Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 12, Number 4 (2017), pp. 977-989

http://www.ripublication.com

On generalized (ϕ, ψ) - weak contractions in intuitionistic fuzzy metric spaces

Antima Sindersiya[1*], Aklesh Pariya[2], Nirmala Gupta[3] and V. H. Badshah[1]

[1] School of Studies in Mathematics, Vikram University, Ujjain (M. P.), India.
[2] Department of Mathematics, Medi - Caps University, Indore (M. P.), India.
[3] Department of Mathematics, Govt. Girls P. G. College, Ujjain (M. P.), India.
**Corresponding Author*

Abstract

In this paper, we generalize the notion of (ϕ, ψ) weak contraction in intuitionistic fuzzy metric spaces and prove some common fixed point theorems in intuitionistic fuzzy metric spaces. Our result generalize recent result of beg et al.[4], Jiao[11], Rafi et al.[18] and reference mentioned there in.

Keywords. Common fixed point,weak contraction, intuitionistic fuzzy metric spaces.

AMS 2010 MSC: 54H25, 47H10.

1.INTRODUCTION AND PRELIMINARIES.

Fuzzy set theory was firstintroduced by Zadeh[26] to describe the situation in which data are imprecise or vague or uncertain. Thereafter the concept of fuzzy set was generalized as intuitionistic fuzzy set by Atanassov[2,3]. The concept of fuzzy metric wasfirst introduced by Kramosil and Michalek[13]but using the idea of intuitionistic fuzzy set, Park[17] introduced the notionof intuitionistic fuzzy metric spaces with the help of continuous t-norms andcontinuous t-conorms, which is a generalization of fuzzy metric space due toGeorge and Veeramani[8].Introducing the contraction mapping with the help of the membership function for fuzzy metric, several authors[9, 16, 22] established the Banach fixedpoint theorem in fuzzy metric spaces. Recently Sugandhi et. al.[21] hasproved common fixed point theorems in intuitionistic fuzzy symmetric spaces for occasionally weakly compatible maps satisfying contractive condition of integral type. In this paper, we prove the common fixed point theorem in intuitionistic fuzzy metric spaces using the notion of (ϕ, ψ) weak contractions.

Definition 1.1[20]. A binary operation $*$: $[0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

1. $*$ is associative and commutative,
2. $*$ is continuous,
3. $a * 1 = a$, for every $a \in [0,1]$,
4. $a * b \leq c * d$ if$a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Examples of continuous t-norm are $a * b = ab$ and $a * b = min(a, b)$.

Definition 1.2[20]. A binary operation$\diamond$: $[0,1] \times [0,1] \to [0,1]$ is a continuous t-conorm if it satisfies the following conditions:

1. $\diamond$ is associative and commutative,
2. $\diamond$ is continuous,
3. $a \diamond 0 = a$, for every $a \in [0,1]$,
4. $a \diamond b \leq c \diamond d$ if $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

Examples of continuous t-conorm are $a \diamond b = min(a + b, 1)$and $a \diamond b = max(a, b)$.

Definition 1.3[17]. The 5-tuple$(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and $M, N$are fuzzy sets in

$X \times X \times (0,\infty)$satisfying the following conditions:

(i) $M(x, y, t) > 0$,

(ii) $M(x, y, t) = 1$ iff $x = y$,

(iii) $M(x, y, t) = M(y, x, t)$

(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

(v) $M(x, y, .) : (0,\infty) \to (0,1]$ is continuous,

(vi) $N(x,y,t) < 1$,

(vii) $N(x,y,t) = 0$ iff $x = y$,

(viii) $N(x,y,t) = N(y,x,t)$,

(ix) $N(x,y,t) \diamond N(y,z,s) \geq N(x,z,t+s)$,

(x) $N(x,y,.) : (0,\infty) \to (0,1]$ is continuous,

(xi) $M(x,y,t) + N(x,y,t) \leq 1$, for all $x,y,z \in X$ and $s,t > 0$.

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x,y,t)$and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 1.1. Every fuzzy metric space $(X,M,*)$ is an intuitionistic fuzzy metric space of the form $(X,M,1-M *,\diamond)$ such that t-norm $*$ and t-conorm $\diamond$ are associated[13], that is

$x \diamond y = 1-((1-x)*(1-y)$ for all$x,y \in X$.

Remark 1.2. In an intuitionistic fuzzy metric space $(X,M,N,*,\diamond)$, $M(x,y,.)$ ais non-decreasing and $N(x,y,.)$ is non- increasing for all $x,y \in X$.

Definition 1.4. A sequence $\{x_n\}$ is an intuitionistic fuzzy metric space is said to be a Cauchy sequence if and only if for each $r \in (0,1)$ and $t > 0$ there exists $n_0 \in \mathbb{N}$ such that

$M(x_n, x_m, t) > 1-r$ and $N(x_n, x_m, t) < r$ for all $n,m \geq n_0$.

Definition 1.5. A sequence $\{x_n\}$ is an intuitionistic fuzzy metric space is convergent to $x \in X$ if, for each $t > 0$, we have $\lim_{n\to\infty} M(x_n,x,t) = 1$ and

$\lim_{n\to\infty} N(x_n,x,t) = 0$.

Definition 1.6. An intuitionistic fuzzy metric space is complete if and only if every Cauchy sequence is convergent. An intuitionistic fuzzy metric space is compact if every sequence contains a convergent subsequence.

It is easy to prove that a metric space (X,d) is complete if and only if the intuitionistic fuzzy metric space $(X,M_d,N_d,*,\diamond)$ is complete.

Definition 1.7[18]. Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space. A mapping

$f: X \to X$ is intuitionistic fuzzy contractive if there exists $k \in (0,1)$ such that

$\frac{1}{M(fx,fy,t)} - 1 \leq k\left(\frac{1}{M(x,y,t)} - 1\right)$ and $N(fx,fy,t) \leq kN(x,y,t)$, for all $x, y \in X$ and $t > 0$.

Definition 1.8[11]. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. A mapping $f: X \to X$ is intuitionistic fuzzy contractive if there exists $k \in (0,1)$ such that

$$\frac{1}{M(fx,fy,t)} - 1 \leq k\left(\frac{1}{M(x,y,t)} - 1\right) \text{ and } \frac{1}{N(fx,fy,t)} - 1 \geq \frac{1}{k}\left(\frac{1}{N(x,y,t)} - 1\right),$$

for all $x, y \in X$ and $t > 0$.

Remark 1.3. Note that the intuitionistic fuzzy contractive definition given in [18] is more general than the intuitionistic fuzzy contractive definition given in [11].

Proposition 1.1[4]. Let (X, d) be a metric space. The mapping $f: X \to X$ is metric contractive on (X, d) with contractive constant k if and only if f is intuitionistic fuzzy contractive, with contractive constant k, on the intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, induced by d.

Recall that a sequence $\{x_n\}$ in a metric space (X, d) is said to be contractive if there exists $k \in (0,1)$ such that $d(x_{n+1}, x_{n+2}) \leq kd(x_n, x_{n+1})$, for all $n \geq 0$.

Definition 1.9[4]. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. We will say that the sequence $\{x_n\}$ in X is intuitionistic fuzzy contractive if there exists $k \in (0,1)$ such that for each $n \geq 0$ and $t > 0$,

$$\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1 \leq k\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right)$$

and

$$N(x_{n+1}, x_{n+2}, t) \leq kN(x_n, x_{n+1}, t).$$

`**Proposition 1.2[4].** Let $(X, M_d, N_d, *, \diamond)$ be an intuitionistic fuzzy metric space induced by the metric d on X. The sequence $\{x_n\}$ in X is contractive in (X, d) if and only if $\{x_n\}$ is intuitionistic fuzzy contractive in $(X, M_d, N_d, *, \diamond)$.

Definition 1.10. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $f, T: X \to X$ be two mappings. A point z in X is called coincidence point(common fixed point) of f and T if $fz = Tz$ $(z = fz = Tz)$.

Beg et al.[4] introduce the notion of ψ -weak contraction in intuitionistic fuzzy metric spaces.

Definition 1.11[4]. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $f, T: X \to X$ be two mappings. The mapping T is called intuitionistic ψ-weak contraction with respect to f if there exists a function $\psi : [0,\infty) \to [0,\infty)$ with $\psi(r) > 0$ for $r > 0$ and $\psi(0) = 0$ such that

$$\frac{1}{M(Tx,Ty,t)} - 1 \leq \left(\frac{1}{M(fx,fy,t)} - 1\right) - \psi\left(\frac{1}{M(fx,fy,t)} - 1\right) \quad (1)$$

and

$$N(Tx,Ty,t) \leq N(fx,fy,t) - \psi(N(fx,fy,t)) \quad (2)$$

hold for all $x, y \in X$ and each $t > 0$. If f is the identity mapping, then T is called intuitionistic ψ-weak contraction.

Definition 1.12[12]. A function ϕ: $[0,\infty) \to [0,\infty)$ is an altering distance function if $\phi(t)$ is monotone non-decreasing and continuous and $\phi(t) = 0$ if and only if $t = 0$.

Khan et. al.[12] initiated the altering distance function and used it for solving fixed point problem in metric spaces. The idea of altering function has been utilize by many authors, we would like the mention the work of [1,6,7,15,16,19,24,25] and Branciari[5] analyze the existence and uniqueness of fixed point theorem satisfying contractive conditions of integral type and gave integral version of the Banach contraction principle, that could be extended to more general contraction conditions[see 23].

Thus, considering the notion of altering distance function, Beg et al.[4] gave the notion of (ϕ, ψ)-weak contraction in intuitionistic fuzzy metric spaces.

Definition 1.13[4]. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and$f, T: X \to X$ be two mappings. The mapping T is called intuitionistic (ϕ, ψ)-weak contraction with respect to f if there exists a function $\psi : [0,\infty) \to [0,\infty)$ with $\psi(r) > 0$ for $r > 0$ and $\psi(0) = 0$ and an altering distance function ϕsuch that

$$\phi\left(\frac{1}{M(Tx,Ty,t)} - 1\right) \leq \phi\left(\frac{1}{M(fx,fy,t)} - 1\right) - \psi\left(\frac{1}{M(fx,fy,t)} - 1\right) \quad (3)$$

and

$$\phi\big(N(Tx,Ty,t)\big) \leq \phi\big(N(fx,fy,t)\big) - \psi(N(fx,fy,t)) \quad (4)$$

hold for all$x, y \in X$ and each $t > 0$. If f is the identity mapping, then T is called intuitionistic (ϕ, ψ)-weak contraction.

Definition 1.14[4]. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $f, T: X \to X$ be two mappings. The mapping T is called intuitionistic (ϕ, ψ)-weak contraction of integral type with respect to f if there exists a function

ψ : [0,∞) →[0,∞) with $\psi(r) > 0$ for $r > 0$ and $\psi(0) = 0$ and an altering distance function ϕ satisfying, for all $x, y \in X$ and each $t > 0$, the conditions

$$\phi\left(\int_0^{\frac{1}{M(Tx,Ty,t)}-1} \varphi(s)ds\right) \le \phi\left(\int_0^{\frac{1}{M(fx,fy,t)}-1} \varphi(s)ds\right) - \psi\left(\int_0^{\frac{1}{M(fx,fy,t)}-1} \varphi(s)ds\right), \quad (5)$$

and

$$\phi\left(\int_0^{N(Tx,Ty,t)} \varphi(s)ds\right) \le \phi\left(\int_0^{N(fx,fy,t)} \varphi(s)ds\right) - \psi\left(\int_0^{N(fx,fy,t)} \varphi(s)ds\right), \quad (6)$$

where φ: [0,∞) →[0,∞) is a Lebesgue integrable function which is summable on each compact subset of [0,∞) and such that for all $\varepsilon > 0$, $\int_0^{\varepsilon} \varphi(s)ds > 0$. If f is the identity mapping, then T is called intuitionistic (ϕ, ψ)-weak contraction of integral type.

Now we define the generalized (ϕ, ψ)-weak contractionsin intuitionistic fuzzy metric spaces as follows.

Definition 1.15. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $f, T: X \to X$ be two mappings. The mapping T is called intuitionistic generalizedψ-weak contraction with respect to f if there exists a function ψ : [0,∞) →[0,∞) with $\psi(r) > 0$ for $r > 0$ and $\psi(0) = 0$ such that

$$\begin{aligned}\frac{1}{M(Tx,Ty,t)} - 1 \le\ & a\left(\frac{1}{M(fx,fy,t)} - 1\right) + b\left(min\left\{\frac{1}{M(fx,Tx,t)}, \frac{1}{M(fx,Ty,t)}\right\} - 1\right) \\ & + c\left(min\left\{\frac{1}{M(Tx,Ty,t)}, \frac{1}{M(Tx,fx,t)}, \frac{1}{M(Ty,fy,t)}\right\} - 1\right) \\ & - \psi\left(min\left\{\frac{1}{M(fx,fy,t)}, \frac{1}{M(fx,Tx,t)}, \frac{1}{M(fx,Ty,t)}, \frac{1}{M(Ty,fy,t)}\right\} - 1\right) \quad (7)\end{aligned}$$

and

$$\begin{aligned}N(Tx,Ty,t) \le\ & a\,N(fx,fy,t) + b\,max\{N(fx,Tx,t), N(fx,Ty,t)\} \\ & + c\,max\{N(Tx,Ty,t), N(Tx,fx,t), N(Ty,fy,t)\} \\ & - \psi(max\{N(fx,fy,t), N(fx,Tx,t), N(fx,Ty,t), N(Ty,fy,t)\}) \quad (8)\end{aligned}$$

hold for all$x, y \in X$ and $a, b, c \in (0,1)$ such that $a + b + c < 1$,for each $t > 0$. If f is the identity mapping, then T is called intuitionistic ψ-weak contraction.

Definition 1.16. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $f, T: X \to X$ be two mappings. The mapping T is called intuitionistic generalized

(ϕ,ψ)-weak contraction with respect to f if there exists a function $\psi : [0,\infty) \to [0,\infty)$ with $\psi(r) > 0$ for $r > 0$ and $\psi(0) = 0$ and an altering distance function ϕ such that

$$\phi\left(\frac{1}{M(Tx,Ty,t)}-1\right) \leq a\,\phi\left(\frac{1}{M(fx,fy,t)}-1\right) + b\phi\left(min\left\{\frac{1}{M(fx,Tx,t)},\frac{1}{M(fx,Ty,t)}\right\}-1\right)$$
$$+ c\,\phi\left(min\left\{\frac{1}{M(Tx,Ty,t)},\frac{1}{M(Tx,fx,t)},\frac{1}{M(Ty,fy,t)}\right\}-1\right)$$
$$- \psi\left(min\left\{\frac{1}{M(fx,fy,t)},\frac{1}{M(fx,Tx,t)},\frac{1}{M(fx,Ty,t)},\frac{1}{M(Ty,fy,t)}\right\}-1\right) \tag{9}$$

and

$$\phi(N(Tx,Ty,t)) \leq a\phi\big(N(fx,fy,t)\big) + b\,\phi(max\{N(fx,Tx,t),N(fx,Ty,t)\})$$
$$+ c\phi(max\{N(Tx,Ty,t),N(Tx,fx,t),N(Ty,fy,t)\})$$
$$- \psi(max\{N(fx,fy,t),N(fx,Tx,t),N(fx,Ty,t),N(Ty,fy,t)\}) \tag{10}$$

hold for all $x,y \in X$ and$a,b,c \in (0,1)$ such that $a+b+c < 1$,for each $t > 0$. If $f$is the identity mapping, then T is called intuitionistic generalized (ϕ,ψ)-weak contraction.

Definition 1.17. Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and $f,T: X \to X$ be two mappings. The mapping T is called intuitionistic generalized

(ϕ, ψ)-weak contraction of integral type with respect to f if there exists a function

$\psi : [0,\infty) \to [0,\infty)$ with $\psi(r) > 0$ for $r > 0$ and $\psi(0) = 0$ and an altering distance function ϕ satisfying, for all $x,y \in X$ and $a,b,c \in (0,1)$ such that $a+b+c < 1$,for each $t > 0$, the conditions

$$\phi\left(\int_0^{\frac{1}{M(Tx,Ty,t)}-1} \varphi(s)ds\right)$$
$$\leq a\phi\left(\int_0^{\frac{1}{M(fx,fy,t)}-1} \varphi(s)ds\right) + b\phi\left(\int_0^{min\left\{\frac{1}{M(fx,Tx,t)},\frac{1}{M(fx,Ty,t)}\right\}-1} \varphi(s)ds\right)$$
$$+c\phi\left(\int_0^{min\left\{\frac{1}{M(Tx,Ty,t)},\frac{1}{M(Tx,fx,t)},\frac{1}{M(Ty,fy,t)}\right\}-1} \varphi(s)ds\right)$$
$$-\psi\left(\int_0^{min\left\{\frac{1}{M(fx,fy,t)},\frac{1}{M(fx,Tx,t)},\frac{1}{M(fx,Ty,t)},\frac{1}{M(Ty,fy,t)}\right\}-1} \varphi(s)ds\right) \tag{11}$$

and

$$\phi\left(\int_0^{N(Tx,Ty,t)}\varphi\,(s)ds\right)$$

$$\leq a\phi\left(\int_0^{N(fx,fy,t)}\varphi(s)ds\right) + b\,\phi\left(\int_0^{\max\{N(fx,Tx,t),N(fx,Ty,t)\}}\varphi(s)ds\right)$$

$$+ c\,\phi\left(\int_0^{\max\{N(Tx,Ty,t),N(Tx,fx,t),N(Ty,fy,t)\}}\varphi(s)ds\right)$$

$$-\psi\left(\int_0^{\max\{N(fx,fy,t),N(fx,Tx,t),N(fx,Ty,t),N(Ty,fy,t)\}}\varphi(s)ds\right), \qquad (12)$$

where $\varphi : [0,\infty) \to [0,\infty)$ is a Lebesgue integrable function which is summable on each compact subset of $[0,\infty)$ and such that for all $\varepsilon > 0$, $\int_0^{\varepsilon}\varphi(s)ds > 0$. If f is the identity mapping, then T is called intuitionistic generalized (ϕ, ψ)-weak contraction of integral type.

Remark 1.1. It is clear that condition (11) and (12) are generalization of conditions (1) to (10) and are special cases of condition (11) and (12).

Main Result

Theorem 2.1. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $T: X \to X$ be generalized intuitionistic (ϕ, ψ)-weak contraction with respect to $f: X \to X$ defined in (1.16). If the range of f contains the range of T and $f(X)$ or $T(X)$ is a complete subset of X then f and T have a unique common fixed point in X provided that ψ is a continuous function.

Proof. Let $x_0 \in X$ be an arbitrary point. Since $T \subset f$, choose a point $x_1 \in X$ such that $Tx_0 = fx_1$. In this similar manner we define a sequence $\{y_n\}$ in X such that $y_n = Tx_n = fx_{n+1}$. Suppose that $y_{n+1} \neq y_n$ for all $n \in \mathbb{N}$, otherwise f and T have a coincidence point. Thus we get

$$\phi\left(\frac{1}{M(y_n,y_{n+1},t)} - 1\right) = \phi\left(\frac{1}{M(Tx_n,Tx_{n+1},t)} - 1\right)$$

$$\leq a\,\phi\left(\frac{1}{M(fx_n,fx_{n+1},t)} - 1\right)$$

$$+ b\,\phi\left(min\left\{\frac{1}{M(fx_n,Tx_n,t)}, \frac{1}{M(fx_n,Tx_{n+1},t)}\right\} - 1\right)$$

$$+ c\,\phi\left(min\left\{\frac{1}{M(Tx_n,Tx_{n+1},t)}, \frac{1}{M(Tx_n,fx_n,t)}, \frac{1}{M(Tx_{n+1},fx_{n+1},t)}\right\} - 1\right)$$

$$-\psi\left(min\left\{\begin{matrix}\frac{1}{M(fx_n,fx_{n+1},t)},\frac{1}{M(fx_n,Tx_n,t)},\frac{1}{M(fx_n,Tx_{n+1},t)},\\ \frac{1}{M(Tx_{n+1},fx_{n+1},t)}\end{matrix}\right\}-1\right)$$

$$\leq a\,\phi\left(\frac{1}{M(y_{n-1},y_n,t)}-1\right)+b\,\phi\left(min\left\{\frac{1}{M(y_{n-1},y_n,t)},\frac{1}{M(y_{n-1},y_{n+1},t)}\right\}-1\right)$$

$$+c\,\phi\left(min\left\{\frac{1}{M(y_n,y_{n+1},t)},\frac{1}{M(y_n,y_{n-1},t)},\frac{1}{M(y_{n+1},y_n,t)}\right\}-1\right)$$

$$-\psi\left(min\left\{\frac{1}{M(y_{n-1},y_n,t)},\frac{1}{M(y_{n-1},y_n,t)},\frac{1}{M(y_{n-1},y_{n+1},t)},\frac{1}{M(y_{n+1},y_n,t)}\right\}-1\right)$$

$$\leq a\,\phi\left(\frac{1}{M(y_{n-1},y_n,t)}-1\right)+b\,\phi\left(\frac{1}{M(y_{n-1},y_n,t)}-1\right)+c\,\phi\left(\frac{1}{M(y_n,y_{n-1},t)}-1\right)$$

$$-\psi\left(\frac{1}{M(y_{n-1},y_n,t)}-1\right)$$

$$\leq (a+b+c)\phi\left(\frac{1}{M(y_{n-1},y_n,t)}-1\right)$$

$$<\ \phi\left(\frac{1}{M(y_{n-1},y_n,t)}-1\right),\text{ where } a+b+c<1\ ,$$

which shows that the ϕ function is non-decreasing implies that

$M(y_n,y_{n+1},t)>M(y_{n-1},y_n,t)$for all $n\in\mathbb{N}$ and hence $\{M(y_{n-1},y_n,t)\}$is a increasing sequence of positive real numbers in (0,1].

Let $S(t)=\lim_{n\to\infty}M(y_{n-1},y_n,t)$, we prove that $S(t)=1$ for all $t>0$. Suppose, there exists

$t>0$ such that $S(t)<1$, then form the above inequality on taking $n\to\infty$, we get

$\phi\left(\frac{1}{S(t)}-1\right)\leq(a+b+c)\,\phi\left(\frac{1}{S(t)}-1\right)-\ \psi\left(\frac{1}{S(t)}-1\right)$ where $a+b+c<1$,

which is a contradiction. Therefore $M(y_n,y_{n+1},t)\to 1$ as $n\to\infty$.

Now for each positive integer p,

$M(y_n,y_{n+p},t)\geq M(y_n,y_{n+1},t/p)*M(y_{n+1},y_{n+2},t/p)*\ldots*M(y_{n+p-1},y_{n+p},t/p)$

It follows that $\lim_{n\to\infty}M(y_n,y_{n+p},t)\geq 1*1*\ldots*1=1$.

Also we have

$\phi(N(y_n,y_{n+1},t))=\phi(N(Tx_n,Tx_{n+1},t))$

$\leq a\,\phi(N(fx_n,fx_{n+1},t))$

$$+b\ \phi(max\{N(fx_{n,}Tx_n,t),N(fx_n,Tx_{n+1},t)\})$$

$$+c\ \phi(max\{N(Tx_n,Tx_{n+1},t),N(Tx_n,fx_n,t),N(Tx_{n+1},fx_{n+1},t)\})$$

$$-\psi\left(max\left\{\begin{matrix}N(fx_n,fx_{n+1},t),N(fx_n,Tx_n,t),N(fx_n,Tx_{n+1},t),\\ N(Tx_{n+1},fx_{n+1},t)\end{matrix}\right\}\right)$$

$$\leq a\ \phi(N(y_{n-1},y_n,t)) + b\ \phi(max\{N(y_{n-1},y_n,t),N(y_{n-1},y_{n+1},t))$$

$$+\ c\ \phi(max\{N(y_n,y_{n+1},t),N(y_n,y_{n-1},t),N(y_{n+1},y_n,t)\})$$

$$-\psi\left(max\left\{\begin{matrix}N(y_{n-1},y_n,t),N(y_{n-1},y_n,t),N(y_{n-1},y_{n+1},t),\\ N(y_{n+1},y_n,t)\end{matrix}\right\}\right)$$

$$\leq a\ \phi(N(y_{n-1},y_n,t)) + b\ \phi(N(y_{n-1},y_{n+1},t)) + c\ \phi(N(y_n,y_{n+1},t))$$

$$-\ \psi(N(y_n,y_{n+1},t))$$

$$< \phi(N(y_{n-1},y_n,t)),$$

which shows that the ϕ function is non-decreasing, implies that $N(y_n,y_{n+1},t) < N(y_{n-1},y_n,t)$ for all $n \in \mathbb{N}$ and hence $\{N(y_{n-1},y_n,t)\}$is a decreasing sequence of positive real numbers in [0,1).

Let $R(t) = \lim_{n\to\infty} N(y_{n-1},y_n,t)$, we prove that $R(t) = 0$ for all $t > 0$. Suppose, there exists

$t > 0$ such that$R(t) > 0$, then form the above inequality on taking $n \to \infty$, we get

$\phi(R(t) \leq (a + b + c)\phi(R(t)) - \psi(R(t))$, where $a + b + c < 1$,

which is a contradiction. Therefore $N(y_n,y_{n+1},t) \to 0$ as $n \to \infty$.

Now for each positive integer p,

$M(y_n,y_{n+p},t) + N(y_n,y_{n+p},t) \leq 1$,

and then $\lim_{n\to\infty}(M(y_n,y_{n+p},t) + N(y_n,y_{n+p},t)) \leq 1$.

Implies that $\lim_{n\to\infty} N(y_n,y_{n+p},t) = 0$.

Hence $\{y_n\}$ is a Cauchy sequence, and since $f(X)$ is complete, sequence $\{y_n\}$ converges to point q in $f(X)$. Similarly$T(X)$ is complete then there exists q in $T(X)$such that $\{y_n\}$ converges to q in $T(X)$.

Let$p \in X$ be an arbitrary point such that $fp = q$. Now we prove that p is a coincidence point of f and T. Then we have

$$\phi\left(\frac{1}{M(Tp,fx_{n+1},t)}-1\right) = \phi\left(\frac{1}{M(Tp,Tx_n,t)}-1\right)$$

$$\leq a\,\phi\left(\frac{1}{M(fp,fx_n,t)}-1\right)+b\,\phi\left(min\left\{\frac{1}{M(fp,Tp,t)},\frac{1}{M(fp,Tx_n,t)}\right\}-1\right)$$

$$+\,c\,\phi\left(min\left\{\frac{1}{M(Tp,Tx_n,t)},\frac{1}{M(Tp,fp,t)},\frac{1}{M(Tx_n,fx_n,t)}\right\}-1\right)$$

$$-\,\psi\,\left(min\left\{\frac{1}{M(fp,fx_n,t)},\frac{1}{M(fp,Tp,t)},\frac{1}{M(fp,Tx_n,t)},\frac{1}{M(Tx_n,fx_n,t)}\right\}-1\right)$$

For every $t>0$, which on taking $n\to\infty$, we have

$$\lim_{n\to\infty} M(Tp,fx_{n+1},t) = \lim_{n\to\infty} M(Tp,Tx_n,t)$$

$$= M(Tp,fp,t) = 1.$$

Hence $fp=Tp=q$.Now we show that $fq=q$. If it is not, then consider

$$\phi\left(\frac{1}{M(fq,q,t)}-1\right) \leq \phi\ \left(\frac{1}{M(Tq,Tp,t)}-1\right)$$

$$\leq a\,\phi\left(\frac{1}{M(fq,fp,t)}-1\right)+b\,\phi\left(min\left\{\frac{1}{M(fq,Tq,t)},\frac{1}{M(fq,Tp,t)}\right\}-1\right)$$

$$+c\phi\left(min\left\{\frac{1}{M(Tq,Tp,t)},\frac{1}{M(Tq,fq,t)},\frac{1}{M(Tp,fp,t)}\right\}-1\right)$$

$$-\psi\left(min\left\{\frac{1}{M(fq,fp,t)},\frac{1}{M(fq,Tq,t)},\frac{1}{M(fq,Tp,t)},\frac{1}{M(Tp,fp,t)}\right\}-1\right)$$

$$\leq a\phi\left(\frac{1}{M(fq,q,t)}-1\right)+b\,\phi\left(min\left\{\frac{1}{M(fq,Tq,t)},\frac{1}{M(fq,q,t)}\right\}-1\right)$$

$$+c\phi\left(min\left\{\frac{1}{M(Tq,q,t)},\frac{1}{M(Tq,fq,t)},\frac{1}{M(q,q,t)}\right\}-1\right)$$

$$-\psi\left(min\left\{\frac{1}{M(fq,q,t)},\frac{1}{M(fq,Tq,t)},\frac{1}{M(fq,q,t)},\frac{1}{M(q,q,t)}\right\}-1\right),$$

this yields a contradiction so that $fq=q$.

The uniqueness of fixed point follows from (9) and (10).This completes the proof.

Theorem 2.2. Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and $T\colon X\to X$ be generalized intuitionistic $(\phi,\,\psi)$-weak contraction of integral type with respect to $f\colon X\to X$. If the range of f contains the range of T and $f(X)$ or $T(X)$ is a complete subset of X then f and T have a unique common fixed point in X provided that ψ is a continuous function.

Proof. Define ξ : $[0,\infty)\to[0,\infty)$ by $\xi=\int_0^x \varphi(s)ds$. So that, condition (11) reduce to condition (7) and condition (12) reduce to condition (8) as $\phi \circ \xi$ is an altering distance function$\psi \circ \xi: [0, \infty) \rightarrow [0, \infty)$ with $\psi(\xi(r)) > 0$ for $r > 0$ and $\psi(\xi(0)) = 0$. Therefore, this theorem follows by Theorem 2.1.

REFERENCES

[1] M. Abbas, M. Imdad and D. Gopal, ϕ-weak contractions in fuzzy metric spaces, Iranian Journal of Fuzzy System 8 (2011), 141-148.

[2] K. Atanassov, Intuitionistic fuzzy sets, In : V. Sgurev editor, VII ITKR'sSession, So_a June, (1983), Central Sci. and Technol Library, BulgarianAcademy of Sciences, (1984).

[3] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986), 87-96.

[4] I. Beg, C. Vetro, D. Gopal and M. Imdad, (ϕ, ψ)-weak contraction in intuitionistic fuzzy metric spaces, Journal of Intelligent & Fuzzy System, 26 (2014), 2497-2504.

[5] A. Branciari, A fixed point theorem for mappings satisfying a general contractive condition of integral type, International Journal of Mathematics and Mathematical Sciences 29 (2006), 531-536.

[6] D. Doric, Common fixed point for generalized (ψ,φ)-weak contractions, Applied Mathematics Letters 22(2009), 1896-1900.

[7] P. N. Dutta and B. S. Choudhary, A generalization of contraction principle in metric spaces, Fixed Point Theory Appl. 2008, Article ID 406368(2008).

[8] A.George and P. Veeramani, On some result in fuzzy metric spaces, Fuzzy Sets Syst., 64 (1994), 395-399.

[9] M. Grabiec, Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems,27 (1988), 385-389.

[10] M. Imdad and J, Ali, A general fixed point theorem in fuzzy metric spaces via an implicit function, Journal of Applied Mathematics and Informatiche 26 (2010), 591-603.

[11] Z. Jiao, On fixed point theorem in intuitionistic fuzzy metric spaces, Journal of Applied Mathematics 2012(2012), Article ID 474983, 9.

[12] M. S. Khan, M. Swaleh and S. Sessa, Fixed point theorems by altering distances between the points, Bulletin of the Australian Mathematical Society 30 (1984), 1-9.

[13] O. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetica, (1975), 326-334.

[14] R. Lowen, Fuzzy set theory, Kluwer Academic Publishers, 1996, Dordrecht.

[15] D. Mihet, On fuzzy contractive mappings in fuzzy metric spaces, Fuzzy Sets and Systems 158 (2007), 915-921.

[16] A. Mohamad, Fixed-point theorems in Intuitionistic fuzzy metric spaces,Chaos, Solitons and Fractals, 34 (2007), 1689-1695.

[17] J. H. Park, Intuitionistic fuzzy metric spaces, Chaos, Solitons and Fractals, 22 (2004), 1039-1046.

[18] M. Rafi and M. S. M. Noorani, Fixed point theorems on intuitionistic fuzzy metric spaces, Iranian Journal of fuzzy System 3(2006), 23-29.

[19] B. E. Rhoades, Some theorems on weakly contractive maps, Nonlinear Analysis 47(2001), 2683-2693.

[20] I. Schweizer and A. Sklar, Statistical metric spaces, Pacific Journal of Mathematics 10 (1960), 314-334.

[21] A. Sugandhi, S. K. Tiwari and A. Pariya, Common fixed point theorems in intuitionistic fuzzy symmetric spaces for occasionally weakly compatible maps satisfying contractive condition of integral type, Annals of Pure and Applied Mathematics 11 (2016), 11-19.

[22] R. Vasuki and P. Veeramani, Fixed point theorems and Cauchy sequences in fuzzy metric spaces, Fuzzy Sets Syst, 135 (2003), 415-417.

[23] C. Vetro, On Branciari's theorem for weakly compatible mappings, Applied Mathematics Letters 23 (2010), 700-705.

[24] C. Vetro, D. Gopal and M. Imdad, Common fixed point theorems for (ϕ,ψ)-weakcontractions fuzzy metric spaces, Indian Journal of Mathematics 52(2010), 573-590.

[25] C. Vetro and P. Vetro, Common fixed points for discontinuous mappings in fuzzy metric spaces, Rendiconti del Circolo Matematico di Palermo 57(2008), 573-590.

[26] L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965), 338-353.