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Learning Evaluation Measures for Fuzzy Association Rule Mining

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Abstract

Association rules are initially discovered in the market basket analysis [1] to identify frequently purchased items by customers. It gives certain regularities and dependencies within a data by finding frequent co-occurrence of items with a set of transactions. Usually support and confidence are the two important quality measures used to assess the quality of association rules. In fuzzy association rule mining, fuzzy support and confidence are used, which are based on t-norm operators. In addition to this opposition measures are also defined using s-norm operators. These measures were arised during the categorisation of transactions into positive and negative examples [5, 6], which we here redefined as true positive and true negative examples. Also we tried to extend these quality measures and studied their properties.

AMS subject classification:
Keywords: Fuzzy association rules, true positive and true negative examples, Support, Opposition, fuzzy average support, fuzzy average opposition.

1. Introduction

Data mining is the process of extracting previously unknown and potentially useful hidden predictive information from large amounts of data [1]. Association rules are initially discovered in the market basket analysis to identify frequently purchased items by customers. It give certain regularities and dependencies within a data by finding frequent co-occurrence of items with a set of transactions and relationships hidden in large data sets. The uncovered relationships can be expressed as association rules or frequent itemsets. In classical association rules, it is not possible to use every data for mining. In most real life applications, the database contains many attributes which are difficult to represent using binary values. In such cases fuzzy sets play a major role. So in the process of association rule mining, fuzzy sets can handle both quantitative and categorical data, providing the necessary support to use uncertain data types with existing algorithms. The approach of quantitative mining allow attributes to be either members or non-members of an interval which tends to make an under or over estimation of values leading to sharp boundary problems. The use of fuzzy sets in association analysis widens the type of relationships between attributes by allowing the intervals to overlap, giving partial memberships to different sets thus avoiding unnatural boundaries in the partitioning of the attribute domain and thus making the interpretation of rules in linguistic terms easier. Thus the obtained results using fuzzy approaches are easy to understand and to apply.

An association rule is of the form $A \rightharpoonup B$, where A and B are attributes or sets of attributes, which tells the idea that *when A occurs in a transaction, B is likely to occur as well*. The strength of association rules can be realized by a number of quality measures. Support and confidence are the two important quality measures used essentially. Support measures the validity of an association rule where as confidence measures the quality of the rule. Thus mining association rules means, to generate all association rules $A \rightharpoonup B$ that have support and confidence greater than the user specified thresholds. These measures can be generalized for fuzzy association rules as well. Here we study about the transaction types and redefined their terminologies as true positive, true negative, false positive, false negative examples to understand the true semantics of the transactions. In this paper we tried to extend some of these measures and studied their properties.

The next section explains the definition of association rules, their support and confidence measures. Sections 3 gives the definition of fuzzy set and fuzzy set operations. Section 4 is devoted to fuzzy association rules, fuzzy support and confidence measures, and fuzzy average support measures. Some of the properties of the defined measures are also discussed here.

2. Association Rules

An association rule gives an efficient way to identify and explore certain dependencies and regularities in a database. Association rule mining was introduced by Agrawal et al.

[1] as a way to discover frequently purchased items by customers in the market basket analysis.

Association rule mining is traditionally performed on a non-empty data table D with binary attributes. Let $I = \{i_1, i_2, \ldots, i_n\}$ be the universe of items. Let $T = \{t_1, t_2, \ldots, t_n\}$ be the transaction database (sets of objects) and let t_i represent the i^{th} transaction set in D such that $t_i \subseteq I$. Each transaction t_i is represented as a binary vector with $t_i(i_j) = 1$ if t_i bought the item i_j and $t_i(_j) = 0$ otherwise. An association rule is a direct association $A \rightharpoonup B$ with $A, B \subset I$ and $A \cap B = \emptyset$, where A is the antecedent and B is the consequent of the rule. A set of items X in I is called an itemset. If X contains k items, then X is called the k-itemset; T_X is the set of transactions that contain the itemset X; $|T|$ is the total number of transactions. Also we denote $\tilde{T}_A$ the set of transactions that does not contain the item A.

The validity and interstingness of an association rule is determined by the quality measures such as support and confidence. Support measures the extend of the simultaneous occurrence (proportion) of the items A and B in the database whereas confidence indicates the proportion of correct application of the rule.

2.1. Support Measures

Definition 2.1. The support count and respectively support of an association rule $A \rightharpoonup B$ is defined as:

$$supp\#(A \rightharpoonup B) = |T_A \cap T_B|$$

and respectively

$$supp(A \rightharpoonup B) = \frac{|T_A \cap T_B|}{|T|}$$

This definition of support count positive examples as it represents the transactions that explicitly support the association expressed by the rule.

De Cock et al. [5, 6] classified transactions with respect of an association rule as positive example, non-positive example, negative example, non-negative example.

In order to explore the true semantics of the transaction classification, we introduce some new terminologies encouraging from the definition of confusion matrix. Thus we define

Definition 2.2. Let $A \rightharpoonup B$ be an association rule and t be a transaction. Then

- t is a true positive example iff $t \in T_A \wedge t \in T_B$.
- t is a true negative example iff $t \notin T_A \vee t \notin T_B$.
- t is a false positive example iff $t \in T_A \wedge t \notin T_B$.
- t is a false negative example iff $t \notin T_A \vee t \in T_B$.

This indicates how effective our expectations. In true positive and true negative example, we got what we expect, according as presence or absence of items. In false positive examples we assume the presence of some items, but it was a false one and in false negative examples, we assume the absence of some items and it appeared to be false.

Based on this classification, we get the following different measures:

- minimum support count: $minsupp\#(A \rightharpoonup B) = |T_A \cap T_B|$
- maximum opposition count: $maxopp\#(A \rightharpoonup B) = |\tilde{T_A} \cup \tilde{T_B}|$
- minimum opposition count: $minopp\#(A \rightharpoonup B) = |T_A \cap \tilde{T_B}|$
- maximum support count: $maxsupp\#(A \rightharpoonup B) = |\tilde{T_A} \cup T_B|$

and the corresponding measures is given by

- minimum support: $minsupp(A \rightharpoonup B) = \dfrac{|T_A \cap T_B|}{|T|}$
- maximum opposition: $maxopp(A \rightharpoonup B) = \dfrac{|\tilde{T_A} \cup \tilde{T_B}|}{|T|}$
- minimum opposition: $minopp(A \rightharpoonup B) = \dfrac{|T_A \cap \tilde{T_B}}{|T|}$
- maximum support: $maxsupp(A \rightharpoonup B) = \dfrac{|\tilde{T_A} \cup T_B|}{|T|}$

Remark 2.3.

$$\begin{aligned} minsupp(A \rightharpoonup B) &\leq maxsupp(A \rightharpoonup B) \\ minopp(A \rightharpoonup B) &\leq maxopp(A \rightharpoonup B) \end{aligned}$$

2.2. Confidence Measures

Definition 2.4. The confidence of a rule $A \rightharpoonup B$ is defined as:

$$conf(A \rightharpoonup B) = \frac{supp\#(A \rightharpoonup B)}{supp\#(A)} \tag{1}$$

Confidence can be treated as the conditional probability $(P(B|A))$ or the relative cardinality of B with respect to A.

Definition 2.5. [Hullermeier, [9]] The confidence measure, n-confidence is defined as:

$$conf_n(A \rightharpoonup B) = \frac{minsupp\#(A \rightharpoonup B)}{minopp\#(A \rightharpoonup B)} \tag{2}$$

Definition 2.6. [DeCook et al., [5]] The pessimistic confidence p-confidence and the optimistic confidence o-confidence are defined as:

$$conf_p(A \rightharpoonup B) = \frac{minsupp\#(A \rightharpoonup B)}{maxopp\#(A \rightharpoonup B)} \tag{3}$$

$$conf_o(A \rightharpoonup B) = \frac{maxsupp\#(A \rightharpoonup B)}{minopp\#(A \rightharpoonup B)} \tag{4}$$

Definition 2.7. [11] Given a pair (M_1, M_2) of quality measures for association rules, with the property $M_1(A \rightharpoonup B) \leq M_2(A \rightharpoonup B)$, the inferior confidence and superior confidence are defined as

(a) inferior confidence

$$conf_*(A \rightharpoonup B) = \frac{\alpha.M_1(A \rightharpoonup B)}{(1-\beta).M_1(A \rightharpoonup B) + \beta.M_2(A \rightharpoonup \tilde{B})} \tag{5}$$

(a) superior confidence

$$conf^*(A \rightharpoonup B) = \frac{\alpha.M_2(A \rightharpoonup B)}{(1-\beta).M_1(A \rightharpoonup B) + \beta.M_1(A \rightharpoonup \tilde{B})} \tag{6}$$

Remark 2.8.

$$conf_p(A \rightharpoonup B) \leq conf_n(A \rightharpoonup B) \leq conf_o(A \rightharpoonup B)$$
$$conf_*(A \rightharpoonup B) \leq conf^*(A \rightharpoonup B)$$

3. Fuzzy Sets and Fuzzy Set Operations

A fuzzy set A in a given universal set X is a mapping from $X \rightarrow [0, 1]$, usually denoted as $A = \{(x, A(x)) : x \in X\}$ where $A(x)$ is called the grade of membership of each $x \in A$. The cardinality of a fuzzy set A in X is defined as $|A| = \Sigma_{x \in X} A(x)$.

A monotonic, associative and commutative mapping from $[0, 1]^2 \rightarrow [0, 1]$ is called $t-norm\ \mathcal{T}$, if it satisfies $\mathcal{T}(x, 1) = x$ for all $x \in [0, 1]$ and a $t-conorm\ \mathcal{S}$ if it satisfies $\mathcal{S}(x, 0) = x$ for all $x \in [0, 1]$. A fuzzy complement $\mathcal{N}$ is a decreasing mapping from $[0, 1] \rightarrow [0, 1]$ satisfying $\mathcal{N}(0) = 1$ and $N(1) = 0$.

For the fuzzy sets A and B in X, the complement, intersection and union can be defined by

$$coA(x) = \tilde{A}(x) = \mathcal{N}(A(x))$$
$$(A \cap_{\mathcal{T}} B)(x) = \mathcal{T}(A(x), B(x))$$
$$(A \cup_{\mathcal{S}} B)(x) = \mathcal{S}(A(x), B(x))$$

4. Fuzzy Association Rules

For quantitative and categorical attributes, binary rules are not satisfactory. As association rule mining deals with only binary transaction data, a new approach emerged out using fuzzy sets to mine quantitative data frequently present in databases efficiently, called fuzzy association rules.

Let $T = \{t_1, t_2, \ldots, t_n\}$ be the transaction database (sets of objects)and let t_i represents the i^{th} transaction in D. Let $I = \{i_1, i_2, \ldots, i_m\}$ be the universe of items. Each attribute i_k will associate with several fuzzy sets. In order to represent the fuzzy sets associated with i_k, we use the notion $F_{i_k} = \{f_{i_k}^1, f_{i_k}^2, \ldots, f_{i_k}^l\}$ where $f_{i_k}^j$ is the j^{th} fuzzy set in F_{i_k}.

Definition 4.1. A fuzzy association rule is of the form: $(X \in F_X) \rightharpoonup (Y \in F_Y)$ where $X, Y \subset I$, $X \cap Y = \emptyset$, $X = \{x_1, x_2, \ldots, x_p\}$ and $Y = \{y_1, y_2, \ldots, y_q\}$ are attributes, and $F_X = \{f_{x_1}, f_{x_2}, \ldots, f_{x_p}\}$ and $F_Y = \{f_{y_1}, f_{y_2}, \ldots, f_{y_q}\}$are fuzzy sets that characterize X and Y respectively. For each fuzzy set f_{ik}^l we can associate a membership function $\mu(f_{ik}^l) : dom(i_k) \rightarrow [0, 1]$ corresponding to the attribute i_k. Fuzzy sets and their corresponding membership functions have to be defined by domain experts.

Now we define the support and confidence measures of fuzzy association rules, extending the crisp association rules. Here we work on simple association rules $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ in which A and B are both attributes, not the sets of attributes.

4.1. Fuzzy Support Measures

Definition 4.2. The fuzzy support count and respectively fuzzy support of a fuzzy association rule $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ is usually defined as:

$$fsupp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \Sigma_{x\in T}(F_A \cap_{\mathcal{T}} F_B)(x)$$

and respectively

$$fsupp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\Sigma_{x\in T}(F_A \cap_{\mathcal{T}} F_B)(x)}{|T|}$$

Definition 4.3. Let $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ be a fuzzy association rule. Then we define:

a) fuzzy minimum support:

$$fminsupp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in T}(F_A \cap_{\mathcal{T}} F_B)(x)}{|T|}$$

b) fuzzy maximum opposition:

$$fmaxopp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in T}(\tilde{F_A} \cup_{\mathcal{S}} \tilde{F_B})(x)}{|T|}$$

c) fuzzy minimum oppposition:

$$fminopp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in T}(F_A \cap_{\mathcal{T}} \tilde{F_B})(x)}{|T|}$$

d) fuzzy maximum support:

$$fmaxopp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in T}(\tilde{F_A} \cup_{\mathcal{S}} F_B)(x)}{|T|}$$

4.2. Fuzzy Average Support Measures

Studying the above definitions in view of averaging operators we define another two measures, fuzzy average support and fuzzy average opposition of the fuzzy association rule.

Definition 4.4. The fuzzy average support and fuzzy average opposition of a fuzzy association rule $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ is defined as:

a) fuzzy average support:

$$favrgsupp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in T}[\lambda(\tilde{F_A} \cup_{\mathcal{S}} F_B)(x) + (1-\lambda)(F_A \cap_{\mathcal{T}} F_B)(x)]}{|T|}$$

where $\lambda \in [0, 1]$

b) fuzzy average opposition:

$$favrgopp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in T}[\lambda(\tilde{F_A} \cup_{\mathcal{S}} \tilde{F_B})(x) + (1-\lambda)(F_A \cap_{\mathcal{T}} \tilde{F_B})(x)]}{|T|}$$

where $\lambda \in [0, 1]$

Similarly we can define the corresponding count measures: $fminsupp\#$, $fmaxopp\#$, $fminopp\#$, $fmaxsupp\#$, $favrgsupp\#$, $favrgopp\#$.

4.3. Fuzzy Confidence Measures

Definition 4.5. The fuzzy confidence of a fuzzy association rule $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ is defined as:

$$fconf(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in T}(F_A \cap_{\mathcal{T}} F_B)(x)}{\Sigma_{x\in T} F_A(x)}$$

Now we define fuzzy version of $conf_n$, $conf_p$, $conf_o$ defined by Hullermeier, [9] and DeCook et al., [5].

Definition 4.6. The fuzzy confidence measures n-confidence, p-confidence and o-confidence of a fuzzy association rule $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ is defined as:

a) fuzzy n-confidence

$$fconf_n(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{fminsupp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)}{fminopp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)} \tag{7}$$

b) fuzzy pessimistic confidence

$$fconf_p(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{fminsupp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)}{fmaxopp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)} \tag{8}$$

c) fuzzy optimistic confidence

$$fconf_o(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{fmaxsupp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)}{fminopp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)} \tag{9}$$

Now we define two more confidence measure fuzzy m-confidence and fuzzy mn-confidence.

Definition 4.7. The fuzzy confidence measure m-confidence of a fuzzy assocaition rule $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ is defined as:

$$fconf_m(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{fmaxsupp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)}{fmaxopp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)} \tag{10}$$

Definition 4.8. The fuzzy confidence measure mn-confidence of a fuzzy assocaition rule $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ is defined as:

$$fconf_{mn}(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{favrgsupp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)}{favrgopp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)} \tag{11}$$

Remark 4.9.

$$i) For \lambda = 0, fconf_{mn} = fconf_n$$
$$ii) For \lambda = 1, fconf_{mn} = fconf_m$$

Definition 4.10. Refer Definition 2.1 The fuzzy inferior confidence and respectively fuzzy superior confidence of the fuzzy association rule $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ is defined as

a) fuzzy inferior confidence
$fconf_*\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$

$$= \frac{\alpha.M_1(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)}{(1-\beta).M_1(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) + \beta.M_2(\langle A, F_A\rangle \rightharpoonup \langle B, \tilde{F_B}\rangle)} \tag{12}$$

b) superior confidence
$fconf^*\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$

$$= \frac{\alpha.M_2(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)}{(1-\beta).M_2(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) + \beta.M_1(\langle A, F_A\rangle \rightharpoonup \langle B, \tilde{F_B}\rangle)} \tag{13}$$

with $\alpha, \beta \in [0, 1]$.

4.4. Properties of Fuzzy Support and Confidence Measures

Remark 4.11. The measures $fconf_*$ and $fconf^*$ defined above represent a general and integrated frame for the quality measures as most of them can be obtained particularising $\mathcal{T}, \mathcal{S}, \mathcal{N}, \alpha, \beta, M_1, M_2$. Thus we get the following

a) For $M_1 = M_2 = fsupp\#, \alpha = \beta = \frac{1}{2}, \mathcal{T}(a,b) = ab$ and $\mathcal{N}(a) = 1 - a$, we have:
$$fconf_* = fconf^* = fconf$$

b) For $M_1 = M_2 = fsupp\#, \alpha = \beta = 1$ we have:
$$fconf_* = fconf^* = fconf_n$$

c) For $M_1 = fminsupp\#$, $M_2 = fmaxsupp\#$ and $\alpha = \beta = 1$ we have:
$$fconf_* = fconf_p$$
$$fconf^* = fconf_o$$

d) For $M_1 = fminsupp\#$, $M_2 = fmaxsupp\#$ and $\alpha = \beta = \frac{1}{2}$ we have:
$$fconf_* = fminsupp$$
$$fconf^* = fmaxsupp$$

e) For $M_1 = fminopp\#$, $M_2 = fmaxopp\#, \alpha = \beta = \frac{1}{2}$ and $\mathcal{N}(a) = 1 - a$ we have:
$$fconf_* = fminopp$$
$$fconf^* = fmaxopp$$

4.5. Properties of Fuzzy average support and Fuzzy average opposition measures

Using the new measures, fuzzy average support and fuzzy average opposition we arrive at the following:

a) For $M_1 = M_2 = favrgsupp\#, \alpha = \beta = \frac{1}{2}, \mathcal{T}(a,b) = ab, \mathcal{S}(a,b) = a+b-ab$ and $\mathcal{N}(a) = 1 - a$, we have:

   i) For $\lambda = 0$,
$$fconf_* = fconf^* = fconf$$

ii) For $\lambda = 1$,

$$\frac{1}{fconf_*} = \frac{1 - fminsupp}{1 - fminopp} + 1 = \frac{1}{fconf^*}$$

b) For $M_1 = M_2 = favrgsupp\#$ and $\alpha = \beta = 1$, we have

i) For $\lambda = 0$,

$$fconf_* = fconf^* = fconf_n = fconf_{mn}$$

ii) For $\lambda = 1$,

$$fconf_* = fconf^* = fconf_m = fconf_{mn}$$

c) For $M_1 = M_2 = favrgopp\#, \alpha = \beta = \frac{1}{2}, \mathcal{T}(a,b) = ab, \mathcal{S}(a,b) = a + b - ab$ and $\mathcal{N}(a) = 1 - a$, we have:

i) For $\lambda = 0$,

$$fconf_* = fconf^* = 1 + fconf$$

ii) For $\lambda = 1$,

$$\frac{1}{fconf_*} = \frac{1 - fminopp}{1 - fminsupp} + 1 = \frac{1}{fconf^*}$$

d) For $M_1 = M_2 = favrgopp\#$ and $\alpha = \beta = 1$, we have

i) For $\lambda = 0$,

$$fconf_* = fconf^* = \frac{1}{fconf_n} = \frac{1}{fconf_{mn}}$$

ii) For $\lambda = 1$,

$$fconf_* = fconf^* = \frac{1}{fconf_m} = \frac{1}{fconf_{mn}}$$

5. Conclusion

In this paper we extend some quality measures defined for crisp association rule to fuzzy association rule. Here we study about the transaction types and redefined their terminologies as true positive, true negative, false positive, false negative examples to understand the true semantics of the transactions. Based on that we defined various support and confidence measures and extended to fuzzy average support and opposition measures. We studied some of the properties of the newly defined measures and compared them with the existing ones.

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