.8,0.1,0.2)	(11,11.05,0.2,0.2) (8,9,1,1)	(4,4.2,0.1,0.2) (8,9,1,1)	(1.25,1.38,0.01,0.02) (2,3,1,1)	(14,15.50,0.2,0.1) (1,2,0.5,0.5)
M/s Venkateswara Infra	(0.07,0.09,0.01,0.01) (0.8,0.9,0.1,0.2)	(11.25,11.90,0.2,0.1) (9,10,1,1)	(3.9,4.15,0.1,0.2) (7,8,0.5,0.5)	(1.35,1.4,0.01,0.02) (2,3,1,1)	(13.9,14.65,0.1,0.1) (1,2,0.5,0.5)
M/s Navya Constructions	(0.06,0.09,0.02,0.01) (0.7,0.8,0.1,0.2)	(10.50,10.90,0.1,0.2) (8,9,1,1)	(3.5,4,0.1,0.1) (8,9,1,1)	(1.34,1.39,0.01,0.02) (1,2,0.5,0.5)	(14,14.9,0.1,0.1) (2,3,1,1)
M/s Hemanthi sai constructions	(0.09,0.12,0.01,0.01) (0.6,0.7,0.1,0.2)	(10.60,11,0.1,0.2) (8,9,1,1)	(3.9,4.2,0.1,0.2) (9,10,1,1)	(1.39,1.42,0.01,0.02) (3,4,0.5,0.5)	(14,14.8,0.1,0.1) (1,2,0.5,0.5)
Dummy	(0,0,0,0) (0,0,0,0)	(0,0,0,0) (0,0,0,0)	(0,0,0,0) (0,0,0,0)	(0,0,0,0) (0,0,0,0)	(0,0,0,0) (0,0,0,0)

Step 2: Dummy row with zero fuzzy cost and fuzzy time is added to make it a balanced one i.e., 5×5 matrix.

Step 3: Using step 3 of the proposed model mentioned in Section 4.2 it is converted into a single fuzzy objective problem as follows:

$$\text{Minimize } \tilde{z} = 0.8\sum_{i=1}^{5}\sum_{j=1}^{5}(\tilde{c}_{ij})x_{ij} + 0.2\sum_{i=1}^{5}\sum_{j=1}^{5}(\tilde{t}_{ij})x_{ij}$$

subject to

$$\sum_{i=1}^{5} x_{ij} = 1, \quad j = 1,2,3,4,5$$

$$\sum_{j=1}^{5} x_{ij} = 1, \quad i = 1,2,3,4,5$$

$$x_{ij} = \begin{cases} 1 \text{ if } i^{th} \text{ person is assigned } j^{th} \text{ work} \\ 0 \quad \text{otherwise} \end{cases}$$

$$x_{ij} = 0 \text{ or } 1, \text{ for all } i=1,2,3,4,5 \text{ and } j=1,2,3,4,5.$$

Here, $w_1 = 0.8$, $w_2 = 0.2$.

Step 4: The fuzzy Multi-objective Assignment Problem is converted into the following crisp linear programming problem as:

$$\text{Minimize } z = 0.8\sum_{i=1}^{5}\sum_{j=1}^{5} R(\tilde{c}_{ij})x_{ij} + 0.2\sum_{i=1}^{5}\sum_{j=1}^{5} R(\tilde{t}_{ij})x_{ij}$$

subject to

$$\sum_{i=1}^{5} x_{ij} = 1, \quad j = 1,2,...,5$$

$$\sum_{j=1}^{5} x_{ij} = 1, \quad i = 1,2,...,5$$

Step 5: Using Section 3, the values $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$ $\forall i, j$ of Table 7a are calculated and given in Table 8.

Table 8: Ranks of Fuzzy Cost and Fuzzy Time for case (i)

	Earth Work	Concrete Work	Brick Masonry	Finishing	Plumbing & Sanitary
SBEC Projects Pvt Ltd	0.07 0.68	9.80 7.70	3.67 7.70	1.18 2.37	12.73 1.48
M/s Venkateswara Infra	0.07 0.77	10.38 8.59	3.61 6.81	1.23 2.37	12.8 1.48
M/s Navya Constructions	0.07 0.68	9.57 7.70	3.40 7.70	1.22 1.48	12.97 2.37
M/s Hemanthi sai constructions	0.09 0.59	9.66 7.70	3.64 8.59	1.25 3.25	12.91 1.48
Dummy	0	0	0	0	0

Step 6: Using step 6 of the proposed model mentioned in Section 4.2 the highest cost and highest time from Table 8 respectively are 12.97 and 8.59. For $w_1 = 0.8$ and $w_2 = 0.2$, the values of a and b are: a = 0.05 and b = 0.02.

Step 7: Using step 7 of the proposed model mentioned in Section 4.2 convert the fuzzy Multi-objective Assignment Problem into the following crisp linear programming as:

$$\text{Minimize } z = (0.01)x_{11} + (0.64)x_{12} + (0.33)x_{13} + (0.10)x_{14} + (0.66)x_{15} + (0.01)x_{21} + (0.69)x_{22} + (0.31)x_{23} + (0.10)x_{24} + (0.66)x_{25} + (0.01)x_{31} + (0.63)x_{32} + (0.32)x_{33} + (0.09)x_{34} + (0.69)x_{35} + (0.01)x_{41} + (0.63)x_{42} + (0.35)x_{43} + (0.12)x_{44} + (0.67)x_{45} + (0)x_{51} + (0)x_{52} + (0)x_{53} + (0)x_{54} + (0)x_{55}$$

subject to

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 1 , \quad x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 1,$$
$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 1 , \quad x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 1,$$
$$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} = 1 , \quad x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 1,$$
$$x_{11} + x_{22} + x_{32} + x_{42} + x_{52} = 1 , \quad x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 1,$$
$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 1 , \quad x_{15} + x_{25} + x_{35} + x_{45} + x_{55} = 1.$$
$$x_{ij} = 0 \text{ or } 1, \text{ for all } i=1,2,3,4,5 \text{ and } j=1,2,3,4,5.$$

Step 8: The optimal solution is $x_{11} = 1$, $x_{23} = 1$, $x_{34} = 1$, x_{42} 1, $x_{55} = 1$.

Step 9: Calculations for minimum total fuzzy assignment cost and the fuzzy assignment time

Contractors	Works	Fuzzy Cost	Fuzzy Time
SBEC Projects Pvt Ltd	Earth Work	(0.08,0.09,0.01,0.01)	(0.7,0.8,0.1,0.2)
M/s Venkateswara Infra	Brick Masonry	(3.9,4.15,0.1,0.2)	(7,8,0.5,0.5)
M/s Navya Constructions	Finishing	(1.34,1.39,0.01,0.02)	(1,2,0.5,0.5)
M/s Hemanthi sai constructions	Concrete Work	(10.60,11,0.1,0.2)	(8,9,1,1)
Total Fuzzy Assignment Cost = (15.92, 16.63, 0.22, 0.43) **Total Fuzzy Assignment Time = (16.7, 19.8, 2.1, 2.2)**			

Similarly for other cases of Section 3, optimal fuzzy Assignment cost and time are calculated and given in Table 9.

Table 9: Optimal fuzzy Assignment for various cases

	Optimal Assignment	**Total Fuzzy Assignment Cost and Total Fuzzy Assignment Time**
Case (ii)	SBEC Projects Pvt Ltd - Earth Work M/s Venkateswara Infra - Brick Masonry M/s Navya Constructions - Finishing M/s Hemanthi sai constructions - Concrete Work	(15.92, 16.63, 0.22, 0.43) (16.7, 19.8, 2.1, 2.2)
Case (iii)	SBEC Projects Pvt Ltd - Earth Work M/s Venkateswara Infra - Brick Masonry M/s Navya Constructions - Finishing M/s Hemanthi sai constructions - Concrete Work	(15.92, 16.63, 0.22, 0.43) (16.7, 19.8, 2.1, 2.2)
Case (iv)	SBEC Projects Pvt Ltd - Earth Work M/s Venkateswara Infra - Brick Masonry M/s Navya Constructions - Finishing M/s Hemanthi sai constructions - Concrete Work	(15.92, 16.63, 0.22, 0.43) (16.7, 19.8, 2.1, 2.2)
Case (v)	SBEC Projects Pvt Ltd - Earth Work M/s Venkateswara Infra - Brick Masonry M/s Navya Constructions - Finishing M/s Hemanthi sai constructions - Concrete Work	(15.92, 16.63, 0.22, 0.43) (16.7, 19.8, 2.1, 2.2)
Case (vi)	SBEC Projects Pvt Ltd - Earth Work M/s Venkateswara Infra - Brick Masonry M/s Navya Constructions - Finishing M/s Hemanthi sai constructions - Concrete Work	(15.92, 16.63, 0.22, 0.43) (16.7, 19.8, 2.1, 2.2)

CONCLUSION

For the validity of the proposed fuzzy Multi-objective Assignment model for linear and nonlinear reference functions, it is tested with the numerical example as presented in section 5 and it is seen that it has the same fuzzy assignment cost and time for all the cases. It is also tested with a practical data in the case of the construction process in the real life situation as given in section 6 and the assignment is done on one-one basis and it has the same fuzzy assignment cost and time in all the cases. This method helps in assigning and implementing the persons to works successfully on one-one basis and to minimize the total fuzzy assignment cost and total fuzzy time of the organization and it is also helpful in assessing the future values even in the uncertainty cases. The total assignment cost and total time are completely expressed by a generalized LR fuzzy number rather than by a crisp value. This method is very simple and fruitful and it is useful for future study of the real world problems.

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