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On Fuzzy Simply Lindelöf Spaces

G. Thangaraj[1] and S. Dharmasaraswathi[2]

[1]*Department of Mathematics, Thiruvalluvar University, Serkkadu, Vellore, Tamilnadu, India - 632 115.*
[2]*Research Scholar, Department of Mathematics, Thiruvalluvar University, Serkkadu, Vellore, Tamilnadu, India - 632 115.*

Abstract

In this paper, the concept of fuzzy simply Lindelof spaces is introduced. Several characterizations of fuzzy simply Lindelof spaces are given.

Keywords: Fuzzy dense set, fuzzy nowhere dense set, fuzzy simply open set, fuzzy second category space, fuzzy submaximal space, fuzzy strongly irresolvable space.
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1. INTRODUCTION

Many generalizations of Lindelof spaces have been introduced and studied by several authors. Among the various covering properties of topological spaces, a lot of attention has been made to those covers which involve open and regular open sets in classical topology. In 1959 **Z.Frolik** [7] introduced and studied the notion of weakly Lindelof spaces. In 1982 **G.Balasubramanian** [2] introduced and studied the notion of nearly Lindeof spaces. In 1984 **S.Willard** and **U.N.B. Dissanayake** [15] gave the notion of almost Lindelof spaces and in 1996 **F. Cammaroto** and **G. Santoro** [5] introduced the notion of weakly regular Lindelof spaces on using regular covers.
In 1965, L.A.**Zadeh** [16] introduced the concept of fuzzy sets as a new approach for modelling uncertainties. In 1968, **C.L.Chang** [6] introduced the concept of fuzzy topological spaces. The paper of Chang proved the way for the subsequent tremendous growth of the numerous fuzzy topological spaces. **A.S.Bin Shahna** [4] introduced the notion of fuzzy Lindelöf spaces and investigated some of their properties. **G. Balasubramanian** and G.Thangaraj [9] introduced and studied fuzzy nearly Lindelof spaces, fuzzy almost Lindelof spaces fuzzy weakly Lindelof spaces. New classes of fuzzy open sets called fuzzy simply open sets are introduced and studied by **G.Thangaraj** and **K.Dinakaran** [13]. In this paper by means of fuzzy simply open sets, the notion of fuzzy simply Lindelof spaces, is introduced and studied. Several examples are given to illustrate the concepts introduced in this paper.

2. PRELIMINARIES

In order to make the exposition self- contained, some basic notions and results used in the sequel are given. In this work (X,T) or simply by X, we will denote a fuzzy topological space due to **Chang** (1968). Let X be a non- empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a function from X into I. The null set 0 is the function from X into I which assumes only the value 0 and the whole fuzzy set 1 is the function from X into I takes 1 only.

Definition 2.1[6]: Let (X,T) be a fuzzy topological space and λ be any fuzzy in (X,T). The interior and the closure of λ are defined as follows

(i) $\text{Int}(\lambda) = \vee \{ \mu / \mu \leq \lambda, \mu \in T\}$
(ii) $\text{Cl}(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1-\mu \in T\}$

Lemma 2.1[1]: For a fuzzy set λ of a fuzzy topological space X,

(i) $1 - \text{int}(\lambda) = \text{cl}(1 - \lambda)$
(ii) $1 - \text{cl}(\lambda) = \text{int}(1 - \lambda)$

Definition 2.2[8]: A fuzzy set λ in a fuzzy topological space (X,T), is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $\text{cl}(\lambda) = 1$, in (X,T).

Definition 2.3[8]: A fuzzy set λ in a fuzzy topological space (X,T), is called fuzzy nowhere dense if there exists no non – zero fuzzy open set μ in (X,T) such that $\mu < \text{cl}(\lambda)$. That is, $\text{intcl}(\lambda) = 0$, in (X,T).

Definition 2.4[14]: The fuzzy boundary of a fuzzy set λ in a fuzzy topological space (X,T) is defined as $\text{Bd}(\lambda) = \text{cl}(\lambda) \wedge \text{cl}(1-\lambda)$.

Definition 2.5[13]: A fuzzy set λ in a fuzzy topological space (X,T), is called a fuzzy simply open set if Bd(λ) is a fuzzy nowhere dense set in (X,T). That is, λ is a fuzzy simply open set in (X,T) if $\text{cl}(\lambda) \wedge \text{cl}(1-\lambda)$, is a fuzzy nowhere dense set in (X,T).

Definition 2.6[9]: A fuzzy topological space (X,T) is said to be fuzzy Lindelöf if every fuzzy open cover of X has a countable subcover. That is, for every fuzzy open cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X, there exists $\{\lambda_{\alpha_n}\}_{n \in N}$ of fuzzy open sets in (X,T) such that $\vee_{n \in N} \{\lambda_{\alpha_n}\} = 1$.

Theorem 2.1[13]: If λ is a fuzzy open and fuzzy dense set in a fuzzy topological space (X,T), then λ is a fuzzy simply open set in (X,T).

Definition 2.7[8]: Let (X,T) be a fuzzy topological space. A fuzzy set λ in a (X,T) is called a fuzzy first category set if $\lambda = \vee_{\alpha=1}^{\infty}(\lambda_\alpha)$,where (λ_α)'s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category.

Definition 2.8[8]: A fuzzy topological space (X,T) is called fuzzy first category space if $1_X = \vee_{\alpha=1}^{\infty}(\lambda_\alpha)$, where (λ_α)'s are fuzzy nowhere dense sets in (X,T). A fuzzy topological space which is not of fuzzy first category is said to be of fuzzy second category.

Definition 2.9[3]: A fuzzy topological space (X,T) is called a fuzzy submaximal space if for each fuzzy set λ in (X,T) such that cl(λ) = 1,then λ ϵ T

Theorem 2.2[13]: If λ is a fuzzy closed set with int(λ) = 0, in a fuzzy topological space (X,T), then λ is a fuzzy simply open set in (X,T).
Theorem 2.3[13]: If λ is a fuzzy simply open set in a fuzzy topological (X,T), then λ ∧ (1 – λ) is a fuzzy nowhere dense set in (X,T).

Theorem 2.4[13]: If λ is a fuzzy nowhere dense set in a fuzzy topological space (X,T), then λ is a fuzzy simply open set in (X,T).

Theorem 2.5[13]: If λ = μ∨δ where μ is a fuzzy open and fuzzy dense set and δ is a fuzzy nowhere dense set in a fuzzy topological space (X,T), then λ is a fuzzy simply open set in (X,T).

Theorem 2.6[13]: If int(λ) = 0 for a fuzzy set λ in a fuzzy strongly irresolvable space (X,T), then λ is a fuzzy simply open set in (X,T).

Theorem 2.7[13]: If λ is a fuzzy simply open set in a fuzzy topological space (X,T), then cl(λ) is a fuzzy simply open set in (X,T).

Lemma 2.2[1]: For a family A = $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X, ∨ cl(λ_α) ≤ cl(∨λ_α). In case A is a finite set, ∨cl(λ_α) = cl (∨λ_α). Also ∨ int (λ_α) ≤ int(∨λ_α).

3. FUZZY SIMPLY LINDELÖF SPACES

Definition 3.1: A fuzzy topological space (X,T) is said to be fuzzy simply Lindelöf if each cover of X by fuzzy simply open sets has a countable subcover. That is, (X,T) is a fuzzy simply Lindelof space if $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$ where intcl [bd(λ_α)] = 0 in (X,T), then $\vee_{n \in N} \{\lambda_{\alpha_n}\} = 1$ in (X,T).

Example 3.1: Let X = {a,b,c}. The fuzzy sets λ, μ, δ, α, β and γ are defined on X as follows:
λ : X → [0,1] is defined as λ(a) = 0.2; λ(b) = 0.3 ; λ(c) = 1:
μ : X → [0,1] is defined as μ(a) = 0.5; μ(b) = 1; μ(c) = 0.7;
δ : X → [0,1] is defined as δ(a) = 1; δ(b) = 0.5 ; δ(c) = 0.6 :
α : X → [0,1] is defined as α(a) = 0.2; α(b) = 0.4; α(c) = 1
β : X → [0,1] is defined as β(a) = 0.5; β(b) = 1; β(c) = 0.6:
γ : X → [0,1] is defined as γ(a) = 1; γ(b) = 0.4; γ(c) = 0.4;

Then T = { 0, λ, μ, δ, $\lambda\vee\mu$, $\lambda\vee\delta$, $\mu\vee\delta$, $\lambda\wedge\mu$, $\lambda\wedge\delta$, $\mu\wedge\delta$, $\lambda\vee(\mu\wedge\delta)$, $\delta\vee(\lambda\wedge\mu)$,$\mu\wedge(\lambda\vee\delta)$,1} is a fuzzy topology on X. On computation,we see that the fuzzy sets λ, μ, δ, $\lambda\vee\mu$, $\lambda\vee\delta$, $\mu\vee\delta$, $\lambda\wedge\mu$, $\lambda\wedge\delta$, $\mu\wedge\delta$, $\lambda\vee(\mu\wedge\delta)$, $\delta\vee(\lambda\wedge\mu)$,$\mu\wedge(\lambda\vee\delta)$ are fuzzy open and fuzzy dense sets in (X,T). Then by theorem 2.1, λ, μ, δ, $\lambda\vee\mu$, $\lambda\vee\delta$, $\mu\vee\delta$, $\lambda\wedge\mu$, $\lambda\wedge\delta$, $\mu\wedge\delta$, $\lambda\vee(\mu\wedge\delta)$, $\delta\vee(\lambda\wedge\mu)$,$\mu\wedge(\lambda\vee\delta)$ are fuzzy simply open sets in (X,T).
Also cl(α)=cl(β)= cl (γ) = cl(1− γ) = 1, cl(1− α) = 1− λ, cl(1−β) = 1− ($\mu\wedge\delta$), int(γ) = int(1−α) = int(1−β) = int(1−γ)=0 int(α) = λ and int(β) = $\mu\wedge\delta$. Now intcl [cl(α)$\wedge$cl(1−α)] = 0, intcl [cl(β)$\wedge$cl(1−β)] = 0, intcl [cl(γ)$\wedge$cl(1−γ)] = 1 $\neq$ 0 and hence α,β are fuzzy simply open sets in (X,T).Now for the cover { λ, $\mu\vee\delta$, $\lambda\vee(\mu\wedge\delta)$, α} of X by fuzzy simply open sets, ($\mu\vee\delta$) $\vee$ ($\lambda\vee(\mu\wedge\delta)$ = 1 and for the cover { λ, β, δ, $\lambda\vee\mu$, $\lambda\vee\delta$} of X by fuzzy simply open sets, $\lambda\vee$ β $\vee\delta$ = 1 implies that (X,T) is a fuzzy simply Lindelof space.

Proposition 3.1: If (X,T) is a fuzzy simply Lindelof space and if $\bigvee_{\alpha\in\Delta}$ {λ_α} = 1 where {λ_α}'s are fuzzy closed sets with int(λ_α) = 0 in (X,T), then $\bigvee_{n\in N}${λ_{α_n}}= 1, in (X,T).

Proof: Let (X,T) be a fuzzy simply Lindelof space. By hypothesis, the fuzzy sets {λ_α}'s are fuzzy closed sets with int(λ_α) = 0 in (X,T).Then by theorem2.2, {λ_α}'s are fuzzy simply open sets in (X,T). Now $\bigvee_{\alpha\in\Delta}$ {λ_α} = 1 where {λ_α}'s are fuzzy simply open sets in (X,T), implies that $\{\lambda_\alpha\}_{\alpha\in\Delta}$ is a fuzzy simply open cover of X. Since (X,T) is a fuzzy simply Lindelof space, there exist a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ of fuzzy simply open sets, for X. Hence $\bigvee_{n\in N}$ {λ_{α_n}}= 1, where (1− λ_{α_n}) $\in$ T and int {λ_{α_n}} = 0, in (X,T).

Proposition 3.2: If (X,T) is a fuzzy simply Lindelof space and if $\bigvee_{\alpha\in\Delta}$ {λ_α} = 1 where $\lambda_\alpha \in$ T and cl(λ_α) = 1 in (X,T), then $\bigwedge_{n\in N}$ {μ_{α_n}}= 0, where (μ_{α_n})'s are fuzzy nowhere dense sets in (X,T).

Proof: Let (X,T) be a fuzzy simply Lindelof space. By hypothesis, the fuzzy sets {λ_α}'s are fuzzy open and fuzzy dense sets in (X,T).Then by theorem 2.1, {λ_α}'s are fuzzy simply open sets in (X,T). Now $\bigvee_{\alpha\in\Delta}$ {λ_α} = 1 where {λ_α}'s are fuzzy simply open sets in (X,T), implies that $\{\lambda_\alpha\}_{\alpha\in\Delta}$ is a fuzzy simply open cover of X. Since (X,T) is a fuzzy simply Lindelof space, there exist a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ of fuzzy simply open sets for X and then $\bigvee_{n\in N}$ {λ_{α_n}}= 1 in (X,T) .Then 1− $\bigvee_{n\in N}$ {λ_{α_n}} = 0 and hence $\bigwedge_{n\in N}$ (1−λ_{α_n}) = 0. Let μ_{α_n} = 1− λ_{α_n}. Now intcl (1− λ_{α_n}) = 1− cl int (λ_{α_n}) = 1− cl(λ_{α_n}) = 1− 1 = 0, and thus (1−λ_{α_n})'s are fuzzy nowhere dense set in (X,T).Hence $\bigwedge_{n\in N}${μ_{α_n}} = 0, where (μ_{α_n})'s are fuzzy nowhere dense sets in (X,T).

Proposition 3.3: If (X,T) is a fuzzy simply Lindelof space, then (X,T) is a fuzzy second category space.

Proof: Let (X,T) be a fuzzy simply Lindelof space and $\{\lambda_\alpha\}_{\alpha \in \Delta}$ be a cover of X by fuzzy simply open sets in (X,T). By theorem 2.3, $\{\lambda_{\alpha_n} \wedge ((1-\lambda_{\alpha_n})\}$'s are fuzzy nowhere dense sets in (X,T).Since (X,T) is a fuzzy simply Lindelof space, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ for X .Then $\bigvee_{n \in N} \{\lambda_{\alpha_n}\} = 1$ in (X,T). Now $[\lambda_{\alpha_n} \wedge (1-\lambda_{\alpha_n})] \leq \lambda_{\alpha_n}$ in (X,T) implies that $\bigvee_{n \in N} [\lambda_{\alpha_n} \wedge (1-\lambda_{\alpha_n})] \leq \bigvee_{n \in N} \{\lambda_{\alpha_n}\}$ and then $\bigvee_{n \in N} [\lambda_{\alpha_n} \wedge (1-\lambda_{\alpha_n})] \leq 1$. Then $\bigvee_{n \in N} [\lambda_{\alpha_n} \wedge ((1-\lambda_{\alpha_n})] \neq 1$,where $\{ \lambda_{\alpha_n} \wedge (1-\lambda_{\alpha_n})\}$'s are fuzzy nowhere dense sets, implies that (X,T) is not a fuzzy first category space and hence (X,T) is a fuzzy second category space.

Proposition 3.4: If $\{\lambda_\alpha\}_{\alpha \in \Delta}$ is a cover of X by fuzzy nowhere dense sets in a fuzzy simply Lindelof space (X,T),then there is a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ for X.

Proof: If $\{\lambda_\alpha\}_{\alpha \in \Delta}$ is a cover of X by fuzzy nowhere dense sets in (X,T), then $\bigvee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$,where $intcl(\lambda_\alpha) = 0$ in (X,T). By theorem 2.4,the fuzzy nowhere dense sets $\{\lambda_\alpha\}$'s are fuzzy simply open sets in (X,T) and thus $\{\lambda_\alpha\}_{\alpha \in \Delta}$ is a cove of X by fuzzy simply open sets. Since (X,T) is a fuzzy simply Lindelof space, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ of X. That is, $\bigvee_{n \in N} \{\lambda_{\alpha_n}\} = 1$ where $intcl(\lambda_{\alpha_n}) = 0$ in (X,T).

Proposition 3.5: If $\bigvee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$ where $\lambda_\alpha = \mu_\alpha \vee \delta_\alpha$ and $\{\mu_\alpha\}$'s are fuzzy open and fuzzy dense sets and $\{\delta_\alpha\}$'s are fuzzy nowhere dense sets in a fuzzy simply Lindelof space (X,T), then $\eta \vee \delta = 1$,where $\eta \in T$ and $cl(\eta) = 1$ and δ is a fuzzy first category set in (X,T).

Proof: Let (X,T) be a fuzzy simply Lindelof space such that $\bigvee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$ and $\lambda_\alpha = \mu_\alpha \vee \delta_\alpha$, where $\mu_\alpha \in T$ and $cl(\mu_\alpha) = 1$ and $intcl(\delta_\alpha) = 0$ in (X,T) By theorem 2.5, $\{\lambda_\alpha\}$'s are fuzzy simply open sets in (X,T). Since (X,T) is a fuzzy simply Lindelof space, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ of X. That is, $\bigvee_{n \in N} \{\lambda_{\alpha_n}\} = 1$.Then, $\bigvee_{n \in N} [\mu_{\alpha_n} \vee \delta_{\alpha_n}] = [\bigvee_{n \in N} (\mu_{\alpha_n})] \vee [\bigvee_{n \in N} (\delta_{\alpha_n})]$ implies that $[\bigvee_{n \in N} (\mu_{\alpha_n})] \vee [\bigvee_{n \in N} (\delta_{\alpha_n})] = 1$. Since $\{\delta_\alpha\}$'s are fuzzy nowhere dense sets in (X,T), $\bigvee_{n \in N} \{\delta_{\alpha_n}\} = \delta$, implies that δ is a fuzzy first category set,in (X,T). Since $(\mu_{\alpha_n}) \in T$, $\bigvee_{n \in N} (\mu_{\alpha_n}) \in T$. Also $cl[\bigvee_{n \in N} (\mu_{\alpha_n})] \geq \bigvee_{n \in N} cl[(\mu_{\alpha_n})]$ implies that $cl [\bigvee_{n \in N} (\mu_{\alpha_n})] \geq \bigvee 1$ and hence $cl[\bigvee_{n \in N} (\mu_{\alpha_n})] = 1$ Let $\eta = \bigvee_{n \in N} (\mu_{\alpha_n})$.Then η is a fuzzy open and fuzzy dense set in (X,T).Thus $\delta \vee \eta = 1$ in (X,T).

Definition 3.2[13]: A fuzzy topological space (X,T) is a fuzzy strongly irresolvable space if, for every fuzzy dense set λ in (X,T), $clint(\lambda) = 1$ in (X,T). That is, $cl(\lambda) = 1$ implies that $clint(\lambda) = 1$, in (X,T).

Proposition 3.6: If $\{\lambda_\alpha\}_{\alpha \in \Delta}$ is a cover of X by fuzzy sets with int(λ_α) = 0, in a fuzzy strongly irresolvable and fuzzy simply Lindelof space (X,T), then $\vee_{n \in N} \{\lambda_{\alpha_n}\} = 1$.

Proof: Suppose that $\vee_{\alpha \in \Delta} \{\lambda_\alpha\}$= 1,where int($\lambda_\alpha$) = 0 in (X,T).Since (X,T) is a fuzzy strongly irresolvable space,by theorem2.6,{ λ_{α_n}}'s are fuzzy simply open sets in (X,T).Since (X,T) is a fuzzy simply Lindelof space, $\vee_{\alpha \in \Delta} \{\lambda_\alpha\}$= 1,where $\{\lambda_\alpha\}$'s are fuzzy simply open sets implies that there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ for X by fuzzy simply open sets. That is, $\vee_{n \in N} \{\lambda_{\alpha_n}\}$= 1,where int($\lambda_{\alpha_n}$) = 0 in (X,T).

Proposition 3.7: If (X,T) is a fuzzy simply Lindelof space and fuzzy submaximal space and if $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$ where $\{\lambda_\alpha\}$'s are fuzzy dense sets in (X,T), then $\wedge_{n \in N} \{\mu_{\alpha_n}\} = 0$, where ($\mu_{\alpha_n}$)'s are fuzzy nowhere dense sets in (X,T).

Proof: Let (X,T) be a fuzzy simply Lindelof and fuzzy submaximal space. By hypothesis $\vee_{\alpha \in \Delta} \{\lambda_\alpha\}$ = 1,where cl(λ_α) = 1 in (X,T).Since (X,T) is a fuzzy submaximal space, the fuzzy dense sets $\{\lambda_\alpha\}$'s are fuzzy open sets in (X,T).Hence $\{\lambda_\alpha\}$'s are fuzzy open sets and fuzzy dense sets in (X,T).Then, by proposition 3.2, $\wedge_{n \in N} \{\mu_{\alpha_n}\}$= 0,where ($\mu_{\alpha_n}$)'s are fuzzy nowhere dense sets in (X,T).

Proposition 3.8: If $\vee_{\alpha \in \Delta} \{\lambda_\alpha\}$= 1,where intcl($\lambda_\alpha$) = 0 in a fuzzy simply Lindelof space (X,T),then there exist a fuzzy first category set λ in (X,T) such that cl(λ) = 1 in (X,T).

Proof: Let (X,T) be a fuzzy simply Lindelof space such that $\vee_{\alpha \in \Delta} \{\lambda_\alpha\}$= 1 and intcl($\lambda_\alpha$) = 0. Now int cl($\lambda_\alpha$) = 0 in (X,T) implies that (λ_α)'s are fuzzy nowhere dense sets in (X,T).Then, by theorem 2.7, { cl(λ_α)}'s are fuzzy simply open sets in (X,T).Now $\lambda_\alpha \leq$ cl(λ_α) implies that $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} \leq \vee_{\alpha \in \Delta}$ cl(λ_α) and then $1 \leq \vee_{\alpha \in \Delta}$ cl(λ_α).That is $\vee_{\alpha \in \Delta}$ cl(λ_α) = 1 where { cl(λ_α)}'s are fuzzy simply open sets in the fuzzy Lindelof space (X,T).Then there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ for X by fuzzy simply open sets in (X,T). That is, $\vee_{n \in N}$ cl(λ_{α_n}) = 1. Now, by lemma 2.2, $\vee_{n \in N}$ cl(λ_{α_n}) ≤ cl ($\vee_{n \in N} \{\lambda_{\alpha_n}\}$) implies that 1≤ cl ($\vee_{n \in N} \{\lambda_{\alpha_n}\}$). That is, cl ($\vee_{n \in N} \{\lambda_{\alpha_n}\}$) = 1.Let $\vee_{n \in N} \{\lambda_{\alpha_n}\}$ = λ.Then λ is a fuzzy first category set in (X,T) such that cl(λ) = 1, in (X,T).

Definition 3.3[10]: A fuzzy topological space (X,T) is a fuzzy resolvable space if there exist a fuzzy dense set λ in (X,T) such that 1 – λ is also a fuzzy dense set in (X,T0. Otherwise (X,T) is called a fuzzy resolvable space.

Definition 3.4[12]: Let (X,T) be a fuzzy topological space.Then (X,T) is called fuzzy Baire space if int($\vee_{\alpha=1}^{\infty}(\lambda_\alpha)$) = 0, where ($\lambda_\alpha$)'s are fuzzy nowhere dense sets in (X,T).

Proposition 3.9: If $\vee_{\alpha \in \Delta} \{\lambda_\alpha\}$= 1where ($\lambda_\alpha$)'s are fuzzy nowhere dense sets in a fuzzy simply Lindelof and fuzzy Baire space (X,T) then (X,T) is a fuzzy resolvable space.

Proof: Let (X,T) be a fuzzy simply Lindelof space such that $\vee_{\alpha\,\epsilon\Delta}$ $\{\lambda_\alpha\}$= 1 where intcl(λ_α) = 0 in (X,T).Then,by proposition 3.8, there exist a fuzzy first category set λ in (X,T) such that cl(λ) = 1. Since (X,T) is a fuzzy Baire space, by theorem int (λ) = 0 in (X,T). Now cl(1 – λ) = 1− int(λ) = 1− 0 = 1.Thus cl(λ) = 1 and cl (1− λ) = 1 in (X,T). Hence (X,T) is a fuzzy resolvable space.

Remark 3.1:
Since each fuzzy open set is a fuzzy simply open set in a fuzzy topological space, fuzzy Lindelof spaces are fuzzy simply Lindelof spaces. But the converse need not be true. For, consider the following example:

Example 3.2: Let X = {a,b,c}.The fuzzy sets λ, μ, δ, α, β and γ are defined on X as follows: λ : X → [0,1] is defined as λ(a) = 0.2; λ(b) =0.3 ; λ(c)=1; μ : X → [0,1] is defined as μ(a) = 0.5; μ(b) = 1; μ(c)=0.7; δ : X → [0,1] is defined as δ(a) = 1; δ(b) =0.5 ; δ(c)=0.6; α : X → [0,1] is defined as α(a) =0.2; α(b) = 0.4; α(c)=1; β : X → [0,1] is defined as β(a) = 0.5; β(b) = 1; β(c)=0.6; γ : X → [0,1] is defined as γ(a) = 1; γ(b) = 0.4; γ(c)=0.6;
Then T = { 0, λ, μ, δ, λ∨μ, λ∨δ, μ∨δ, λ∧μ, λ∧δ, μ∧δ, λ∨(μ∧δ), μ∨(λ∧δ), δ∨(λ∧μ), μ∧(λ∨δ),1} is a fuzzy topology on X. On computation λ, μ, δ, λ∨μ, λ∨δ, μ∨δ, λ∧μ, λ∧δ, μ∧δ, λ∨(μ∧δ), μ∨(λ∧δ), δ∨(λ∧μ),μ∧(λ∨δ),α,β,γ and 1 are fuzzy open and fuzzy dense sets in (X,T).Also cl (α) = cl (β) = cl (γ) = 0, cl (1− γ) = 1− (λ ∧ δ), cl (1− α) = 1− λ, cl (1−β) = 1− (μ∧δ).Clearly (X,T) is a fuzzy simply Lindelof space, since, for each cover on X by fuzzy simply open sets, there exist a countable subcover for X. But (X,T) is not a fuzzy Lindelof space. Since λ∨μ∨δ = 1 and μ∨δ ≠ 1 in (X,T).

Proposition 3.10: If $\vee_{\alpha\epsilon\Delta}$ $\{\lambda_\alpha\}$= 1,where (λ_α)'s are fuzzy open sets in a fuzzy simply Lindelof and fuzzy hyper connected space (X,T) then there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\,\epsilon\,N}$ for X.

Proof: Let (X,T) be a fuzzy simply Lindelof and fuzzy hyper connected space such that $\vee_{\alpha\,\epsilon\Delta}$ $\{\lambda_\alpha\}$= 1,where $\lambda_\alpha\ \epsilon$ T. Since (X,T) is a fuzzy hyper connected space, the fuzzy open sets (λ_α)'s are fuzzy dense sets in (X,T).Then (λ_α)'s are fuzzy open and fuzzy dense sets in (X,T).Then, by theorem 2.1, (λ_α)'s are fuzzy simply open sets in (X,T).Since (X,T) is a fuzzy simply Lindelof space, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\,\epsilon\,N}$ for X.

Remark 3.2: In view of the above proposition, one will have the following result "If a fuzzy simply Lindelof space is a fuzzy hyper connected space, then it is a fuzzy Lindelof space".

REFERENCES

[1]. K.K. Azad, *On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly contiunity*, J. Math. Anal. Appl, 52 (1981),14-32.

[2]. G. Balasubramanian, *On Some Generalizations of Compact Spaces*, Glasnik Math.17, No. 37(1982), 367 - 380.

[3]. G. Balasubramanian, *Maximal Fuzzy Topologies*, Kybernetika, 31(5)(1995), 459-464.

[4]. A.S. Bin Shahna, *On Fuzzy Compactness and Fuzzy Lindelofness,* Bull. Calcutta Math.Soc. 83(1991),146 - 150.

[5]. F.Cammaroto and G.Santoro, *Some counterexamples and properties on Generalizations of Lindelof Spaces*, Inter.J.Math. & Math. Sci.19, no.4(1996), 737 - 746.

[6] . C.L.Chang, *Fuzzy Topological Spaces*, J. Math. Anal., 24, (1968), 182 -190.

[7]. Z. Frolik *Generalizations of Compact and Lindelof Spaces*, Czech. Math.J ., 9, No.84 (1959), 172 - 217.

[8]. G. Thangaraj and G. Balasubramanian, *On Somewhat Fuzzy Continuous Functions*, J. Fuzzy. Math, 11(2)(2003), 725-736.

[9]. G. Thangaraj and G.Balasubramanian, *On Some Generalizations of Fuzzy Lindelof Spaces,* J. Fuzzy. Math, Vol.15,No. 3, 2007,1-7.

[10]. G.Thangaraj and G.Balasubramanian, *On fuzzy resolvable and irresolvable spaces*, Fuzzy sets, Rough sets and Multi-valued Operations and Applications,1(2) (2009), 173 -180.

[11] . G.Thangaraj and V. Seenivasan *On Fuzzy Strongly Irresolvable Spaces*, Proc. Nat. Conf. on Fuzzy Math. and Graph Theory, Jamal Mohamed College, Trichy, Tamilnadu, India (2008), 1-7.

[12]. G.Thangaraj and S.Anjalmose, *On fuzzy Baire spaces*, J. Fuzzy. Math., 21(3)(2013), 667-676.

[13]. G. Thangaraj and K. Dinakaran, *On Fuzzy Simply Continuous Functions*, J. Fuzzy. Math, Vol 25, No.4, 2016, 99-124.

[14]. R.H. Warren. *Neighbourhood, bases and Continuity in Fuzzy Topological Spaces*, Rocky Mountain Jour. Math. (1978) .459 – 470.

[15]. S.Willard and U.N.B. Dissanayake, *The Almost Lindelof Degree*., Canad. Math Bull. 27, No. 4 (1984), 452 – 455.

[16]. L.A. Zadeh, *Fuzzy Sets*, Inform. and Control, Vol. 8,(1965), 338 – 353.