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Intuitionistic Fuzzy γ Generalized Continuous Mappings

Prema S[1] and Jayanthi D[2]

Department of Mathematics, Avinashilingam University,
Coimbatore, Tamil Nadu, India.

Abstract

In this paper we have introduced intuitionistic fuzzy γ generalized continuous mappings and investigated some of their properties. Also we have provided some characterization of intuitionistic fuzzy γ generalized continuous mappings.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy γ generalized $T_{1/2}$ space, intuitionistic fuzzy γ generalized continuous mappings.

1. INTRODUCTION

Atanassov [1] introduced intuitionistic fuzzy sets. Coker [2] introduced intuitionistic fuzzy topological spaces. Prema and Jayanthi[8] introduced intuitionistic fuzzy γ generalized closed sets. In this paper we have introduced intuitionistic fuzzy γ generalized continuous mappings and investigated some of their properties. Also we have provided some characterization of intuitionistic fuzzy γ generalized continuous mappings.

2. PRELIMINARIES

Definition 2.1: [1] An intuitionistic fuzzy set (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element x ∈ X to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each x ∈ X. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X. An intuitionistic fuzzy set A in X is simply denoted by A = ⟨x, μ_A, ν_A⟩ instead of denoting A = {⟨x, $\mu_A(x)$, $\nu_A(x)$⟩: x ∈ X}.

Definition 2.2: [1] Let A and B be two IFSs of the form A = {⟨x, $\mu_A(x)$, $\nu_A(x)$⟩: x ∈ X} and B = {⟨x, $\mu_B(x)$, $\nu_B(x)$⟩ : x ∈ X}. Then,

(a) A ⊆ B if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all x ∈ X,

(b) A = B if and only if A ⊆ B and A ⊇ B,

(c) A^c = {⟨x, $\nu_A(x)$, $\mu_A(x)$⟩: x ∈ X},

(d) A ∪ B = {⟨x, $\mu_A(x) \vee \mu_B(x)$, $\nu_A(x) \wedge \nu_B(x)$⟩: x ∈ X},

(e) A ∩ B = {⟨x, $\mu_A(x) \wedge \mu_B(x)$, $\nu_A(x) \vee \nu_B(x)$⟩: x ∈ X}.

The intuitionistic fuzzy sets $0_\sim$ = ⟨x, 0, 1⟩ and $1_\sim$ = ⟨x, 1, 0⟩ are respectively the empty set and the whole set of X.

Definition 2.3: [2] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i: i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [7] Let (X, τ) be an IFTS and A= ⟨x, μ_A, ν_A ⟩ be an IFS in X. Then the intuitionistic fuzzy γ interior and intuitionistic fuzzy γ closure are defined by

γint(A) = ∪{ G/ G is an IFγOS in X and G ⊆ A },

γcl(A) = ∩{K/ K is an IFγCS in X and A ⊆ K}.

Note that for any IFS A in (X, τ), we have $\gamma cl(A^c) = (\gamma int(A))^c$ and $\gamma int(A^c) = (\gamma cl(A))^c$.

Definition 2.5: [8] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy γ generalized closed set (IFγGCS for short) if γcl(A) ⊆ U whenever A ⊆ U and U is an IFγOS in (X, τ).

The complement A^c of an IFγGCS A in an IFTS (X, τ) is called an intuitionistic fuzzy γ generalized open set (IFγGOS for short) in X.

Definition 2.6: [4] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy continuous (IF continuous for short) mapping if $f^{-1}(B) \in$ IFO(X) for every $B \in \sigma$.

Definition 2.7: [8] An IFTS (X ,τ) is an intuitionistic fuzzy $\gamma_\gamma T_{1/2}$ (IF$\gamma_\gamma T_{1/2}$ in short) space if every IFγGCS is an IFγCS in X.

Definition 2.8: [3] An intuitionistic fuzzy point (IFP in short) written as $p_{(\alpha, \beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Theorem 2.9: [8] For any IFS A in (X ,τ) where X is an IFγ_γ $T_{1/2}$ space, A ∈ IFγGO(X) iff for every IFP $p_{(\alpha,\beta)} \in A$, there exist an IFγGOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

3. INTUITIONISTIC FUZZY γ GENERALIZED CONTINUOUS MAPPINGS

In this section we have introduced intuitionistic fuzzy γ generalized continuous mappings and investigated some of their properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy γ generalized continuous (IFγG continuous for short) mapping if $f^{-1}(V)$ is an IFγGCS in (X, τ) for every IFCS V of (Y, σ).

For the sake of simplicity, we shall use the notation A = ⟨x, (μ_a, μ_b), (ν_a, ν_b)⟩ instead of A = ⟨x, (a/μ_a, b/μ_b), (a/ν_a, b/ν_b)⟩ in the following examples. Similarly we shall use the notation B = ⟨y, (μ_u, μ_v), (ν_u, ν_v)⟩ instead of B = ⟨y, (u/μ_u, v/μ_v), (u/ν_u, v/ν_v)⟩ in the following examples.

Example 3.2: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩, G_2 = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩, G_3 = ⟨y, (0.7_u, 0.8_v), (0.3_u, 0.2_v)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS G_3^c = ⟨y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)⟩ is an IFCS in Y. Then $f^{-1}(G_3^c)$ = ⟨x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)⟩ is an IFS in X.

IFγO(X)= {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\nu_a < 0.4$ or $\nu_b < 0.3$, $\nu_a \geq 0.5$ whenever $\nu_b < 0.6$, $0.4 \leq \nu_a \leq 0.5$ whenever $\nu_b \leq 0.4$, $\mu_a \geq 0.5$, $\mu_b \geq 0.6$, $0.5 \leq \nu_a < 0.6$ whenever $\nu_b \geq 0.6$, $\mu_a \geq 0.4$, $\mu_b \geq 0.3$ and $\nu_a \geq 0.6$ whenever $\nu_b \geq 0.4$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\mu_a < 0.4$ or $\mu_b < 0.3$, $\mu_a \geq 0.5$ whenever $\mu_b < 0.6$, $0.4 \leq \mu_a \leq 0.5$ whenever $\mu_b \leq 0.4$, $\nu_a \geq 0.5$, $\nu_b \geq 0.6$, $0.5 \leq \mu_a < 0.6$ whenever $\mu_b \geq 0.6$, $\nu_a \geq 0.4$, $\nu_b \geq 0.3$ and $\mu_a \geq 0.6$ whenever $\mu_b \geq 0.4$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Hence $f^{-1}(G_3^c)$ is an IFγGCS in (X, τ). Therefore f is an IFγG continuous mapping.

Theorem 3.3: Every IF continuous mapping[4] is an IFγG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF continuous mapping . Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IFγGCS[8], $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.4: In Example 3.2, f is an IFγG continuous mapping, but since $f^{-1}(G_3^c)$ is not an IFCS in X, as $cl(f^{-1}(G_3^c)) = G_1^c \neq f^{-1}(G_3^c)$, f is not an IF continuous mapping.

Theorem 3.5: Every IFS continuous mapping[6] is an IFγG continuous mapping in (X, τ) but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFS continuous mapping . Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IFγGCS[8], $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.6: In Example 3.2, f is an IFγG continuous mapping, but since $f^{-1}(G_3^c)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3^c))) = int(G_1^c) = G_2 \not\subseteq f^{-1}(G_3^c)$, f is not an IFS continuous mapping.

Theorem 3.7: Every IFP continuous mapping[6] is an IFγG continuous mapping in (X, τ) but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFP continuous mapping . Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFγGCS[8], $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.8: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_3^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

IFγO(X)= $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$/ either $\nu_a < 0.4$ or $\nu_b < 0.3$, $\nu_a \geq 0.5$ whenever $\nu_b < 0.6$, $0.4 \leq \nu_a \leq 0.5$ whenever $\nu_b \leq 0.4$, $\mu_a \geq 0.5$, $\mu_b \geq 0.6$, $0.5 \leq \nu_a < 0.6$ whenever $\nu_b \geq 0.6$, $\mu_a \geq 0.4$, $\mu_b \geq 0.3$ and $\nu_a \geq 0.6$ whenever $\nu_b \geq 0.4$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$.

Then, IFγC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0, 1], \nu_a \in [0,1], \nu_b \in [0,1]$/ either $\mu_a < 0.4$ or $\mu_b < 0.3$, $\mu_a \geq 0.5$ whenever $\mu_b < 0.6$, $0.4 \leq \mu_a \leq 0.5$ whenever $\mu_b \leq 0.4$, $\nu_a \geq 0.5$, $\nu_b \geq 0.6$, $0.5 \leq \mu_a < 0.6$ whenever $\mu_b \geq 0.6$, $\nu_a \geq 0.4$, $\nu_b \geq 0.3$ and $\mu_a \geq 0.6$ whenever $\mu_b \geq 0.4$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$.

Then f is an IFγG continuous mapping, but since $f^{-1}(G_3^c)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3^c))) = cl(G_2) = G_1^c \nsubseteq f^{-1}(G_3^c)$, f is not an IFP continuous mapping.

Theorem 3.9: Every IFα continuous mapping[6] is an IFγG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFα continuous mapping . Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IFγGCS[8], $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.10: In Example 3.2, f is an IFγG continuous mapping, but not an IFα continuous mapping, since $f^{-1}(G_3^c)$ is not an IFαCS in X, as $cl(int(cl(f^{-1}(G_3^c)))) = cl(int(G_1^c)) = cl(G_2) = G_1^c \nsubseteq f^{-1}(G_3^c)$.

Theorem 3.11: Every IFγ continuous mapping[5] is an IFγG continuous mapping but not conversely in general.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be an IFγ continuous mapping . Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFγCS in X. Since every IFγCS is an IFγGCS[8], $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.12: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS $G_3^c = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)\rangle$ is an IFS in X. Then

IFγO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]/$ either $\nu_a < 0.5$ or $\nu_b < 0.6$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$.

Then IFγC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]/$ either $\mu_a < 0.5$ or $\mu_b < 0.6$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$.

Hence $f^{-1}(G_3^c)$ is an IFγGCS in (X, τ). Therefore f is an IFγG continuous mapping. But since $f^{-1}(G_3^c)$ is not an IFγCS in X, as $int(cl(f^{-1}(G_3^c))) \cap cl(int(f^{-1}(G_3^c))) = 1_\sim \cap 1_\sim = 1_\sim \not\subseteq f^{-1}(G_3^c)$, f is not an IFγ continuous mapping.

Theorem 3.13: Every IFSP continuous mapping[9] is an IFγG continuous mapping but not conversely in general.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be an IFSP continuous mapping . Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSPCS in X. Since every IFSPCS is an IFγGCS[8], $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.14: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS $G_3^c = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)\rangle$ is an IFS in X. Then

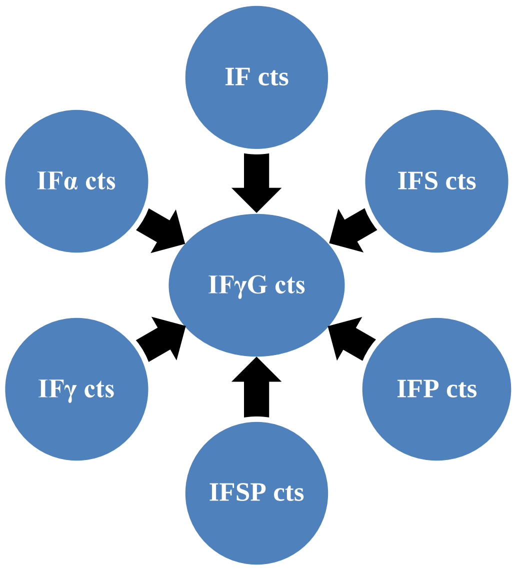
IFγO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]/$ either $\nu_a < 0.5$ or $\nu_b < 0.6$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$.

Then IFγC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]/$ either $\mu_a < 0.5$ or $\mu_b < 0.6$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$.

Hence $f^{-1}(G_3^c)$ is an IFγGCS in (X, τ). Therefore f is an IFγG continuous mapping.

Since IFPC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1]/$ either $\mu_a < 0.5$ or $\mu_b < 0.6$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$, there exists no IFPCS B in X such that int(B) $\subseteq$ f $^{-1}$ (G_3^c) $\subseteq$ B in X, f $^{-1}$ (G_3^c) is not an IFSPCS in X. Hence f is not an IFSP continuous mapping.

The relation between various type of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts' means continuous.



Theorem 3.15: A mapping f: (X, τ) → (Y, σ) is an IFγG continuous mapping if and only if the inverse image of each IFOS in Y is an IFγGOS in X.

Proof: Necessity: Let A be an IFOS in Y. This implies A^c is an IFCS in Y. Then f $^{-1}$ (A^c) is an IFγGCS in X, by hypothesis. Since f $^{-1}$ (A^c) = (f $^{-1}$ (A))^c, f $^{-1}$ (A) is an IFγGOS in X.

Sufficiency: Let A be an IFCS in Y. Then A^c is an IFOS in Y. By hypothesis f $^{-1}$ (A^c) is IFγGOS in X. Since f $^{-1}$ (A^c) = (f $^{-1}$ (A))^c , (f $^{-1}$ (A))^c is an IFγGOS in X. Therefore f $^{-1}$ (A) is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Theorem 3.16: If f: (X, τ) → (Y, σ) is an IFγG continuous mapping then for each IFP $p_{(\alpha,\beta)}$ of X and each A ∈ σ such that f($p_{(\alpha,\beta)}$) $_q$ A, there exists an IFγGOS B of X such that $p_{(\alpha,\beta)}$ $_q$ B and f(B) ⊆ A.

Proof: Let $p_{(\alpha,\beta)}$ be an IFP of X and A ∈ σ such that f($p_{(\alpha,\beta)}$) $_q$ A. Put B = f $^{-1}$ (A). Then by hypothesis, B is an IFγGOS in X such that $p_{(\alpha,\beta)}$ $_q$ B and f(B) = f (f $^{-1}$ (A)) ⊆ A.

Theorem 3.17: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an IFγG continuous mapping, then f is an IFγ continuous mapping if X is an IFγ_{γ} $T_{1/2}$ space.

Proof: Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFγGCS in X, by hypothesis. Since X is an IF$\gamma_{\gamma}T_{1/2}$ space, $f^{-1}(V)$ is an IFγCS in X. Hence f is an IFγ continuous mapping.

Theorem 3.18: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an IFγG continuous mapping, then f is an IF continuous mapping if X is an IFγ_c $T_{1/2}$ space.

Proof: Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFγGCS in X, by hypothesis. Since X is an IFγ_c $T_{1/2}$ space, $f^{-1}(V)$ is an IFCS in X. Hence f is an IF continuous mapping.

Theorem 3.19: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an IFγG continuous mapping and g: $(Y, \sigma) \rightarrow (Z, \delta)$ is an IF continuous mapping then $g \, o \, f : (X, \tau) \rightarrow (Z, \delta)$ is an IFγG continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFCS in Y, by hypothesis. Since f is an IFγG continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFγGCS in X. Hence $g \, o \, f$ is an IFγG continuous mapping.

Theorem 3.20: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IFγ_{γ} $T_{1/2}$ spaces:

(i) f is an IFγG continuous mapping

(ii) $f^{-1}(B)$ is an IFγGOS in X for each IFOS B in Y

(iii) for each IFP $p_{(\alpha,\beta)}$ in X and for every IFOS B in Y such that $f(p_{(\alpha,\beta)}) \in B$, there exists an IFγGOS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

Proof: (i) ⇒ (ii) is obvious from the Theorem 3.15.

(ii) ⇒ (iii) Let B be any IFOS in Y and let $p_{(\alpha,\beta)} \in X$. Given $f(p_{(\alpha,\beta)}) \in B$. By hypothesis $f^{-1}(B)$ is an IFγGOS in X. Take $A = f^{-1}(B)$. Then $p_{(\alpha,\beta)} \in f^{-1}(B) = A$. This implies $p_{(\alpha,\beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) ⇒ (i) Let A be an IFCS in Y. Then its complement, say B is an IFOS in Y. Let $p_{(\alpha,\beta)} \in X$ and $f(p_{(\alpha,\beta)}) \in B$. Then there exists an IFγGOS, say C in X such that $p_{(\alpha,\beta)} \in C$ and $f(C) \subseteq B$. Therefore $p_{(\alpha,\beta)} \in C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$ and hence $f^{-1}(B)$ is an IFγGOS in X[8]. That is $f^{-1}(A^c)$ is an IFγGOS in X and hence $f^{-1}(A)$ is an IFγGCS in X. Thus f is an IFγG continuous mapping.

Theorem 3.21: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an $IF\gamma_\gamma T_{1/2}$ space:

(i) f is an IFγG continuous mapping,

(ii) If B is an IFOS in Y then $f^{-1}(B)$ is an IFγGOS in X,

(iii) $f^{-1}(int(B)) \subseteq (int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))))$ for every IFS B in Y.

Proof: (i) ⇒ (ii) is obviously true by Theorem 3.15.

(ii) ⇒ (iii) Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IFγGOS in X. Since X is an $IF\gamma_\gamma T_{1/2}$ space, $f^{-1}(int(B))$ is an IFOS in X and hence an IFγOS in X. Therefore $f^{-1}(int(B)) \subseteq (int(cl(f^{-1}(intB))) \cap cl(int(f^{-1}(intB)))) \subseteq (int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))))$.

(iii) ⇒ (i) Let B be an IFCS in Y. Then its complement, say A is an IFOS in Y, then int(A) = A. Now by hypothesis $f^{-1}(int(A)) \subseteq (int(cl(f^{-1}(A))) \cap cl(int(f^{-1}(A))))$.This implies $f^{-1}(A) \subseteq (int(cl(f^{-1}(A))) \cap cl(int(f^{-1}(A))))$. Hence $f^{-1}(A)$ is an IFγOS in X. Since every IFγOS is an IFγGOS, $f^{-1}(A)$ is an IFγGOS in X. Thus $f^{-1}(B)$ is an IFγGCS in X, since $f^{-1}(A) = f^{-1}(B^c)$. Hence f is an IFγG continuous mapping.

Theorem 3.22: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IF\gamma_\gamma T_{1/2}$ spaces:

(i) f is an IFγG continuous mapping,

(ii) $(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B)))) \subseteq f^{-1}(\gamma cl(B))$ for each IFCS B in Y,

(iii) $f^{-1}(\gamma int(B)) \subseteq (int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B))))$ for each IFOS B of Y,

(iv) $f(int(cl(A)) \cap cl(int(A))) \subseteq cl(f(A))$ for each IFS A of X.

Proof: (i) ⇒ (ii) Let B be an IFCS in Y. Then $f^{-1}(B)$ is an IFγGCS in X, by hypothesis. Since X is an $IF\gamma_\gamma T_{1/2}$ space, $f^{-1}(B)$ is an IFγCS in X. Therefore $(int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B)))) \subseteq f^{-1}(B) = f^{-1}(\gamma cl(B))$.

(ii) ⇒ (iii) can be easily proved by taking complement in (ii).

(iii) ⇒ (iv) Let A ∈ X. Then B = f(A) in Y and therefore $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Here int(f(A)) = int(B) is an IFOS in Y. Then (iii) implies that $f^{-1}(\gamma int(int(B))) \subseteq (int(cl(f^{-1}(int(B)))) \cup cl(int(f^{-1}(int(B))))) \subseteq (int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B))))$. Now $(int(cl(A^c)) \cup cl(int(A^c)))^c \subseteq (int(cl(f^{-1}(B)^c)) \cup cl(int(f^{-1}(B)^c)))^c \subseteq (f^{-1}(\gamma int(int(B^c))))^c$. Therefore $int(cl(A)) \cap cl(int(A)) \subseteq f^{-1}(\gamma cl(cl(B)))$. Now $f(int(cl(A)) \cap cl(int(A))) \subseteq f(f^{-1}(\gamma cl(cl(B)))) \subseteq cl(B) = cl(f(A))$.

(iv) ⇒ (i) Let B be any IFCS in Y, then $f^{-1}(B)$ is an IFS in X. By hypothesis $f(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B)))) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B) = B$. Now $(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}$

^{1}(B))))⊆ f $^{-1}$ (f((int(cl(f $^{-1}$(B))) ∩ cl(int(f $^{-1}$(B)))))) ⊆ f $^{-1}$ (B). This implies f $^{-1}$ (B) is an IFγCS and hence it is an IFγGCS in X. Thus f is an IFγG continuous mapping.

Theorem 3.23: A mapping f: (X, τ) → (Y, σ) is an IFγG continuous mapping if cl(int(cl(f $^{-1}$ (A)))) ⊆ f $^{-1}$ (cl(A)) for every IFS A in Y.

Proof: Let A be an IFOS in Y then A^c is an IFCS in Y. By hypothesis, cl(int(cl(f $^{-1}$(A^c)))) ⊆ f $^{-1}$ (cl(A^c)) = f $^{-1}$ (A^c). Now (int(cl(int(f $^{-1}$ (A)))))^c = cl(int(cl(f $^{-1}$ (A^c)))) ⊆ f $^{-1}$ (A^c) = (f $^{-1}$ (A))^c. This implies f $^{-1}$ (A) ⊆ int(cl(int(f $^{-1}$ (A)))). Hence f $^{-1}$ (A) is an IFαOS and hence it is an IFγGOS. Therefore f is an IFγG continuous mapping, by Theorem 3.15.

Theorem 3.24: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an IFγ_γ $T_{1/2}$ space:

(i) f is an IFγG continuous mapping,

(ii) f $^{-1}$ (B) is an IFγGCS in X for every IFCS B in Y,

(iii) (int(cl(f $^{-1}$ (A))) ∩ cl(int(f $^{-1}$ (A)))) ⊆ f $^{-1}$ (cl(A)) for every IFS A in Y.

Proof: (i) ⇒ (ii) is obvious from Definition 3.1.

(ii) ⇒ (iii) Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, f $^{-1}$ (cl(A)) is an IFγGCS in X. Since X is an IFγ_γ $T_{1/2}$ space, f $^{-1}$ (cl(A)) is an IFγCS. Therefore (int(cl(f $^{-1}$ (cl(A)))) ∩ cl(int(f $^{-1}$ (cl(A))))) ⊆ f $^{-1}$ (cl(A)). Now (int(cl(f $^{-1}$ (A))) ∩ cl(int(f $^{-1}$ (A)))) ⊆ (int(cl(f $^{-1}$ (cl(A)))) ∩ cl(int(f $^{-1}$ (cl(A))))) ⊆ f $^{-1}$ (cl(A)).

(iii) ⇒ (i) Let A be an IFCS in Y. By hypothesis (int(cl(f $^{-1}$ (A))) ∩ cl(int(f $^{-1}$ (A)))) ⊆ f $^{-1}$ (cl(A)) = f $^{-1}$ (A). This implies f $^{-1}$ (A) is an IFγCS in X and hence it is an IFγGCS. Thus f is an IFγG continuous mapping.

Theorem 3.25: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y that satisfies f $^{-1}$(int(B)) = int(cl(f $^{-1}$(B))) for every IFS B in Y. Then f is an IFγG continuous mapping.

Proof: Let B be an IFOS in Y. Then int(B) = B, by hypothesis f $^{-1}$(B) = int(cl(f $^{-1}$(B))). This implies f $^{-1}$(B) is an IFROS in X. Therefore it is an IFγGOS in X. Hence f is an IFγG continuous mapping.

Theorem 3.26: Let f: (X, τ) → (Y, σ) be an IFγG continuous mapping, then f is an IFP continuous mapping if X is an IFγ_p $T_{1/2}$ space.

Proof: Since every IFγGCS is an IFPCS in an IF$\gamma_p T_{1/2}$ space, the proof is obvious.

Theorem 3.27: If f: (X, τ) → (Y, σ) is an IFγG continuous mapping and g: (Y, σ) → (Z, δ) is an IFG continuous mapping and Y is an IF$T_{1/2}$ space, then g *o* f : (X, τ) → (Z, δ) is an IFγG continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFGCS in Y, by hypothesis. Since Y is an IF$T_{1/2}$ space, $g^{-1}(V)$ is an IFCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is an IFγGCS in X, by hypothesis. Hence g *o* f is an IFγG continuous mapping.

Theorem 3.28: Let f : (X, τ) → (Y, σ) be an IFγG continuous mapping, then

(i) f is an IF continuous mapping if X is an IF$\gamma_c T_{1/2}$ space

(ii) f is an IFP continuous mapping if X is an IF$\gamma_p T_{1/2}$ space

Proof: (i) Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFγGCS in X, by hypothesis. Since X is an IF$\gamma_c T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous mapping.

(ii) Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFγGCS in X, by hypothesis. Since X is an IF$\gamma_p T_{1/2}$ space, $f^{-1}(A)$ is an IFPCS in X. Hence f is an IFP continuous mapping.

Theorem 3.29: Let f : (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if X is an IF$\gamma_c T_{1/2}$ space:

(i) f is an IFγG continuous mapping

(ii) for each IFP $p_{(\alpha,\beta)}$ in X and for every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFγGOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$

(iii) for each IFP $p_{(\alpha,\beta)}$ in X and for every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFγGOS B in X such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$

Proof: (i) ⇒ (ii) Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since f is an IFγG continuous mapping, $f^{-1}(C)$ = B (say), is an IFγGOS in X and $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(C) \subseteq f^{-1}(A)$. Therefore $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.

(ii) ⇒ (iii) Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFγGOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$, by hypothesis. Therefore $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(iii) ⇒ (i) Let B be an IFOS in Y and let $p_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(p_{(\alpha,\beta)}) \in B$. Therefore B is an IFN of $f(p_{(\alpha,\beta)})$. Then by hypothesis there exists an IFγGOS A in X such that

$p_{(\alpha,\beta)} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(B)$ is an IFγGOS [8] in X. Hence f is an IFγG continuous mapping.

Definition 3.30: Let (X, τ) be an IFTS. The γ generalized closure (γgcl(A) in short) for any IFS A is defined as follows.

$\gamma gcl(A) = \cap \{K / K$ is an IFγGCS in X and $A \subseteq K\}$

Remark 3.31: If A is an IFγGCS, then γgcl(A) = A, but the converse is not true as in does not exist for IFγGCS.

Theorem 3.32: Let f: (X, τ) → (Y, σ) be an IFγG continuous mapping. Then the following conditions hold:

(i) $f(\gamma gcl(A)) \subseteq cl(f(A))$, for every IFS A in X.

(ii) $\gamma gcl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, for every IFS B in Y.

Proof: (i) Let A be any IFS in X. Then f(A) is an IFS in Y and cl(f(A)) is an IFCS in Y. Since f is an IFγG continuous mapping, $f^{-1}(cl(f(A)))$ is an IFγGCS in X. That is $\gamma gcl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$. Therefore $f(\gamma gcl(A)) \subseteq f(\gamma gcl(f^{-1}(f(A)))) \subseteq f(\gamma gcl(f^{-1}(cl(f(A)))) \subseteq f(f^{-1}(cl(f(A))) \subseteq cl(f(A))$.

(ii) Replacing A by $f^{-1}(B)$ in (i), we get $f(\gamma gcl(f^{-1}(B))) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B)$. Hence $\gamma gcl(f^{-1}(B)) \subseteq f^{-1}(f(\gamma gcl(f^{-1}(B))) \subseteq f^{-1}(cl(B))$, for every IFS B in Y.

4. INTUITIONISTIC FUZZY γ GENERALIZED IRRESOLUTE MAPPINGS

In this section we have introduced intuitionistic fuzzy γ generalized irresolute mappings and studied some of their properties.

Definition 4.1: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy γ generalized irresolute (IFγG irresolute for short) mapping if $f^{-1}(V)$ is an IFγGCS in (X, τ) for every IFγGCS V of (Y, σ).

Example 4.2: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.5_b), (0.4_a, 0.4_b)\rangle$ $G_3 = \langle y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then

$IF\gamma O(X) = \{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]/$ either $\nu_a < 0.5$ or $\nu_b < 0.5$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$.

IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\mu_a < 0.5$ or $\mu_b < 0.5$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1$}.

IFγO(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$/ either $\nu_u < 0.5$ or $\nu_v < 0.4$, $\nu_u \ge 0.5$ whenever $\nu_v \ge 0.6$, $0.5 \le \nu_u \le 0.6$ whenever $0.4 < \nu_v < 0.6$ and $\mu_u \ge 0.5$, $\mu_v \ge 0.4$, $0 \le \mu_u + \nu_u \le 1$ and $0 \le \mu_v + \nu_v \le 1$}.

IFγC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0, 1]$, $\mu_v \in [0, 1]$, $\nu_u \in [0, 1]$, $\nu_v \in [0, 1]$/ either $\mu_u < 0.5$ or $\mu_v < 0.4$, $\mu_u \ge 0.5$ whenever $\mu_v \ge 0.6$, $0.5 \le \mu_u < 0.6$ whenever $0.4 \le \mu_v < 0.6$ and $\nu_u \ge 0.5$, $\nu_v \ge 0.4$, $0 \le \mu_u + \nu_u \le 1$ and $0 \le \mu_v + \nu_v \le 1$}.

Let A = ⟨y, (0.4_u, 0.4_v), (0.4_u, 0.6_v)⟩. Then A is an IFγGCS in Y and f^{-1}(A) = ⟨x, (0.4_a, 0.4_b), (0.4_a, 0.6_b)⟩ is also an IFγGCS in (X, τ). Therefore f is an IFγG irresolute mapping.

Theorem 4.3: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping, then f is an IFγG continuous mapping but not conversely in general.

Proof: Let f be an IFγG irresolute mapping. Let V be any IFCS in Y. Then V is an IFγGCS in Y and by hypothesis f^{-1}(V) is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 4.4: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)⟩, G_2 = ⟨x, (0.5_a, 0.5_b), (0.4_a, 0.4_b)⟩ G_3 = ⟨y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)⟩. Then τ = {$0_\sim$, G_1, G_1, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then

IFγO(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\nu_a < 0.5$ or $\nu_b < 0.5$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1$}.

IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\mu_a < 0.5$ or $\mu_b < 0.5$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1$}.

IFγO(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$/ either $\nu_u < 0.5$ or $\nu_v < 0.4$, $\nu_u \ge 0.5$ whenever $\nu_v \ge 0.6$, $0.5 \le \nu_u \le 0.6$ whenever $0.4 < \nu_v < 0.6$ and $\mu_u \ge 0.5$, $\mu_v \ge 0.4$, $0 \le \mu_u + \nu_u \le 1$ and $0 \le \mu_v + \nu_v \le 1$}.

IFγC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0, 1]$, $\mu_v \in [0, 1]$, $\nu_u \in [0, 1]$, $\nu_v \in [0, 1]$/ either $\mu_u < 0.5$ or $\mu_v < 0.4$, $\mu_u \ge 0.5$ whenever $\mu_v \ge 0.6$, $0.5 \le \mu_u < 0.6$ whenever $0.4 \le \mu_v < 0.6$ and $\nu_u \ge 0.5$, $\nu_v \ge 0.4$, $0 \le \mu_u + \nu_u \le 1$ and $0 \le \mu_v + \nu_v \le 1$}.

Then f is an IFγG continuous mapping but not an IFγG irresolute mapping, since the IFS A = ⟨y, (0.5_u, 0.5_v), (0.5_u, 0.4_v)⟩ is an IFγGCS in Y but f^{-1}(A) = ⟨x, (0.5_a, 0.5_b),

$(0.5_a, 0.4_b)\rangle$ is not an IFγGCS in X, since $\gamma cl(f^{-1}(A)) = 1_\sim \nsubseteq G_1, G_2$, but $f^{-1}(A) \subseteq G_1, G_2$.

Theorem 4.5: A mapping f: $(X, \tau) \to (Y, \sigma)$ is an IFγG irresolute mapping if and only if the inverse image of each IFγGOS in Y is an IFγGOS in X.

Proof: Necessity: Let A be an IFγGOS in Y. This implies A^c is an IFγGCS in Y. Then $f^{-1}(A^c)$ is an IFγGCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFγGOS in X.

Sufficiency: Let A be an IFγGCS in Y. Then A^c is an IFγGOS in Y. By hypothesis $f^{-1}(A^c)$ is IFγGOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an IFγGOS in X. Therefore $f^{-1}(A)$ is an IFγGCS in X. Hence f is an IFγG irresolute mapping.

Theorem 4.6: Let f: $(X, \tau) \to (Y, \sigma)$ and g: $(Y, \sigma) \to (Z, \delta)$ be IFγG irresolute mappings then g *o* f : $(X, \tau) \to (Z, \delta)$ is an IFγG irresolute mapping.

Proof: Let V be an IFγGCS in Z. Then $g^{-1}(V)$ is an IFγGCS in Y. Since f is an IFγG irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFγGCS in X, by hypothesis. Hence g *o* f is an IFγG irresolute mapping.

Theorem 4.7: Let f: $(X, \tau) \to (Y, \sigma)$ be an IFγG irresolute mapping and g: $(Y, \sigma) \to (Z, \delta)$ is an IFγG continuous mapping then g *o* f : $(X, \tau) \to (Z, \delta)$ is an IFγG continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFγGCS in Y. Since f is an IFγG irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFγGCS in X. Hence g *o* f is an IFγG continuous mapping.

Theorem 4.8: Let f: $(X, \tau) \to (Y, \sigma)$ be an IFγG continuous mapping and g: $(Y, \sigma) \to (Z, \delta)$ be an IFγG irresolute mapping then g *o* f : $(X, \tau) \to (Z, \delta)$ is an IFγG irresolute mapping, where Y is an $IF\gamma_cT_{1/2}$ space.

Proof: Let V be an IFγGCS in Z. Then $g^{-1}(V)$ is an IFγGCS in Y as g is an IFγG irresolute. Since Y is an $IF\gamma_cT_{1/2}$space, $g^{-1}(V)$ is an IFCS in Y. Since f is an IFγG continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFγGCS in X. Hence g *o* f is an IFγG irresolute mapping.

Theorem 4.9: Let f: $(X, \tau) \to (Y, \sigma)$ be an IFγG continuous mapping then f is an IFγG irresolute mapping if (Y, σ) be an $IF\gamma_cT_{1/2}$space.

Proof: Let F be an IFγGCS in Y. Since Y is an IF$\gamma_c T_{1/2}$space, F is an IFCS in Y. By hypothesis f^{-1} (F) is an IFγGCS in X. Therefore f is an IFγG irresolute mapping.

Theorem 4.10: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping. Then f^{-1}(B) ⊆γint (f^{-1}(int(cl(B)))) for every IFγGOS B in Y, if X and Y are IF$\gamma_p T_{1/2}$ spaces.

Proof: Let B be IFγGOS in Y. Then by hypothesis f^{-1} (B) is an IFγGOS in X. Since X is an IF$\gamma_p T_{1/2}$ space, f^{-1} (B) is an IFPOS in X. Since every IFPOS is an IFγOS, γint(f^{-1}(B)) = f^{-1}(B). Since Y is an IF$\gamma_p T_{1/2}$ space, B is an IFPOS in Y and B ⊆ int(cl(B)). Hence f^{-1} (B) = γint(f^{-1} (B)) ⊆ γint(f^{-1}(int(cl(B)))).

Theorem 4.11: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IF$\gamma_\gamma T_{1/2}$ space:

(i) f is an IFγG irresolute mapping
(ii) f^{-1} (B) is an IFγGOS in X for every IFγGOS B in Y
(iii) f^{-1} (γint(B)) ⊆ γint(f^{-1} (B)) for every IFS B in Y
(iv) γcl(f^{-1} (B)) ⊆ f^{-1} (γcl(B)) for every IFS B in Y

Proof: (i) ⇒ (ii) is obvious from Theorem 4.5.

(ii) ⇒ (iii) Let A be an IFS in Y. Since γint(A) is an IFγOS in Y, it is an IFγGOS in Y. Therefore f^{-1} (γint(A)) is an IFγGOS in X, by hypothesis. Since X is an IF$\gamma_\gamma T_{1/2}$ space, f^{-1} (γint(A)) is an IFγOS in X. Hence f^{-1} (γint(A)) = γint(f^{-1} (γint(A))) ⊆ γint(f^{-1} (A)).

(iii) ⇒ (iv) it is obvious by taking complement in (iii).

(iv) ⇒ (i) Let A be an IFγGCS in Y. Since Y is an IF$\gamma_\gamma T_{1/2}$ space, A is an IFγCS in Y and γcl(B) = B. Hence f^{-1} (A) = f^{-1} (γcl(A)) ⊇ γcl(f^{-1} (A)) ⊇ f^{-1} (A). Therefore γcl(f^{-1}(A)) = f^{-1} (A). This implies f^{-1} (A) is an IFγCS and hence it is an IFγGCS in X. Thus f is an IFγG irresolute mapping.

Theorem 4.12: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping. Then f is an IF pre irresolute mapping, if (X, τ) is an IF$\gamma_p T_{1/2}$ space.

Proof: Let B be an IFPCS inY. Then B is an IFγGCS in Y. Since f is an IFγG irresolute mapping, f^{-1}(B) is an IFγGCS in X. Since X is an IF$\gamma_p T_{1/2}$ space, f^{-1}(B) is an IFPCS in X. Hence f is an IF pre irresolute mapping.

Theorem 4.13: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping.Then f^{-1}(B) ⊆ γint (f^{-1}(int(cl(B)) ∪ cl(int(B)))) for every IFγGOS B in Y, if X and Y are IF$\gamma_\gamma T_{1/2}$ spaces.

Proof: Let B be IFγGOS in Y. Then by hypothesis f^{-1} (B) is an IFγGOS in X. Since X is an IFγ$_\gamma$T$_{1/2}$ space, f^{-1} (B) is an IFγOS in X. Therefore γint(f^{-1}(B)) = f^{-1}(B). Since Y is an IFγ$_\gamma$T$_{1/2}$ space, B is an IFγOS in Y and B ⊆ int(cl(B)) ∪ cl(int(B)). Now f^{-1} (B) = γint(f^{-1} (B)) ⊆ γint (f^{-1}(int(cl(B)) ∪ cl(int(B)))).

Theorem 4.14: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping. Then f^{-1}(B) ⊆ γint(f^{-1}(int(cl(f^{-1}(B))) ∪ cl(int(f^{-1}(B))))) for every IFγGOS B in Y, if X and Y are IFγ$_\gamma$T$_{1/2}$ spaces.

Proof: Let B be IFγGOS in Y. Then by hypothesis f^{-1} (B) is an IFγGOS in X. Since X is IFγ$_\gamma$T$_{1/2}$ space, f^{-1} (B) is an IFγOS in X. Therefore γint(f^{-1}(B)) = f^{-1}(B) and f^{-1}(B) ⊆ (int(cl(f^{-1}(B))) ∪ cl(int (f^{-1}(B)))). Hence f^{-1}(B) = γint(f^{-1}(B)) ⊆ γint(f^{-1}(int(cl(f^{-1}(B))) ∪ cl(int(f^{-1}(B))))).

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