Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 12, Number 4 (2017), pp. 1007-1015

http://www.ripublication.com

Intuitionistic Fuzzy Subfields of a Field with Respect to (T,S)-norm

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Abstract

In this paper, we made an attempt to study the algebraic nature of intuitionistic fuzzy subfield of a field with respect to (T,S)-norm.

Keywords: **T**-norm, S-norm, fuzzy subset, intuitionistic fuzzy subset, fuzzy subfield, intuitionistic fuzzy subfield of a field with respect to (T,S)-norm.

INTRODUCTION

Most of the problems in engineering, medical science, economics, environments, etc. have various uncertainties. To exceed these uncertainties, some kind of theories were given like theory of fuzzy sets, intuitionistic fuzzy sets and so on. Fuzzy set was introduction by L.A.Zadeh[24], several researchers explored on the generalization of the concept of fuzzy sets. Prof. K.T. Atanassov, a Bulgarian Engineer, introduced a new component which determines the degree of non-membership also in defining Intuitionistic fuzzy Subset(IFS) theory. In 1983, he came across A.Kauffmann's book ''Introduction to the theory of fuzzy subsets'' Academic Press, New York, 1975, then he tried to introduce intuitionistic fuzzy subsets to study the properties of the new objects so defined. George Gargov named new sets as the ''Intuitionistic fuzzy subsets'', as their fuzzification denies the law of the excluded middle, $A \cup A^c =$

X. This has encouraged Prof. K.T.Atanassov to continue his work on intuitionistic fuzzy subsets. The notion of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas.R[8, 9]. In this paper, we introduce the some theorems in intuitionistic fuzzy subfield of a field with respect to (T,S)-norm.

1. PRELIMINARIES:

1.1 Definition: A T-norm is a binary operations T: $[0, 1]\times[0, 1] \rightarrow [0, 1]$ satisfying the following requirements;

(i) $T(0, x) = 0$, $T(1, x) = x$ (boundary condition)

(ii) $T(x, y) = T(y, x)$ (commutativity)

(iii) $T(x, T(y, z)) = T(T(x,y), z)$(associativity)

(iv) if $x \leq y$ and $w \leq z$, then $T(x, w) \leq T(y, z)$(monotonicity).

1.2 Definition: A S-norm is a binary operation S: $[0, 1]\times[0, 1] \rightarrow [0, 1]$ satisfying the following requirements;

(i) $0 S x = x$, $1 S x = 1$ (boundary condition)

(ii) $x S y = y S x$ (commutativity)

(iii) $x S (y S z) = (x S y) S z$ (associativity)

(iv) if $x \leq y$ and $w \leq z$, then $x S w \leq y S z$ (monotonicity).

1.3 Definition: Let X be a non-empty set. A **fuzzy subset** A of X is a function $A : X \rightarrow [0, 1]$.

1.4 Definition: Let (F, +, ·) be a field. A fuzzy subset A of F is said to be a f**uzzy subfield** of F if the following conditions are satisfied:

(i) $A(x-y) \geq \min(A(x), A(y))$, for all x and y in F,

(ii) $A(xy) \geq \min(A(x), A(y))$, for all x and y in F,

(iii) $A(x^{-1}) \geq A(x)$, for all x in $F-\{0\}$, where 0 is the additive identity element of F.

1.5 Definition: An **intuitionistic fuzzy subset(IFS**) A of a set X is defined as an object of the form A= {⟨ x, $\mu_A(x)$, $\nu_A(x)$ ⟩ / x∈X }, where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element x in X respectively and for every x in X satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.6 Example: Let X = { a, b, c } be a set. Then A={⟨ a, 0.5, 0.3⟩, ⟨b, 0.4, 0.1⟩, ⟨ c, 0.5, 0.4 ⟩ } is an intuitionistic fuzzy subset of X.

1.7 Definition: Let (F, +, ·) be a field. An intuitionistic fuzzy subset A of F is said to be an **intuitionistic fuzzy subfield of a field F with respect to (T, S)-norm (intuitionistic (T, S)-fuzzy subfield of a field)** of F if the following conditions are satisfied:

(i) $\mu_A(x+y) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in F,

(ii) $\mu_A(-x) \geq \mu_A(x)$, for all x in F,

(iii) $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in F,

(iv) $\mu_A(x^{-1}) \geq \mu_A(x)$, for all $x \neq 0$ in F,

(v) $\nu_A(x+y) \leq S(\nu_A(x), \nu_A(y))$, for all x and y in F,

(vi) $\nu_A(- x) \leq \nu_A(x)$, for all x in F,

(vii) $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y))$, for all x and y in F,

(viii) $\nu_A(x^{-1}) \leq \nu_A(x)$, for all $x \neq 0$ in F.

2-PROPERTIES OF INTUITIONISTIC (T, S)-FUZZY SUBFIELDS:

2.1 Theorem: If A is an intuitionistic (T, S)-fuzzy subfield of a field (F, +, ·), then $\mu_A(-x) = \mu_A(x)$, for all x in F and $\mu_A(x^{-1}) = \mu_A(x)$, for all $x \neq 0$ in F and $\nu_A(-x) = \nu_A(x)$, for all x in F and $\nu_A(x^{-1}) = \nu_A(x)$, for all $x \neq 0$ in F and $\mu_A(x) \leq \mu_A(0)$, for all x in F and $\mu_A(x) \leq \mu_A(1)$, for all $x \neq 0$ in F and $\nu_A(x) \geq \nu_A(0)$, for all x in F and $\nu_A(x) \geq \nu_A(1)$, for all $x \neq 0$ in F, where 0 and 1 are identity elements in F.

Proof: For x in F and 0, 1 are identity elements in F. Now, $\mu_A(x) = \mu_A(-(-x)) \geq \mu_A(-x) \geq \mu_A(x)$. Therefore, $\mu_A(-x) = \mu_A(x)$, for all x in F. And $\mu_A(x) = \mu_A((x^{-1})^{-1}) \geq \mu_A(x^{-1}) \geq \mu_A(x)$. Therefore, $\mu_A(x^{-1}) = \mu_A(x)$, for all $x \neq 0$ in F. And, $\nu_A(x) = \nu_A(-$

$(-x)) \leq \nu_A(-x) \leq \nu_A(x)$. Therefore, $\nu_A(-x) = \nu_A(x)$, for all x in F. Also, $\nu_A(x) = \nu_A((x^{-1})^{-1}) \leq \nu_A(x^{-1}) \leq \nu_A(x)$. Therefore, $\nu_A(x^{-1}) = \nu_A(x)$, for all $x \neq 0$ in F. Now, $\mu_A(0) = \mu_A(x-x) \geq T(\mu_A(x), \mu_A(-x)) = \mu_A(x)$. Therefore, $\mu_A(0) \geq \mu_A(x)$ for all x in F. And $\mu_A(1) = \mu_A(xx^{-1}) \geq T(\mu_A(x), \mu_A(x^{-1})) = \mu_A(x)$. Therefore, $\mu_A(1) \geq \mu_A(x)$, for all $x \neq 0$ in F. And, $\nu_A(0) = \nu_A(x-x) \leq S(\nu_A(x), \nu_A(-x)) = \nu_A(x)$. Therefore, $\nu_A(0) \leq \nu_A(x)$, for all x in F. And, $\nu_A(1) = \nu_A(xx^{-1}) \leq S(\nu_A(x), \nu_A(x^{-1})) = \nu_A(x)$. Therefore, $\nu_A(1) \leq \nu_A(x)$, for all $x \neq 0$ in F.

2.2 Theorem: If A is an intuitionistic (T, S)-fuzzy subfield of a field (F, +, ·), then (i) $\mu_A(x-y) = \mu_A(0)$ gives $\mu_A(x) = \mu_A(y)$, for all x and y in F, (ii) $\mu_A(xy^{-1}) = \mu_A(1)$ gives $\mu_A(x) = \mu_A(y)$, for all x and $y \neq 0$ in F, (iii) $\nu_A(x-y) = \nu_A(0)$ gives $\nu_A(x) = \nu_A(y)$, for all x and y in F and (iv) $\nu_A(xy^{-1}) = \nu_A(1)$ gives $\nu_A(x) = \nu_A(y)$, for all x and $y \neq 0$ in F, where 0 and 1 are identity elements in F.

Proof: Let x and y in F and 0, 1 are identity elements in F. (i) Now, $\mu_A(x) = \mu_A(x-y+y) \geq T(\mu_A(x-y), \mu_A(y)) = T(\mu_A(0), \mu_A(y)) = \mu_A(y) = \mu_A(x-(x-y)) \geq T(\mu_A(x-y), \mu_A(x)) = T(\mu_A(0), \mu_A(x)) = \mu_A(x)$. Therefore, $\mu_A(x) = \mu_A(y)$, for all x, y in F. (ii) Now, $\mu_A(x) = \mu_A(xy^{-1}y) \geq T(\mu_A(xy^{-1}), \mu_A(y)) = T(\mu_A(1), \mu_A(y)) = \mu_A(y) = \mu_A((xy^{-1})^{-1}x) \geq T(\mu_A(xy^{-1}), \mu_A(x)) = T(\mu_A(1), \mu_A(x)) = \mu_A(x)$. Therefore, $\mu_A(x) = \mu_A(y)$, for all x and $y \neq 0$ in F. (iii) And $\nu_A(x) = \nu_A(x-y+y) \leq S(\nu_A(x-y), \nu_A(y)) = S(\nu_A(0), \nu_A(y)) = \nu_A(y) = \nu_A(x-(x-y)) \leq S(\nu_A(x-y), \nu_A(x)) = S(\nu_A(0), \nu_A(x)) = \nu_A(x)$. Therefore, $\nu_A(x) = \nu_A(y)$, for all x and y in F. (iv) Also, $\nu_A(x) = \nu_A(xy^{-1}y) \leq S(\nu_A(xy^{-1}), \nu_A(y)) = S(\nu_A(1), \nu_A(y)) = \nu_A(y) = \nu_A((xy^{-1})^{-1}x) \leq S(\nu_A(xy^{-1}), \nu_A(x)) = S(\nu_A(1), \nu_A(x)) = \nu_A(x)$. Therefore, $\nu_A(x) = \nu_A(y)$, for all $x \neq 0$ and $y \neq 0$ in F.

2.3 Theorem: If A is an intuitionistic (T, S)-fuzzy subfield of a field (F, +, ·), then H = { x / x∈F: $\mu_A(x) = 1$, $\nu_A(x) = 0$ } is either empty or a subfield of F.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(-y)) \geq T(\mu_A(x), \mu_A(y)) = T(1, 1) = 1$. Therefore, $\mu_A(x-y) = 1$, for all x and y in H. Also $\mu_A(xy^{-1}) \geq T(\mu_A(x), \mu_A(y^{-1})) \geq T(\mu_A(x), \mu_A(y)) = T(1, 1) = 1$. Therefore, $\mu_A(xy^{-1}) = 1$, for all x and $y \neq e$ in H. And, $\nu_A(x-y) \leq S(\nu_A(x), \nu_A(-y)) \leq S(\nu_A(x), \nu_A(y)) = S(0, 0) = 0$. Therefore, $\nu_A(x-y) = 0$, for all x and y in H. And, $\nu_A(xy^{-1}) \leq S(\nu_A(x), \nu_A(y^{-1})) \leq S(\nu_A(x), \nu_A(y)) = S(0, 0) = 0$. Therefore, $\nu_A(xy^{-1}) = 0$, for all x and $y \neq e$ in H. We get $x-y$, xy^{-1} in H. Therefore, H is a subfield of F. Hence H is either empty or a subfield of F.

2.4 Theorem: If A is an intuitionistic (T, S)-fuzzy subfield of a field (F, +, ·), then H = { ⟨ x, $\mu_A(x)$ ⟩ : 0 < $\mu_A(x) \leq 1$ and $\nu_A(x) = 0$ } is either empty or a T-fuzzy subfield of F.

Proof: It is trivial.

2.5 Theorem: If A is an intuitionistic (T, S)-fuzzy subfield of a field (F, +, ·) then H = { ⟨ x, $\mu_A(x)$ ⟩ : 0 < $\mu_A(x) \leq 1$} is either empty or a T-fuzzy subfield of F.

Proof: It is trivial.

2.6 Theorem: If A is an intuitionistic (T, S)-fuzzy subfield of a field (F, +, ·), then H = { ⟨ x, $\nu_A(x)$ ⟩ : 0 < $\nu_A(x) \leq 1$} is either empty or an anti S-fuzzy subfield of F.

Proof: It is trivial.

2.7 Theorem: If A is an intuitionistic (T, S)-fuzzy subfield of a field (F, +, ·), then H ={ x∈F: $\mu_A(x) = \mu_A(e) = \mu_A(e^{l})$ and $\nu_A(x) = \nu_A(e) = \nu_A(e^{l})$ } is either empty or a subfield of F, where e and e^{l} are identity elements of F.

Proof: It is trivial.

2.8 Theorem: Let A be an intuitionistic (T, S)-fuzzy subfield of a field (F, +, ·). Then (i) if $\mu_A(x-y) = 1$, then $\mu_A(x) = \mu_A(y)$, for all x and y in F and if $\mu_A(xy^{-1}) = 1$, then $\mu_A(x) = \mu_A(y)$, for all x and y≠ e in F, (ii) if $\nu_A(x-y) = 0$, then $\nu_A(x) = \nu_A(y)$, for all x and y in F and if $\nu_A(xy^{-1}) = 0$, then $\nu_A(x) = \nu_A(y)$, for all x and y≠ e in F, where e and e^{l} are identity elements of F.

Proof: Let x and y in F. (i) Now, $\mu_A(x) = \mu_A(x-y+y) \geq T(\mu_A(x-y), \mu_A(y)) = T(1, \mu_A(y)) = \mu_A(y) = \mu_A(-y) = \mu_A(-x+x-y) \geq T(\mu_A(-x), \mu_A(x-y)) = T(\mu_A(-x), 1) = \mu_A(-x) = \mu_A(x)$. Therefore, $\mu_A(x) = \mu_A(y)$, for all x and y in F. And, $\mu_A(x) = \mu_A(xy^{-1}y) \geq T(\mu_A(xy^{-1}), \mu_A(y)) = T(1, \mu_A(y)) = \mu_A(y) = \mu_A(y^{-1}) = \mu_A(x^{-1}xy^{-1}) \geq T(\mu_A(x^{-1}), \mu_A(xy^{-1})) = T(\mu_A(x^{-1}), 1) = \mu_A(x^{-1}) = \mu_A(x)$. Therefore, $\mu_A(x) = \mu_A(y)$, for all x≠ e and y≠ e in F. (ii) Now, $\nu_A(x) = \nu_A(x-y+y) \leq S(\nu_A(x-y), \nu_A(y)) = S(0, \nu_A(y)) = \nu_A(y) = \nu_A(-y) = \nu_A(-x+x-y) \leq S(\nu_A(-x), \nu_A(x-y)) = S(\nu_A(-x), 0) = \nu_A(-x) = \nu_A(x)$. Therefore, $\nu_A(x) = \nu_A(y)$, for all x and y in F. And, $\nu_A(x) = \nu_A(xy^{-1}y) \leq S(\nu_A(xy^{-1}), \nu_A(y)) = S(0, \nu_A(y)) = \nu_A(y) = \nu_A(y^{-1}) = \nu_A(x^{-1}xy^{-1}) \leq S(\nu_A(x^{-1}), \nu_A(xy^{-1})) = S(\nu_A(x^{-1}), 0) = \nu_A(x^{-1}) = \nu_A(x)$. Therefore, $\nu_A(x) = \nu_A(y)$, for all x≠ e and y≠ e in F.

2.9 Theorem: If A is an intuitionistic (T, S)-fuzzy subfield of a field (F, +, ·) , then (i) if $\mu_A(x-y) = 0$, then either $\mu_A(x) = 0$ or $\mu_A(y) = 0$, for all x and y in F and if $\mu_A(xy^{-}$

^{1}) = 0, then either $\mu_A(x)$= 0 or $\mu_A(y)$= 0, for all x and y≠ e in F, (ii) if $\nu_A(x-y)$ =1, then either $\nu_A(x)$ = 1or $\nu_A(y)$ = 1, for all x and y in F and if $\nu_A(xy^{-1})$ =1, then either $\nu_A(x)$ = 1or $\nu_A(y)$ = 1, for all x and y≠ e in F, where e and e^{l}are identity elements of F.

Proof: Let x and y in F. (i) By the definition $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$ which implies that $0 \geq T(\mu_A(x), \mu_A(y))$. Therefore, either $\mu_A(x) = 0$ or $\mu_A(y) = 0$, for all x and y in F. And, by the definition $\mu_A(xy^{-1}) \geq T(\mu_A(x), \mu_A(y))$ which implies that $0 \geq T(\mu_A(x), \mu_A(y))$. Therefore, either $\mu_A(x) = 0$ or $\mu_A(y) = 0$, for all x and y≠ e in F. (ii) By the definition $\nu_A(x-y) \leq S(\nu_A(x), \nu_A(y))$ which implies that $1\leq S(\nu_A(x), \nu_A(y))$. Therefore, either $\nu_A(x)$ = 1or $\nu_A(y)$ = 1, for all x and y in F. And by the definition $\nu_A(xy^{-1}) \leq S(\nu_A(x), \nu_A(y))$ which implies that $1\leq S(\nu_A(x), \nu_A(y))$. Therefore, either $\nu_A(x)$ = 1or $\nu_A(y)$ = 1, for all x and y≠ e in F.

2.10 Theorem: Let (F, +, ·) be a field. If A is an intuitionistic (T, S)-fuzzy subfield of F, then $\mu_A(x+y) = T(\mu_A(x), \mu_A(y))$, for all x and y in F and $\mu_A(xy) = T(\mu_A(x), \mu_A(y))$, for all x and y ≠ 0 in F and $\nu_A(x+y) = S(\nu_A(x), \nu_A(y))$, for all x and y in F and $\nu_A(xy) = S(\nu_A(x), \nu_A(y))$, for all x and y≠ 0 in F with $\mu_A(x) \neq \mu_A(y)$ and $\nu_A(x)\neq\nu_A(y)$, where 0 and 1 are identity elements of F.

Proof: Let x and y belongs to F. Assume that $\mu_A(x) > \mu_A(y)$ and $\nu_A(x) < \nu_A(y)$. Now, $\mu_A(y) = \mu_A(-x+x+y) \geq T(\mu_A(-x), \mu_A(x+y)) \geq T(\mu_A(x), \mu_A(x+y)) = \mu_A(x+y) \geq T(\mu_A(x), \mu_A(y)) = \mu_A(y)$. Therefore, $\mu_A(x+y) = \mu_A(y) = T(\mu_A(x), \mu_A(y))$, for all x and y in F. And, $\mu_A(y) = \mu_A(x^{-1}xy) \geq T(\mu_A(x^{-1}), \mu_A(xy)) \geq T(\mu_A(x), \mu_A(xy)) = \mu_A(xy) \geq T(\mu_A(x), \mu_A(y)) = \mu_A(y)$. Therefore, $\mu_A(xy) = \mu_A(y) = T(\mu_A(x), \mu_A(y))$, for all x≠ 0 and y in F. And, $\nu_A(y) = \nu_A(-x+x+y) \leq S(\nu_A(-x), \nu_A(x+y)) \leq S(\nu_A(x), \nu_A(x+y)) = \nu_A(x+y) \leq S(\nu_A(x), \nu_A(y)) = \nu_A(y)$. Therefore, $\nu_A(x+y) = \nu_A(y) = S(\nu_A(x), \nu_A(y))$, for all x and y in F. And, $\nu_A(y) = \nu_A(x^{-1}xy) \leq S(\nu_A(x^{-1}), \nu_A(xy)) \leq S(\nu_A(x), \nu_A(xy)) = \nu_A(xy) \leq S(\nu_A(x), \nu_A(y)) = \nu_A(y)$. Therefore, $\nu_A(xy) = \nu_A(y) = S(\nu_A(x), \nu_A(y))$, for all x≠ 0 and y in F.

2.11 Theorem: If A and B are any two intuitionistic (T, S)-fuzzy subfields of a field (F, +, ·), then their intersection A∩B is an intuitionistic (T, S)-fuzzy subfield of F.

Proof: Let x and y belong to F, A = $\{ \langle x, \mu_A(x), \nu_A(x) \rangle / x\in F\}$ and B = $\{\langle x, \mu_B(x), \nu_B(x)\rangle / x\in F\}$. Let C = A∩B and C = $\{\langle x, \mu_C(x), \nu_C(x) \rangle / x\in F \}$, where $\mu_C(x) = \min \{ \mu_A(x), \mu_B(x) \}$ and $\nu_C(x) = \max \{\nu_A(x), \nu_B(x) \}$. (i) $\mu_C(x-y) = \min \{\mu_A(x-y), \mu_B(x-y)\} \geq \min \{ T(\mu_A(x), \mu_A(y)), T(\mu_B(x), \mu_B(y)) \} \geq T(\min (\mu_A(x), \mu_B(x)), \min(\mu_A(y), \mu_B(y))) = T(\mu_C(x), \mu_C(y))$. Therefore, $\mu_C(x-y) \geq T(\mu_C(x), \mu_C(y))$, for all x and y in F. (ii) $\mu_C(xy^{-1}) = \min (\mu_A(xy^{-1}), \mu_B(xy^{-1})) \geq \min (T(\mu_A(x), \mu_A(y)), T(\mu_B(x), \mu_B(y))) \geq$

$T(\min(\mu_A(x), \mu_B(x)), \min(\mu_A(y), \mu_B(y))) = T(\mu_C(x), \mu_C(y))$. Therefore, $\mu_C(xy^{-1}) \geq T(\mu_C(x), \mu_C(y))$, for all x and $y \neq 0$ in F. (iii) $\nu_C(x-y) = \max(\nu_A(x-y), \nu_B(x-y)) \leq \max(S(\nu_A(x), \nu_A(y)), S(\nu_B(x), \nu_B(y))) \leq S(\max(\nu_A(x), \nu_B(x)), \max(\nu_A(y), \nu_B(y))) = S(\nu_C(x), \nu_C(y))$. Therefore, $\nu_C(x-y) \leq S(\nu_C(x), \nu_C(y))$, for all x and y in F. (iv) $\nu_C(xy^{-1}) = \max(\nu_A(xy^{-1}), \nu_B(xy^{-1})) \leq \max(S(\nu_A(x), \nu_A(y)), S(\nu_B(x), \nu_B(y))) \leq S(\max(\nu_A(x), \nu_B(x)), \max(\nu_A(y), \nu_B(y))) = S(\nu_C(x), \nu_C(y))$. Therefore, $\nu_C(xy^{-1}) \leq S(\nu_C(x), \nu_C(y))$, for all x and $y \neq 0$ in F. Hence $A \cap B$ is an intuitionistic (T, S)-fuzzy subfield of a field F.

2.12 Theorem: The intersection of a family of intuitionistic (T, S)-fuzzy subfields of a field $(F, +, \cdot)$ is an intuitionistic (T, S)-fuzzy subfield of F.

Proof: It is trivial.

2.13 Theorem: Let A be an intuitionistic (T, S)-fuzzy subfield of a field $(F, +, \cdot)$. If $\mu_A(x) < \mu_A(y)$ and $\nu_A(x) > \nu_A(y)$, for some x and y in F, then (i) $\mu_A(x+y) = \mu_A(x) = \mu_A(y+x)$, for all x and y in F and $\mu_A(xy) = \mu_A(x) = \mu_A(yx)$, for all x and $y \neq 0$ in F, (ii) $\nu_A(x+y) = \nu_A(x) = \nu_A(y+x)$, for all x and y in F and $\nu_A(xy) = \nu_A(x) = \nu_A(yx)$, for all x and $y \neq 0$ in F.

Proof: Let A be an intuitionistic (T, S)-fuzzy subfield of a field F. (i) Also we have $\mu_A(x) < \mu_A(y)$ and $\nu_A(x) > \nu_A(y)$, for some x and y in F, $\mu_A(x+y) \geq T(\mu_A(x), \mu_A(y)) = \mu_A(x)$ and $\mu_A(x) = \mu_A(x+y-y) \geq T(\mu_A(x+y), \mu_A(-y)) \geq T(\mu_A(x+y), \mu_A(y)) = \mu_A(x+y)$. Therefore, $\mu_A(x+y) = \mu_A(x)$, for all x and y in F. Hence $\mu_A(x+y) = \mu_A(x) = \mu_A(y+x)$, for all x and y in F. And, $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y)) = \mu_A(x)$ and $\mu_A(x) = \mu_A(xyy^{-1}) \geq T(\mu_A(xy), \mu_A(y^{-1})) \geq T(\mu_A(xy), \mu_A(y)) = \mu_A(xy)$. Therefore, $\mu_A(xy) = \mu_A(x)$, for all x and $y \neq 0$ in F. Hence $\mu_A(xy) = \mu_A(x) = \mu_A(yx)$, for all x and $y \neq 0$ in F. (ii) Now, $\nu_A(x+y) \leq S(\nu_A(x), \nu_A(y)) = \nu_A(x)$ and $\nu_A(x) = \nu_A(x+y-y) \leq S(\nu_A(x+y), \nu_A(-y)) \leq S(\nu_A(x+y), \nu_A(y)) = \nu_A(x+y)$. Therefore, $\nu_A(x+y) = \nu_A(x)$, for all x and y in F. Hence $\nu_A(x+y) = \nu_A(x) = \nu_A(y+x)$, for all x and y in F. Now, $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y)) = \nu_A(x)$ and $\nu_A(x) = \nu_A(xyy^{-1}) \leq S(\nu_A(xy), \nu_A(y^{-1})) \leq S(\nu_A(xy), \nu_A(y)) = \nu_A(xy)$. Therefore, $\nu_A(xy) = \nu_A(x)$, for all x and $y \neq 0$ in F. Hence $\nu_A(xy) = \nu_A(x) = \nu_A(yx)$, for all x and $y \neq 0$ in F.

2.14 Theorem: Let A be an intuitionistic (T, S)-fuzzy subfield of a field $(F, +, \cdot)$. If $\mu_A(x) < \mu_A(y)$ and $\nu_A(x) < \nu_A(y)$, for some x and y in F, then (i) $\mu_A(x+y) = \mu_A(x) = \mu_A(y+x)$, for all x and y in F and $\mu_A(xy) = \mu_A(x) = \mu_A(yx)$, for all x and $y \neq 0$ in F (ii) $\nu_A(x+y) = \nu_A(y) = \nu_A(y+x)$, for all x and y in F and $\nu_A(xy) = \nu_A(y) = \nu_A(yx)$, for all x and $y \neq 0$ in F.

proof: It is trivial.

2.15 Theorem: Let A be an intuitionistic (T, S)-fuzzy subfield of a field (F, +, ·). If $\mu_A(x) > \mu_A(y)$ and $\nu_A(x) > \nu_A(y)$, for some x and y in F, then (i) $\mu_A(x+y) = \mu_A(y) = \mu_A(y+x)$, for all x and y in F and $\mu_A(xy) = \mu_A(y) = \mu_A(yx)$, for all x and $y\neq 0$ in F, (ii) $\nu_A(x+y) = \nu_A(x) = \nu_A(y+x)$, for all x and y in F and $\nu_A(xy) = \nu_A(x) = \nu_A(yx)$, for all x and $y\neq 0$ in F.

Proof: It is trivial.

2.16 Theorem: Let A be an intuitionistic (T, S)-fuzzy subfield of a field (F, +, ·). If $\mu_A(x) > \mu_A(y)$ and $\nu_A(x) < \nu_A(y)$, for some x and y in F, then (i) $\mu_A(x+y) = \mu_A(y) = \mu_A(y+x)$, for all x and y in F and $\mu_A(xy) = \mu_A(y) = \mu_A(yx)$, for all x and $y\neq 0$ in F (ii) $\nu_A(x+y) = \nu_A(y) = \nu_A(y+x)$, for all x and y in F and $\nu_A(xy) = \nu_A(y) = \nu_A(yx)$, for all x and $y\neq 0$ in F.

Proof: It is trivial.

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