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On Pairwise Fuzzy P-Spaces

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Abstract

In this paper, several characterizations of pairwise fuzzy P-spaces are studied and the conditions under which fuzzy bitopological spaces become pairwise fuzzy P-spaces, are investigated.

Keywords: Pair wise fuzzy dense set, pair wise fuzzy nowhere dense set, pairwise fuzzy G_δ -set , pairwise fuzzy F_σ-set , pairwise fuzzy first category set , pairwise fuzzy Baire space, pairwise fuzzy submaximal space.

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1. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets , fuzzy set operations was introduced by **L.A. Zadeh** [14] in his classical paper in the year 1965. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. The concepts of fuzzy topology was defined by **C. L. Chang** [3] in the year 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then, much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

In 1954 **L.Gillman** and **M.Henriksen** [5] defined and characterized the classes of P-spaces. **A. K. Mishra** [7] introduced the concepts of P-spaces as a generalization of ω_α-additive spaces of **Sikorski** [8] and **L.W.Cohen** and **C.Goffman** [4]. In 1989,

Kandil [6] introduced the concept of fuzzy bitopological spaces as a generalization of fuzzy topological spaces. The concept of pairwise fuzzy P-spaces was introduced by **G.Thangaraj** and **V.Chandiran** in [10]. In this paper several characterizations of pairwise fuzzy P-spaces are studied and the conditions under which fuzzy bitopological spaces become pairwise fuzzy P-spaces, are investigated.

2. PRELIMINARIES

In order to make the exposition self- contained, some basic notions and results used in the sequel are given. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to CHANG (1968). Let X be a non- empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a function from X into I. The null set 0_X is the function from X into I which assumes only the value 0 and the whole fuzzy set 1_X is the function from X into I which takes 1 only. By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X.

Definition 21 : Let λ and μ be fuzzy sets in X. Then, for all $x \in$ X,

(i) $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$,

(ii) $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$,

(iii) $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max\{\lambda(x), \mu(x)\}$,

(iv) $\delta = \lambda \wedge \mu \Leftrightarrow \psi(x) = \min\{\lambda(x), \mu(x)\}$,

(v) $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i (\lambda_i)$ and intersection $\delta = \wedge_i (\lambda_i)$, are defined respectively as

(vi) $\psi(x) = sup_i \{\lambda_i(x) / x \in X\}$,

(vii) $\delta(x) = inf_i \{\lambda_i(x) / x \in X\}$.

Definition 2.2 : Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior and the closure of λ are defined respectively as follows :

(i). $\text{int}(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu \in T\}$ and

(ii) . $\text{cl}(\lambda) = \wedge\{\mu / \lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.1 [1] : For a fuzzy set λ of a fuzzy topological space X,

(i). $1 - \text{Int}(\lambda) = \text{Cl}(1 - \lambda)$, (ii). $1 - \text{Cl}(\lambda) = \text{Int}(1 - \lambda)$.

Definition 2.3 [10] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ (i = 1,2). The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a pairwise fuzzy closed set.

Definition 2.4 [9] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T_1} cl_{T_2}(\lambda) = cl_{T_2} cl_{T_1}(\lambda) = 1$, in (X, T_1, T_2).

Definition 2.5 [13] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $int_{T_1} cl_{T_2}(\lambda) = int_{T_2} cl_{T_1}(\lambda) = 0$, in (X, T_1, T_2).

Definition 2.6 [13]: Let (X,T_1,T_2) be a fuzzy bitopological space. A fuzzy set λ in(X,T_1,T_2) is called a pairwise fuzzy first category set if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are pairwise fuzzy nowhere dense sets in (X, T_1,T_2). Any other fuzzy set in (X,T_1,T_2) is said to be a pairwise fuzzy second category set in (X,T_1,T_2).

Definition 2.7 [13] : If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X,T_1,T_2), then the fuzzy set (1 – λ) is called a pairwise fuzzy residual set in (X,T_1,T_2).

Definition 2.8 [10] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy G_δ -set if $\lambda = \bigwedge_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'s$ are pairwise fuzzy open sets in (X,T_1,T_2).

Definition 2.9 [10] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy F_σ -set if $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'s$ are pairwise fuzzy closed sets in (X,T_1,T_2).

2. PAIRWISE FUZZY P-SPACES

Definition 3.1 [10] : A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy P-space if every non- zero pairwise fuzzy G_δ-set in (X, T_1, T_2), is a pairwise fuzzy open set in (X, T_1, T_2). That is, if $\lambda = \bigwedge_{k=1}^{\infty}(\lambda_k)$, where ($(\lambda_k)'s$ are pairwise fuzzy open sets in (X,T_1,T_2), then λ is a pairwise fuzzy open set in (X, T_1, T_2).

Example 3.1 : Let X = { a, b, c }. Consider the fuzzy sets λ , μ , δ and β, are defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.3$; $\lambda(b) = 0.7$; $\lambda(c) = 0.9$,

$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.7$; $\mu(b) = 0.5$; $\mu(c) = 0.3$,

$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0.5$; $\delta(b) = 0.9$; $\delta(c) = 0.7$,

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.8$; $\beta(b) = 0.5$; $\beta(c) = 0.3$.

Then, $T_1 = \{ 0, \lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta,$

$\lambda \vee [\mu \wedge \delta], \mu \vee [\lambda \wedge \delta], \delta \wedge [\lambda \vee \mu], [\lambda \vee \mu \vee \delta], 1 \}$ and

$T_2 = \{ 0, \lambda, \delta, \beta, \lambda \vee \delta, \lambda \vee \beta, \delta \vee \beta, \lambda \wedge \delta, \lambda \wedge \delta, \lambda \wedge \beta, \delta \wedge \beta,$

$\lambda \vee [\delta \wedge \beta], \beta \vee [\lambda \wedge \delta], \delta \wedge [\lambda \vee \beta], [\lambda \vee \delta \vee \beta], 1 \}$ are fuzzy topologies on X. On computation $\lambda, \delta, \lambda \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee [\mu \wedge \delta], \delta \wedge [\lambda \vee \mu], \lambda \wedge \beta, \delta \wedge \beta, \lambda \vee [\delta \wedge \beta], \delta \wedge [\lambda \vee \beta]$ and 1 , are pairwise fuzzy open sets in (X,T_1,T_2). Now, the fuzzy sets

$\alpha = \lambda \wedge [\lambda \wedge \mu] \wedge (\delta \wedge [\lambda \vee \mu]) \wedge (\lambda \vee [\delta \wedge \beta])$ and

$\gamma = [\lambda \vee \delta] \wedge (\lambda \vee [\delta \wedge \beta]) \wedge (\delta \wedge [\lambda \vee \beta])$, are pairwise fuzzy G_δ-sets in (X,T_1,T_2) and $\alpha \in T_i$ (i = 1,2) and $\gamma \in T_i$ (i = 1,2) shows that **the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy P-space.**

Example 3.2 : Let X = { a, b, c}. Consider the fuzzy sets λ, μ, γ and β, are defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.8$; $\lambda(b) = 0.6$; $\lambda(c) = 0.7$,

$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.6$; $\mu(b) = 0.9$; $\mu(c) = 0.8$,

$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.7$; $\gamma(b) = 0.5$; $\gamma(c) = 0.9$,

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.7$; $\beta(b) = 0.4$; $\beta(c) = 0.9$.

Then, $T_1 = \{ 0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \lambda \wedge \gamma, \mu \wedge \gamma,$

$(\lambda \vee [\mu \wedge \gamma]), (\mu \vee [\lambda \wedge \gamma]), (\gamma \vee [\lambda \wedge \mu]), (\lambda \wedge [\mu \vee \gamma]), (\mu \wedge [\lambda \vee \gamma]),$

$(\gamma \wedge [\lambda \vee \mu]), [\lambda \vee \mu \vee \gamma], [\lambda \wedge \mu \wedge \gamma], 1 \}$, and

$T_2 = \{ 0, \lambda, \mu, \beta, \lambda \vee \mu, \lambda \vee \beta, \mu \vee \beta, \lambda \wedge \mu, \lambda \wedge \beta, \mu \wedge \beta, (\lambda \vee [\mu \wedge \beta]),$

$(\mu \vee [\lambda \wedge \beta]), (\beta \vee [\lambda \wedge \mu]), (\lambda \wedge [\mu \vee \beta]), (\mu \wedge [\lambda \vee \beta]),$

$(\beta \wedge [\lambda \vee \mu]), [\lambda \vee \mu \vee \beta], [\lambda \wedge \mu \wedge \beta], 1\}$, are fuzzy topologies on X. On computation λ, μ, $[\lambda \vee \mu]$, $[\lambda \vee \gamma]$, $[\mu \vee \gamma]$, $[\lambda \wedge \mu]$,

$(\lambda \vee [\mu \wedge \gamma])$, $(\mu \vee [\lambda \wedge \gamma])$, $(\gamma \vee [\lambda \wedge \mu])$, $(\lambda \wedge [\mu \vee \gamma])$, $(\mu \wedge [\lambda \vee \gamma])$, $[\lambda \vee \mu \vee \gamma]$, , $[\lambda \vee \beta]$, $[\mu \vee \beta]$, $(\lambda \vee [\mu \wedge \beta])$, $(\mu \vee [\lambda \wedge \beta])$, $(\beta \vee [\lambda \wedge \mu])$, $(\lambda \wedge [\mu \vee \beta])$, $(\mu \wedge [\lambda \vee \beta])$ and 1, are pairwise fuzzy open sets in (X,T_1,T_2). Now, the fuzzy set $\alpha = (\lambda \vee [\mu \wedge \gamma]) \wedge (\mu \vee [\lambda \wedge \gamma]) \wedge (\gamma \vee [\lambda \wedge \mu])$ is a pairwise fuzzy G_δ-set in (X,T_1,T_2). But α is not a pair wise fuzzy open set in (X,T_1,T_2) and hence the fuzzy bitopological space ***(X,T_1,T_2) is not a pairwise fuzzy P-space.***

Proposition 3.1 : If λ is a pairwise fuzzy F_σ- set in a pairwise fuzzy P-space (X,T_1,T_2), then λ is a pairwise fuzzy closed set in (X,T_1,T_2).

Proof : Let λ be a pair wise fuzzy F_σ- set in (X,T_1,T_2) . Then, $1-\lambda$ is a

pairwise fuzzy G_δ-set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy P-space, the pairwise fuzzy G_δ-set $(1-\lambda)$ is a pairwise fuzzy open set in (X,T_1,T_2). Hence λ is a pairwise fuzzy closed set in (X,T_1,T_2).

Proposition 3.2 : If the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy P-space , then $cl_{T_i}(\bigvee_{k=1}^{\infty}(\lambda_k))) = \bigvee_{k=1}^{\infty} cl_{T_i}(\lambda_k)$, $(i = 1,2)$ where (λ_k)'s are pairwise fuzzy closed sets in (X,T_1,T_2).

Proof: Let the fuzzy sets (λ_k)'s ($k = 1$ to ∞) be pairwise fuzzy closed sets in (X,T_1,T_2). Then, $cl_{T_i}(\lambda_k) = (\lambda_k)$, $(i = 1,2)$ in (X,T_1,T_2). Let $\lambda = \bigvee_{k=1}^{\infty}(\lambda_k)$. Then, λ is a pairwise fuzzy F_σ- set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy P-space, by proposition 3.1, λ is a pairwise fuzzy closed set in (X,T_1,T_2). Then, $cl_{T_i}(\lambda) = \lambda (i = 1,2)$. Now $cl_{T_i}(\bigvee_{k=1}^{\infty}(\lambda_k)) == \bigvee_{k=1}^{\infty}(\lambda_k) = \bigvee_{k=1}^{\infty} cl_{T_i}(\lambda_k)$ and hence $cl_{T_i}(\bigvee_{k=1}^{\infty}(\lambda_k))) = \bigvee_{k=1}^{\infty} cl_{T_i}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy closed sets in (X,T_1,T_2).

Proposition 3.3: If the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy P-space , then $int_{T_i}(\bigwedge_{k=1}^{\infty}(\lambda_k)) = \bigwedge_{k=1}^{\infty} int_{T_i}(\lambda_k))$ where (λ_k)'s are pairwise fuzzy open sets in (X,T_1,T_2).

Proof Let the fuzzy sets (λ_k)'s ($k = 1$ to ∞) be pairwise fuzzy open sets in (X,T_1,T_2). Then, $(1-\lambda_k)$'s ($k = 1$ to ∞) are pairwise fuzzy closed sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy P-space, by proposition 3.2, $cl_{T_i}(\bigvee_{k=1}^{\infty}(1-\lambda_k))) = \bigvee_{k=1}^{\infty} cl_{T_i}(1-\lambda_k)$, in (X,T_1,T_2). Then, by lemma 2.1, $cl_{T_i}[1-(\bigwedge_{k=1}^{\infty}(\lambda_k))] = \bigvee_{k=1}^{\infty}(1 - int_{T_i}(\lambda_k)$ and hence $1 - int_{T_i}(\bigwedge_{k=1}^{\infty}(\lambda_k)) = 1 - \bigwedge_{k=1}^{\infty} int_{T_i}(\lambda_k))$. Therefore $int_{T_i}(\bigwedge_{k=1}^{\infty}(\lambda_k)) = \bigwedge_{k=1}^{\infty} int_{T_i}(\lambda_k))$ where (λ_k)'s are pairwise fuzzy open sets in (X,T_1,T_2).

4. PAIRWISE FUZZY P-SPACES and PAIRWISE FUZZY SUBMAXIMAL SPACES

Definition 4.1 [12]: A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy submaximal space if each pairwise fuzzy dense set in (X,T_1,T_2) is a pairwise fuzzy open set in (X,T_1,T_2) . That is, if λ is a pairwise fuzzy dense set in a fuzzy bitopological space (X,T_1,T_2), then $\lambda \in T_i$ (i = 1,2).

Proposition 4.1: If (λ_i)'s are pairwise fuzzy dense sets in a pairwise fuzzy submaximal and pairwise fuzzy P-space (X,T_1,T_2), then $\wedge_{i=1}^{\infty} (\lambda_i)$ is a pairwise fuzzy open set in (X,T_1,T_2).

Proof : Let (λ_i)'s (i = 1 to ∞) be pairwise fuzzy dense sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy submaximal space, (λ_i)'s are pairwise fuzzy open sets in (X,T_1,T_2). Then, $\wedge_{i=1}^{\infty} (\lambda_i)$ is a pairwise fuzzy G_δ –set in (X,T_1,T_2). Since the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy P-space , $\wedge_{i=1}^{\infty} (\lambda_i)$ is a pairwise fuzzy open set in (X,T_1,T_2).

Proposition 4.2 : If each pairwise fuzzy G_δ- set is a pairwise fuzzy dense set

in a pairwise fuzzy submaximal space (X,T_1,T_2) , then (X,T_1,T_2) is a pairwise

fuzzy P-space.

Proof : Let λ be a pairwise fuzzy G_δ- set in (X,T_1,T_2). By hypothesis, λ is a pairwise fuzzy dense set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy submaximal space, the pairwise fuzzy dense set λ is a pairwise fuzzy open set in (X,T_1,T_2). Hence the pairwise G_δ- set in (X,T_1,T_2) is a pairwise fuzzy open set in (X,T_1,T_2). Thus, the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy P-space.

Proposition 4.3 : If $\mathrm{int}_{T_i}\mathrm{int}_{T_j}(\lambda) = 0$, (i, j = 1,2 and $i \neq j$) where λ is a pairwise fuzzy F_σ- set in a pairwise fuzzy submaximal space (X,T_1,T_2), then (X,T_1,T_2) is a pairwise fuzzy P-space.

Proof : Let λ be a pairwise fuzzy G_δ- set in (X,T_1,T_2). Then, $1 - \lambda$ is a pairwise fuzzy F_σ -set in (X,T_1,T_2). By hypothesis, $\mathrm{int}_{T_i}\mathrm{int}_{T_j}(1 - \lambda) = 0$, (i,j = 1,2 and $i \neq j$) in (X,T_1,T_2). This implies that $1 - \mathrm{cl}_{T_i}\mathrm{cl}_{T_j}(\lambda) = 0$ and thus $\mathrm{cl}_{T_i}\mathrm{cl}_{T_j}(\lambda) = 1$. Hence λ is a pairwise fuzzy dense set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy submaximal space, the pairwise fuzzy dense set λ is a pairwise open set in (X,T_1,T_2). Hence the pairwise fuzzy G_δ- set λ in (X,T_1,T_2) is a pairwise fuzzy open set in (X,T_1,T_2). Therefore (X,T_1,T_2) is a pairwise fuzzy P-space.

Proposition 4.4 : If each pairwise fuzzy F_σ-set is a pairwise fuzzy nowhere dense set in a pairwise fuzzy submaximal space (X,T_1,T_2), then (X,T_1,T_2) is a pairwise fuzzy P-space.

Proof : Let λ be a pairwise fuzzy F_σ- set in a pairwise fuzzy submaximal

space (X, T_1, T_2) such that $\text{int}_{T_i}\text{cl}_{T_J}(\lambda) = 0$, (i, j = 1, 2 and i ≠ j). But

$\text{int}_{T_i}(\lambda) \leq \text{int}_{T_i}\text{cl}_{T_J}(\lambda)$, implies that $\text{int}_{T_i}(\lambda) \leq 0$. That is, $\text{int}_{T_i}(\lambda) = 0$.

Then, $\text{int}_{T_i}\text{int}_{T_j}(\lambda) = \text{int}_{T_i}(\lambda) = 0$, (i, j = 1,2 and i ≠ j).Thus, $\text{int}_{T_i}\text{int}_{T_j}(\lambda) = 0$,

for a pairwise fuzzy F_σ set λ in a pairwise fuzzy submaximal space (X,T_1,T_2).

Then , by proposition 4.3, the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy P-space.

Proposition 4.5 : If $\text{cl}_{T_i}\text{int}_{T_j}(\lambda) = 1$, (i, j =1, 2 and i ≠ j) for each pairwise fuzzy G_δ- set λ in a pairwise fuzzy submaximal space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy P-space.

Proof : Let λ be a pairwise fuzzy F_σ- set in the fuzzy bitopological space (X,T_1,T_2).Then, $1-\lambda$ is a pairwise fuzzy G_δ- set in (X,T_1,T_2). By hypothesis, $\text{cl}_{T_i}\text{int}_{T_j}(1-\lambda) = 1$,(i, j =1, 2 and i ≠ j) in (X,T_1,T_2).This implies that $1 - \text{int}_{T_i}\text{cl}_{T_j}(\lambda)=1$ in (X,T_1,T_2) and hence $\text{int}_{T_i}\text{cl}_{T_j}(\lambda) = 0$, in (X,T_1,T_2) .Then λ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2).Thus, the pairwise fuzzy F_σ set λ is a pairwise fuzzy nowhere dense set in a pairwise fuzzy submaximal space (X,T_1,T_2). Hence, by proposition 4.4, the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy P-space.

Proposition 4.6 : If λ is a pairwise fuzzy residual set in a pairwise fuzzy submaximal space (X,T_1,T_2) , then λ is a pairwise fuzzy G_δ –set in (X,T_1,T_2).

Proof : Let λ be a pairwise fuzzy residual set in a pairwise fuzzy submaximal

space (X, T_1, T_2). Then, $1 - \lambda$ is a pairwise fuzzy first category set in

(X,T_1,T_2) and hence $1 - \lambda = \vee_{k=1}^{\infty}(\lambda_k)$**,** where the fuzzy sets (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Since (λ_k)'s are pairwise

fuzzy nowhere dense sets in (X,T_1,T_2), $\mathrm{int}_{T_i}\mathrm{cl}_{T_J}(\lambda_k) = 0$, (i, j = 1, 2 and i ≠ j).

But , $\mathrm{int}_{T_i}(\lambda_k) \leq \mathrm{int}_{T_i}\mathrm{cl}_{T_J}(\lambda_k)$, implies that $\mathrm{int}_{T_i}(\lambda_k) \leq 0$. That is, $\mathrm{int}_{T_i}(\lambda_k) = 0$.

Thus, $\mathrm{int}_{T_i}\mathrm{int}_{T_j}(\lambda_k) = 0$, in (X,$T_1$,$T_2$). Then, $\mathrm{cl}_{T_i}\mathrm{cl}_{T_j}(1-\lambda_k) = 1 - \mathrm{int}_{T_i}\mathrm{int}_{T_j}(\lambda_k) = 1 - 0 = 1$ .Hence $(1-\lambda_k)$'s are pairwise fuzzy dense sets in (X,T_1,T_2).Since (X, T_1, T_2) is a pairwise fuzzy submaximal space, the pairwise fuzzy dense sets $(1-\lambda_k)$'s are pairwise fuzzy open sets in (X,T_1,T_2). Then, $(\lambda_k)'$s are pairwise fuzzy closed sets in (X, T_1, T_2). Hence $1-\lambda = \bigvee_{k=1}^{\infty}(\lambda_k)$, where the fuzzy sets (λ_k)'s are pairwise fuzzy closed sets in (X,T_1,T_2), implies that $(1-\lambda)$ is a pairwise fuzzy F_σ -set in (X,T_1,T_2). Therefore λ is a pairwise fuzzy G_δ -set in (X,T_1,T_2).

Proposition 4.7 : If λ is a pairwise fuzzy residual set in a pairwise fuzzy submaximal and pairwise fuzzy P-space (X,T_1,T_2), then λ is a pairwise fuzzy open set in (X,T_1,T_2).

Proof : Let λ be a pairwise fuzzy residual set in a pairwise fuzzy submaximal space (X,T_1,T_2). Since the fuzzy bitopological space (X,T_1,T_2) is a fuzzy submaximal space, by proposition 4.6, λ is a pairwise fuzzy G_δ –set in (X,T_1,T_2).Since (X,T_1,T_2) is a pairwise fuzzy P-space, the pairwise fuzzy G_δ-set λ is a pairwise fuzzy open set in (X,T_1,T_2). Therefore, the pairwise fuzzy residual set λ in a pairwise fuzzy submaximal and pairwise fuzzy P-space (X,T_1,T_2), is a pairwise fuzzy open set in (X,T_1,T_2).

Proposition 4.8 : If λ is a pairwise fuzzy nowhere dense set in a pairwise submaximal space (X,T_1,T_2), then λ is a pairwise fuzzy closed set in (X,T_1,T_2).

Proof : Let λ be a pairwise fuzzy nowhere dense set in the fuzzy bitopological

space (X,T_1,T_2). Then, $\mathrm{int}_{T_i}\mathrm{cl}_{T_J}(\lambda) = 0$, (i, j = 1, 2 and i ≠ j). But , $\mathrm{int}_{T_i}(\lambda) \leq \mathrm{int}_{T_i}\mathrm{cl}_{T_J}(\lambda)$, implies that $\mathrm{int}_{T_i}(\lambda) \leq 0$. That is, $\mathrm{int}_{T_i}(\lambda) = 0$. Thus, $\mathrm{int}_{T_i}\mathrm{int}_{T_j}(\lambda) = 0$, in (X,$T_1$,$T_2$). Then, $\mathrm{cl}_{T_i}\mathrm{cl}_{T_j}(1-\lambda) = 1 - \mathrm{int}_{T_i}\mathrm{int}_{T_j}(\lambda) = 1 - 0 = 1$ .Hence $(1-\lambda)$ is a pairwise fuzzy dense set in (X,T_1,T_2).Since (X, T_1, T_2) is a pairwise fuzzy submaximal space, the pairwise fuzzy dense set $(1-\lambda)$ is a pairwise fuzzy open set in (X,T_1,T_2). Thus, λ is a pairwise fuzzy closed set in (X,T_1,T_2).

Proposition 4.9 : If (λ_k)'s (k =1 to ∞) are pairwise fuzzy nowhere dense sets in a pairwise fuzzy submaximal and pairwise fuzzy P-space (X,T_1,T_2) such that $\bigvee_{k=1}^{\infty}(\lambda_k) \neq 1$, then $\mathrm{cl}_{T_i}[\bigvee_{k=1}^{\infty}(\lambda_k)] = \bigvee_{k=1}^{\infty}(\lambda_k)$, in (X,T_1, T_2).

Proof : Let (λ_k)'s (k = 1 to ∞) be pairwise fuzzy nowhere dense sets in (X,T_1,T_2). Since the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy submaximal space , by proposition 4.8, $(\lambda_k)'s$ are pairwise fuzzy closed sets in (X,T_1,T_2) and hence $(1-\lambda_k)'s$ are pairwise fuzzy open sets in (X,T_1,T_2). Let $\lambda = \wedge_{k=1}^{\infty}(1-\lambda_k)$. Then, λ is a non- zero pairwise fuzzy G_{δ}- set in (X,T_1,T_2). [For, if λ = 0, then $\wedge_{k=1}^{\infty}(1-\lambda_k) = 0$, will imply that $\vee_{k=1}^{\infty}(\lambda_k) = 1$, which is a contradiction to the hypothesis]. Since (X,T_1,T_2) is a pairwise fuzzy P-space, the pairwise fuzzy G_{δ}- set λ is a pairwise fuzzy open set in (X,T_1,T_2) and hence , $\mathrm{int}_{T_i}(\lambda) = \lambda$. This implies that $\mathrm{int}_{T_i}(\wedge_{k=1}^{\infty}(1-\lambda_k)) = \wedge_{k=1}^{\infty}(1-\lambda_k)$. Then , $1 - \mathrm{cl}_{T_i}[\vee_{k=1}^{\infty}(\lambda_k)] = 1 - \vee_{k=1}^{\infty}(\lambda_k)$ in (X,T_1,T_2). Hence $\mathrm{cl}_{T_i}[\vee_{k=1}^{\infty}(\lambda_k)] = \vee_{k=1}^{\infty}(\lambda_k)$, in (X,T_1,T_2).

Proposition 4.10 : If λ is a pairwise fuzzy first category set in a pairwise fuzzy submaximal and pairwise fuzzy P-space (X,T_1,T_2) , then λ is a pairwise fuzzy closed set in (X,T_1,T_2).

Proof : Let λ (≠ 1) be a pairwise fuzzy first category set in (X,T_1,T_2).

Then, $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense

sets in (X,T_1,T_2).). Since the fuzzy bitopological space (X,T_1,T_2) is a pairwise

fuzzy submaximal and pairwise fuzzy P-space , by proposition 4.10, we have

$\mathrm{cl}_{T_i}[\vee_{k=1}^{\infty}(\lambda_k)] = \vee_{k=1}^{\infty}(\lambda_k)$, in(X,T_1,T_2) and hence $\mathrm{cl}_{T_i}(\lambda) = \lambda$. Thus the

pairwise fuzzy first category set λ is a pairwise fuzzy closed set in (X,T_1,T_2).

Theorem 4.1 [11] : If a fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy Baire space, then,

(i). $\mathrm{int}_{T_i}(\lambda) = 0$, (i =1,2) , for each pairwise fuzzy first category set λ in (X,T_1,T_2)
(ii). $\mathrm{cl}_{T_i}(\lambda) = 1$, (i = 1,2), for each pairwise fuzzy residual set λ in (X,T_1,T_2).

Proposition 4.11 : If the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy Baire space and pairwise fuzzy submaximal space, then each pairwise fuzzy first category set is a pairwise fuzzy nowhere dense set in (X,T_1,T_2).

Proof : Let λ be a pairwise fuzzy first category set in (X,T_1,T_2). Since the

fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy submaximal and

pairwise fuzzy P-space , by proposition 4.10, λ is a pairwise fuzzy closed set in (X,T_1,T_2). Then $\mathrm{cl}_{T_i}(\lambda) = \lambda$, (i, = 1, 2), in (X,T_1,T_2). Also since (X,T_1,T_2) is a pairwise fuzzy Baire space, by theorem 4.1, int $_{Ti}(\lambda) = 0$,(i, = 1, 2), for the pairwise fuzzy first category set λ in (X, T_1, T_2) .Now $\text{int}_{T_i}\text{cl}_{T_j}(\lambda) = \text{int}_{T_i}(\lambda) = 0$, (i, = 1, 2) . Therefore, the pairwise fuzzy first category set λ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2).

Definition 4.2.[13] : A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy D-Baire space if every pairwise fuzzy first category set is a pairwise fuzzy nowhere dense set in (X, T1, T2). That is, if λ is a pairwise fuzzy first category set in (X,T_1,T_2), then $\text{int}_{T_i}\text{cl}_{T_j}(\lambda) = 0$,, (i,= 1, 2), in (X,$T_1$,$T_2$).

Remark: In view of the above proposition one will have the following result : "' Every pairwise fuzzy submaximal and pairwise fuzzy Baire space (X,T_1,T_2) is a pairwise fuzzy D-Baire space". For, if λ is a pairwise fuzzy first category set in (X,T_1,T_2), then by proposition 4.11, λ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2) and hence (X,T_1,T_2) is a pairwise fuzzy D-Baire space.

Proposition 4.12 : If λ is a pairwise fuzzy residual set in a pairwise fuzzy submaximal , pairwise fuzzy Baire and pairwise fuzzy P-space (X,T_1,T_2), then λ is a pairwise fuzzy open and pairwise fuzzy dense set in (X,T_1,T_2).

Proof : Let λ be a pairwise fuzzy residual set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy submaximal and pairwise fuzzy P-space , by proposition 4.8, λ is a pairwise fuzzy open set in (X,T_1,T_2). Also since (X,T_1,T_2) is a pairwise fuzzy Baire space, by theorem 4.1, cl $_{Ti}(\lambda) = 1$,(i, = 1, 2), for the pairwise fuzzy residual set λ in (X, T_1, T_2) and hence λ is a pairwise fuzzy open and a pairwise fuzzy dense set in (X,T_1,T_2).

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