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On Fuzzy Regular Generalized Weakly Closed Sets In Fuzzy Topological Space

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Abstract

In this paper, we introduce the concept of fuzzy regular generalized weakly (fuzzy rgw-closed) closed sets in fuzzy topological spaces, which is followed by fuzzy regular weakly (fuzzy rw-closed) closed sets. We also investigate there fundamental properties and compare it with some other types of fuzzy sets.

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1. INTRODUCTION

In twentieth century, mathematicians defined the concepts of sets and functions to represent problems. In many circumstances the solutions using this concept are meaningless. This difficulty was overcome by the fuzzy concept. Almost all mathematical, Engineering, Medicine, etc. concepts have been redefined using fuzzy sets. In view of the fact that set theory is the cornerstone of modern mathematics, a new and more general framework of mathematics was established. Fuzzy mathematics is just a kind of mathematics developed in this framework, and fuzzy topology is just a kind of topology developed on fuzzy sets. Fuzzy topology is a generalization of topology in classical mathematics, but it also has its own marked characteristics. Also it can deepen the understanding of basic structure of classical mathematics, offer new methods and results, and obtain significant results of classical mathematics. Fuzzy topological space was introduced in 1968 by C.L.Chang as a generalization of topological spaces. The paper was published in1968 under the title "Fuzzy topological spaces" [2].This was the beginning of fuzzy topology. This paper by C.L. Chang [2] attracted mathematicians all over the globe and the ball began to roll. Here is what we see today. Google scholar lists 1335 papers with title "fuzzy topology" or "fuzzy topological spaces", and Google Books lists 132 books with title "fuzzy topology" or "fuzzy topological spaces".

The purpose of this paper is to introduce a new class of fuzzy sets called *fuzzy regular generalized weakly closed (*fuzzy rgw-closed) sets and investigate certain basic properties of these fuzzy sets.

2. PRELIMINARIES

Definition 2.1 [2] A family τ of fuzzy sets of X is called fuzzy topology on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection. The elements of τ are called fuzzy open sets and there complements are called fuzzy closed sets.

Definition 2.2 For a fuzzy set α of X, the closure *Cl α* and the interior *Int α* of α are defined respectively, as

$Cl\ \alpha = \wedge\{ \mu : \mu \geq \alpha, 1-\mu \in \tau \}$ and

$\text{Int}\ \alpha = \vee\{ \mu : \mu \leq \alpha, \mu \in \tau \}$

Definition 2.3 [20] A fuzzy set *A* is said to be *fuzzy semi-open set* if and only if there exists a fuzzy open set α such that $\alpha \leq A \leq Cl(\alpha)$,equivalently $Cl(Int(A)) \geq A$. And *A* is called a *fuzzy semi-closed* set if $Int(Cl(A)) \leq A$.

Definition 2.4 [20] A fuzzy set α of a fuzzy topological space *X* is called fuzzy regular open set of *X* if $int(cl(\alpha)) = \alpha$

Definition 2.5 [20] A fuzzy set *α* of a fuzzy topological space X is called fuzzy regular closed set of X if $Cl(Int(\alpha)) = \alpha$.

Definition 2.5 [23] A fuzzy set *α* of a fuzzy topological space *X* is said to be a *fuzzy regular semiopen* set in fuzzy topological space X if there exists a fuzzy regular open set μ in X such that $\mu \leq \alpha \leq cl(\mu)$.

Definition 2.6 [6] A fuzzy set α of a fuzzy topological space X is called fuzzy regular w-closed if *Cl(α)* $\leq \mu$ whenever $\alpha \leq \mu$ and μ is fuzzy regular semiopen in fuzzy topological space X.

Definition 2.7 [4] A fuzzy set *A* in a fuzzy topological space X is called *generalized fuzzy closed*, if cl(A) $\leq$ U whenever A $\leq$U and U is fuzzy open.

Remark 3.3. Every fuzzy closed set is a fuzzy regular weakly-closed set in a fuzzy topological space.[6]

Remark 3.4. Every fuzzy regular closed set is a fuzzy closed set but not conversely.[6]

Remark 3.8. Every fuzzy-θg-closed set is fuzzy-θ-closed and every fuzzy-θ-closed set is fuzzy closed [21].

Remark 3.10. Every fuzzy regular semi-open set in a fuzzy topological space X is fuzzy semi- open. [6]

3. FUZZY REGULAR GENERALIZED WEAKLY -- CLOSED SETS

Definition3. A fuzzy set α in a fuzzy topological space (X,τ) is called *fuzzy regular generalized weakly* closed set (fuzzy rgw-closed) if *Cl(Int(α))* $\leq \mu$ whenever $\alpha \leq \mu$ and μ is fuzzy regular semi-open in X.

Example 3.1. Let X={a, b, c, d} be a space with fuzzy topology τ = {1,0,φ, α, β, γ},where φ, α, β, γ : X → [0,1] are defined as

$$\alpha(x) = \begin{cases} 1 & if\ x = a \\ 0 & otherwise \end{cases}$$

$$\beta(x) = \begin{cases} 1 & if\ x = b \\ 0 & otherwise \end{cases}$$

$$\varphi(x) = \begin{cases} 1 & if\ x = a, b \\ 0 & otherwise \end{cases}$$

$$\gamma(x) = \begin{cases} 1 & if\ x = a, b, c \\ 0 & otherwise \end{cases}$$

Then (X,τ) is a fuzzy topological space. In this fuzzy topological space the fuzzy set μ : X → [0,1] defined by

$$\mu(x) = \begin{cases} 1 \;\; if\ x = d \\ 0 \;\; otherwise \end{cases}$$

Then μ is a fuzzy regular generalized weakly closed set in fuzzy topological space (X,τ).

Theorem 3.2. *Every fuzzy regular weakly (fuzzy rw)-closed set is fuzzy regular generalized weakly (fuzzy rgw)-closed set but not conversely.*

Proof. Suppose α is a fuzzy regular weakly-closed(fuzzy rw-closed) set in X. Then Cl(α) ≤ U whenever α ≤ U, where U is fuzzy regular semi-open in X. We have the standard result Int(α) ≤ α ≤ Cl(α), implies Cl(Int(α)) ≤ Cl(α). So we have Cl(Int(α)) ≤ Cl(α) ≤ U whenever α ≤ U, where U is regular semi open. α is satisfying the condition of fuzzy rgw-closed set, implies α is a fuzzy regular generalized weakly (fuzzy rgw)-closed set in X.

Converse, In fuzzy topological space (X,τ) defined in Example 3.1 the fuzzy set μ : X → [0,1] defined by

$$\mu(x) = \begin{cases} 1\ if\ x = c \\ 0\ otherwise \end{cases}$$

Then μ is *fuzzy regular generalized weakly closed*(fuzzy rgw) set in this fuzzy topolog-ical space (X,τ), but it is not a *fuzzy regular weakly(fuzzy rw)* closed set in (X, τ).

Theorem 3.5. *Every fuzzy closed set is a fuzzy regular generalized weakly-closed (fuzzy rgw-*

closed) set, but not conversely.

Proof. According to *Remark 3.3* every fuzzy closed set is fuzzy regular weakly closed and by Theorem 3.2 every fuzzy rw-closed set is fuzzy rgw-closed. So we have every fuzzy-closed set is fuzzy rgw-closed.

Converse, Let X={a, b ,c} be a space with fuzzy topology τ = {1, 0, α, β}, where α,β : X→ [0, 1] are defined as

$$\alpha(x) = \begin{cases} 1 \; if \; x = a \\ 0 \; otherwise \end{cases}$$

$$\beta(x) = \begin{cases} 1 \; if \; x = a, b \\ 0 \; otherwise \end{cases}$$

Then (X,τ) is a fuzzy topological space and β is a fuzzy rgw-closed set in (X, τ) but not fuzzy closed.

Theorem 3.6. Every fuzzy regular closed set is fuzzy rgw-closed but not conversely.

Proof. By Remark 3.4, every fuzzy regular closed set is a fuzzy closed set and by Theorem 3.5, every fuzzy closed set is fuzzy rgw-closed but not conversely. So we have every fuzzy regular closed set is fuzzy rgw-closed.

Converse, In fuzzy topological space (X,τ) defined in Example3.1, if we define fuzzy set

$$\mu(x) = \begin{cases} 1 \;\; if \; x = a, b \\ 0 \;\; otherwise \end{cases}$$

then μ is a fuzzy rgw-closed set but not fuzzy regular closed in (X,τ).

Theorem 3.7. Every fuzzy α-closed set is fuzzy rgw-closed but not conversely.

Proof. Suppose α is any arbitrary fuzzy α-closed set in fuzzy topological space (X,τ), suppose α ≤ A and A is fuzzy regular semi open. so Cl(Int(α)) ≤ α.We have α ≤Cl(α) implies Cl(Int(α)) ≤ Cl(Int(Cl(α))) ≤ α Implies α is fuzzy rgw-closed set in (X,τ). Converse, In fuzzy topological space (X,τ) defined in Example3.1 if we define fuzzy set

$$\gamma(x) = \begin{cases} 1 \;\; if \; x = a, b, c \\ 0 \;\; otherwise \end{cases}$$

Then γ is a fuzzy rgw-closed but not fuzzy α-closed set in fuzzy topological space (X,τ).

Theorem 3.9. Every fuzzy-θ-closed set is fuzzy rgw-closed but not conversely.

Proof. As every fuzzy-θ-closed set is fuzzy closed by Remark 3.8 and by Theorem 3.5, every fuzzy-closed set is fuzzy rgw-closed. Implies every fuzzy- θ -closed set is fuzzy rgw-closed.

Converse In fuzzy topological space (X,τ) defined in Example3.1 if we define fuzzy set

$$\alpha(x) = \begin{cases} 1 & if\ x = a, b \\ 0 & otherwise \end{cases}$$

Then α is fuzzy rgw-closed but not closed. To show α is not fuzzy-θ-closed, we prove it by contradiction. Let α is fuzzy-θ-closed, so it must be fuzzy closed by *Remark 3.8*, but this is a contradiction as α is not closed. Thus α is not fuzzy-θ-closed.

Theorem 3.11. Every fw-closed set is frw-closed.

Proof. Suppose A is fw-closed. So Cl(A)≤B whenever A≤ B and B is fuzzy semi-open. Since every fuzzy regular semi-open set in X is fuzzy semi-open by *Remark 3.10*.Implies Cl(A) ≤ B whenever A ≤ B and B is fuzzy regular semi-open. So A is fuzzy rw-closed.

Theorem 3.12. Every fuzzy w-closed set is fuzzy rgw-closed but not conversely.

Proof. According to Theorem3.11 every fuzzy w-closed is fuzzy rw-closed and by Theorem 3.2 every fuzzy rw-closed set is fuzzy rgw-closed but not conversely.

Converse, In fuzzy topological space (X,τ) defined in Example3.1 if we define fuzzy set

$$\beta(x) = \begin{cases} 1 & if\ x = c \\ 0 & otherwise \end{cases}$$

Then β is a fuzzy rgw-closed set but not fuzzy rw-closed.

Result, From the above discussion and known results we have the following table of implications

TABLE A

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	*1*	*0*	*1*	*1*	*1*	*1*	*1*
B	*1*	*1*	*1*	*1*	*1*	*1*	*1*
C	*0*	*0*	*1*	*0*	*0*	*0*	*0*
D	*0*	*0*	*0*	*1*	*0*	*0*	*0*
E	*0*	*0*	*0*	*0*	*1*	*1*	*0*
F	*0*	*0*	*0*	*0*	*0*	*1*	*0*
G	*0*	*0*	*0*	*1*	*0*	*1*	*1*

In the above table, A, B, C, D, E, F, and G denote fuzzy closed sets, fuzzy regular closed sets, fuzzy generalized closed sets, fuzzy semi-closed sets, fuzzy rw-closed sets, fuzzy rgw-closed sets, and fuzzy α-closed sets. Also 1 denotes 'implies' and 0 denotes 'does not imply'.

Theorem 3.13. The union of two fuzzy rgw-closed sets is a fuzzy rgw-closed set.

Proof. Suppose A and B are two fuzzy rgw-closed sets, and α be a regular semi-open set such that $A\cup B\leq\alpha$.So we have $A \leq \alpha$ and $B \leq \alpha$. Implies $Cl(Int(A)) \leq \alpha$ and $Cl(Int(B)) \leq \alpha$, or

$Cl(Int(A)) \cup Cl(Int(B)) \leq \alpha$ or $Cl(Int(A) \cup Int(B)) \leq \alpha$, which implies $Cl(Int(A\cup B))\leq\alpha$. Implies $A\cup B$ is a fuzzy rgw-closed.

Theorem 3.14. If α and β are fuzzy rgw-closed sets in fuzzy topological space X, then $A \wedge B$ need not be a fuzzy rgw-closed set in general as we can see from the following example.

Example 3.15 In fuzzy topological space (X,τ) defined in Example3.1 if we define fuzzy set

$$\alpha(x) = \begin{cases} 1 & if\ x = a, b, c \\ 0 & otherwise \end{cases}$$

$$\beta(x) = \begin{cases} 1 & if\ x = a, c, d \\ 0 & otherwise \end{cases}$$

Then α and β are fuzzy rgw-closed sets in fuzzy topological space (X,τ). Let $\gamma = \alpha \wedge \beta$. Then

$$\gamma(x) = \begin{cases} 1 & if\ x = a, c \\ 0 & otherwise \end{cases}$$

Then $\gamma = \alpha \wedge \beta$ is not a fuzzy rgw-closed set in fuzzy topological space (X,τ).

Theorem 3.16. If A is a fuzzy rgw-closed subset of (X,τ) such that A ≤ B ≤ Cl(Int(A)), then B is a fuzzy rgw-closed set in (X,τ).

Proof. Suppose A is a fuzzy rgw-closed subset of (X,τ) such that A ≤B ≤Cl(Int(A)).Suppose U is fuzzy regular semi-open set in (X,τ) such that B ≤ U, then A ≤ U. Now A is fuzzy rgw-closed subset of (X,τ), implies Cl(Int(A)) ≤ U.

As B ≤ Cl(Int(A)), implies Cl(Int(B)) ≤ Cl(Int(Cl(Int(A)))) ≤ Cl(Int(A)) ≤ U. So we have Cl(Int(B)) ≤ U, whenever B ≤ U where U is a fuzzy regular semiopen, Implies that B is a fuzzy rgw-closed set in (X,τ).

Theorem 3.17. If a subset A of a fuzzy topological space (X,τ) is both fuzzy regular semi-open and fuzzy rgw-closed, then it is fuzzy regular closed.

Proof. Suppose A is both fuzzy regular semi-open and fuzzy rgw-closed. Now A ≤ A and also A is fuzzy rgw-closed implies Cl(Int(A)) ≤ A.

Now from Remark 3.10 every fuzzy regular semi-open set is fuzzy semi-open, i,e A ≥ Cl(Int(A)),So we have A = Cl(Int(A)). Implies that A is fuzzy regular closed.

Theorem 3.18. Let A be fuzzy regular semi-open and fuzzy rgw-closed in fuzzy topological space (X,τ). If B is fuzzy regular closed in (X,τ). Then A ∧ B is a fuzzy rgw-closed set in (X,τ).

Proof. Suppose A is fuzzy regular semi-open and fuzzy rgw-closed in fuzzy topological space (X,τ).So by Theorem 3.17 A is fuzzy regular closed. Also given that B is a fuzzy regular closed set. So A ∧ B is fuzzy regular closed, and from table A, we have every fuzzy regular closed set is fuzzy closed, Implies A ∧ B is fuzzy closed and by Theorem3.5 Every fuzzy closed set is fuzzy rgw-closed. So A ∧ B is a fuzzy rgw-closed set in (X,τ).

Theorem 3.19. In a fuzzy topological space (X,τ), If {φ,X} are the only fuzzy regular semi-open sets of X, then every subset of X is a fuzzy rgw-closed set.

Proof. Let (X,τ) be a fuzzy topological space and {φ,X} are the only fuzzy regular semi-open sets. Let A be any arbitrary subset of (X,τ). If A = φ, then X is a fuzzy rgw-closed set in (X,τ). Now if A ≠ φ, then X is the only fuzzy regular semi-open set containing A. Implies Cl(Int(A)) ≤ X. So A is a fuzzy rgw-closed set in X.

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