

On Anti Fuzzy Graphs

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Abstract

In this paper, we introduce the concept of an anti fuzzy graph. Some types of anti fuzzy graphs are discussed with examples. Further we introduce the concept of degrees on anti fuzzy graph. Based on the results we characterize the regular and irregular anti fuzzy graph. Some types of regular and irregular anti fuzzy graph are discussed. Some basic theorems and results are obtained.

Keywords: Fuzzy graph, vertex degrees, distance, regular fuzzy graph, irregular fuzzy graph.

Mathematical Classification: 05C72, 05C07, 05C12, 05C99.

I. INTRODUCTION

The concept of fuzzy graph was first introduced by Kaufmann[1] from the fuzzy relation introduced by Zedah[2]. Although Rosenfield[3] introduced another elaborated definition, including fuzzy vertex and fuzzy edge, and also introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. A.Nagoorgani and K.Radha[4] was introduced the concept of regular fuzzy graph and A.Nagoorgani and S.R. Latha [5] was introduced concept of irregular fuzzy graph. In this paper, we introduce the concept of anti fuzzy graphs. we discussed some types of anti fuzzy graphs. Based on the concept of degrees on anti fuzzy graph, we classify the types regular, irregular anti fuzzy graph and derived some theorems and results.

II. AN ANTI FUZZY GRAPHS

Definition 2.1

A fuzzy graph $G=(\sigma, \mu)$ is said to be an anti fuzzy graph with a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$, where for all $u, v \in V$, we have $\mu(u,v) \geq \sigma(u) \vee \sigma(v)$ and it is denoted by $G_A(\sigma, \mu)$.

Note

μ is considered as reflexive and symmetric. In all examples σ is chosen suitably. ie., undirected anti fuzzy graphs are only considered.

Definition 2.2

An anti fuzzy graph $H_A=(\tau,\rho)$ is called an anti fuzzy subgraph of $G_A(\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for all $u \in V$ and $\rho(u,v) \leq \mu(u,v)$ for all $u,v \in V$.

Definition 2.3

An anti fuzzy graph $H_A=(\tau,\rho)$ is called an spanning anti fuzzy subgraph of $G_A(\sigma, \mu)$ if $\tau(u) = \sigma(u)$ for all $u \in V$.

Example 2.4

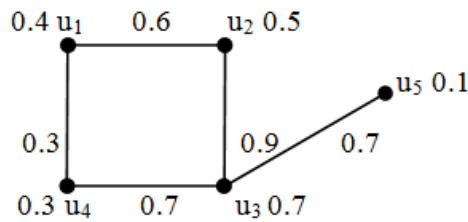


Fig.1 Anti Fuzzy graph

In Fig.1, Anti fuzzy subgraph of G_A is $\tau=\{u_1|0.4, u_2|0.5, u_4|0.2\}$ and $\rho=\{(u_1,u_2)|0.6, (u_4,u_1)|0.5\}$. Spanning anti fuzzy subgraph of G_A is $\tau=\{u_1|0.4, u_2|0.5, u_3|0.7, u_4|0.3, u_5|0.1\}$ and $\rho=\{(u_1,u_2)|0.6, (u_2,u_3)|0.9, (u_3,u_4)|0.7, (u_3,u_5)|0.7\}$.

Definition 2.5

Let $G_A=(\sigma,\mu)$ be an anti fuzzy graph and τ be an anti fuzzy subset of σ . ie., $\tau(u) \leq \sigma(u)$ for all $u \in V$ then the anti fuzzy subgraph of $G_A=(\sigma,\mu)$ induced by τ is the maximal anti fuzzy subgraph of $G_A=(\sigma,\mu)$ that has fuzzy vertex set τ then anti fuzzy subgraph is $H(\tau,\rho)$ where $\rho(u,v)=\tau(u) \vee \tau(v) \vee \mu(u,v)$ for all $u,v \in V$.

Definition 2.6

The underlying crisp graph of an anti fuzzy graph $G_A = (\sigma, \mu)$ is denoted by $G_A^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in V / \sigma(u) > 0\}$ and $\mu^* = \{(u,v) \in V \times V / \mu(u,v) > 0\}$.

Definition 2.7

An anti fuzzy graph $G_A = (\sigma, \mu)$ is a strong anti fuzzy graph if $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v) \in \mu^*$ and is a complete anti fuzzy graph if $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v) \in \sigma^*$. Two nodes u and v are said to be neighbors if $\mu(u,v) > 0$.

Definition 2.8

Two vertices u and v in G are called adjacent if $(\frac{1}{2})[\sigma(u) \vee \sigma(v)] \geq \mu(u,v)$.

Definition 2.9

An arc (u,v) is said to be a strong edge, if $\mu(u,v) \geq \mu^\infty(u,v)$ and if $\mu(u,v) > 0$ then node v is said to be the strong neighbor of u .

Definition 2.10

A path P in an anti fuzzy graph is a sequence of distinct vertices $u_0, u_1, u_2, \dots, u_n$ such that $\mu(u_{i-1}, u_i) > 0$, $1 \leq i \leq n$. here $n \geq 0$ is called the length of the path P . The consecutive pairs (u_{i-1}, u_i) are called the edges of the path. The degree of membership of a weakest arc is defined as its strength.

Definition 2.11

A cycle in G_A is said to be an anti fuzzy cycle if it contains more than one weakest arc.

Definition 2.12

An isomorphism between two fuzzy graphs G_{A_1} and G_{A_2} is a bijective map $h : V_1 \rightarrow V_2$ that satisfies $\sigma_1(u) = \sigma_2(h(u)) \forall u \in V_1$ and

$$\mu_1(u, v) = \mu_2(h(u), h(v)) \forall u, v \in V_1.$$

And we write $G_{A_1} \approx G_{A_2}$. An automorphism of G_A is an isomorphism of G_A with itself.

Definition 2.13

The complement of an anti fuzzy graph $G_A = (\sigma, \mu)$ is an anti fuzzy subgraph $\overline{G_A} = (\overline{\sigma}, \overline{\mu})$ where $\overline{\sigma} = \sigma$ and $\overline{\mu}(u, v) = 0$ if $\mu(u, v) > 0$ and $\overline{\mu}(u, v) = (\sigma(u) \vee \sigma(v))$. It given that $\overline{G_A} = G_A$ if G_A is a strong anti fuzzy graph.

Example 2.14

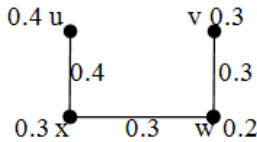


Fig.2 Anti fuzzy graph G_A

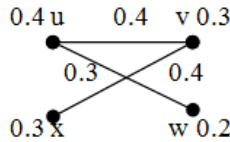


Fig.3 $\overline{G_A}$

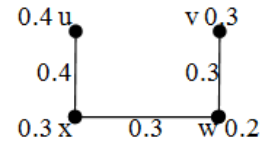


Fig.4 $\overline{G_A}$

The automorphism group of G_A and $\overline{G_A}$ are identical. But considering the definition of an anti fuzzy graph $\mu(u, v) \geq \sigma(u) \vee \sigma(v)$. From the observation, we induce to modify the definition of complement of anti fuzzy graph.

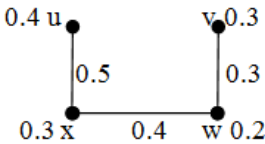


Fig.5 Anti fuzzy graph G_A

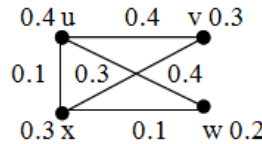


Fig.6 $\overline{G_A}$

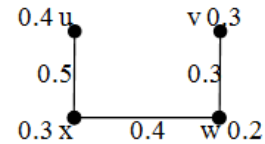


Fig.7 $\overline{G_A}$

The complement of an anti fuzzy graph $G_A(\sigma, \mu)$ is an anti fuzzy graph $\overline{G_A} = (\overline{\sigma}, \overline{\mu})$ where $\overline{\sigma} = \sigma$ and $\overline{\mu}(u, v) = \mu(u, v) - (\sigma(u) \vee \sigma(v))$ for all u, v in V . and the complement of complement of an anti fuzzy graph is denoted by $\overline{\overline{G_A}} = (\overline{\overline{\sigma}}, \overline{\overline{\mu}})$ where $\overline{\overline{\sigma}} = \sigma$ and $\overline{\overline{\mu}}(u, v) = \overline{\mu}(u, v) + (\sigma(u) \vee \sigma(v))$ for all u, v in V .

Definition 2.15

Let $G_A = (\sigma, \mu)$ be an anti fuzzy graph. The degree of a vertex $\sigma(u)$ of an anti fuzzy graph is sum of degree of membership of all those edges which are incident on vertex $\sigma(u)$ and is denoted by $d_{G_A}(\sigma(u)) = d(\sigma(u)) = \sum_{u \neq v} \mu(u, v) = \sum_{uv \in E} \mu(u, v)$.

Example 2.16

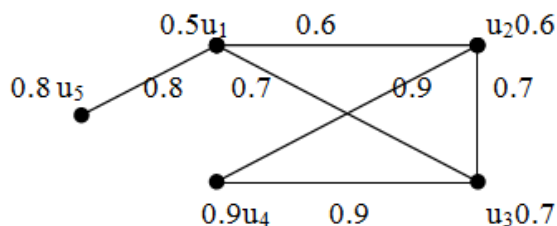


Fig.8 Anti fuzzy graph G_A

In Fig.8, Degree of vertex $\sigma(u_1)$ = Degree of membership of all those edges which are incident on a vertex $\sigma(u_1)$

$$\begin{aligned}
 &= \mu(u_1, u_2) + \mu(u_1, u_3) + \mu(u_1, u_5) \\
 &= 0.6 + 0.7 + 0.8 \\
 &= 2.1
 \end{aligned}$$

i.e., $d(\sigma(u_1)) = 2.1,$

Similarly, $d(\sigma(u_2)) = 2.2, d(\sigma(u_3)) = 2.3, d(\sigma(u_4)) = 1.8, d(\sigma(u_5)) = 0.8.$

Theorem 2.17

If $G_A: (\sigma, \mu)$ is an anti fuzzy graph with $G_A(V, E).$

Then $\sum_{i=1}^n d(\sigma(v_i)) = 2 \sum_{i=1}^n \mu(v_i, v_{i+1}).$

Proof:

Consider $G_A(\sigma, \mu)$ is an anti fuzzy graph with $G(V, E).$ Let us consider G_A has n vertices such as $\sigma(v_1), \sigma(v_2), \dots, \sigma(v_n).$ We know that each edges are incident on two vertices then by the definition of degree of any vertex of an anti fuzzy graph, the sum of the degree of vertices in anti fuzzy graph $G_A(\sigma, \mu)$ is twice the degree of membership of edges in $G_A.$ Thus

$$\sum_{i=1}^n d(\sigma(v_i)) = 2 \sum_{i=1}^n \mu(v_i, v_{i+1}).$$

In Fig.8, $d(\sigma(u_1)) = 2.1, d(\sigma(u_2)) = 2.2, d(\sigma(u_3)) = 2.3, d(\sigma(u_4)) = 1.8, d(\sigma(u_5)) = 0.8.$

$$\begin{aligned}
 \sum_{i=1}^n \mu(v_i, v_{i+1}) &= 0.6 + 0.7 + 0.7 + 0.9 + 0.9 + 0.8 \\
 &= 4.6
 \end{aligned}$$

$$\begin{aligned} \sum_{i=1}^n d(\sigma(v_i)) &= 2.1 + 2.2 + 2.3 + 1.8 + 0.8 \\ &= 9.2 \\ &= 2(4.6) \\ &= 2 \sum_{i=1}^n \mu(v_i, v_{i+1}) \end{aligned}$$

Definition 2.18

$G_A(\sigma, \mu)$ is an anti fuzzy graph then the distance $d(\sigma(u), \sigma(v))$ between two of its vertices $\sigma(u)$ and $\sigma(v)$ is the length of shortest path between them, i.e., $d(\sigma(u), \sigma(v)) = \min [\sum_{u,v \in V} \mu(u, v)]$.

Example 2.19

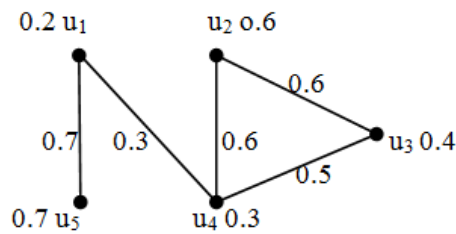


Fig.9 Anti fuzzy graph G_A

In Fig. 9, consider the path from u_1 to u_3 . Then there exists two path between $\sigma(u_1)$ and $\sigma(u_3)$. They are

- 1) $\mu(u_1, u_4), \mu(u_4, u_3)$
- 2) $\mu(u_1, u_4), \mu(u_4, u_2), \mu(u_2, u_3)$

The distance between the vertices $\sigma(u_1)$ and $\sigma(u_3)$,

Via path 1, is $\mu(u_1, u_4) + \mu(u_4, u_3) = 0.3 + 0.5 = 0.8$

Via path 2, is $\mu(u_1, u_4) + \mu(u_4, u_2) + \mu(u_2, u_3) = 0.3 + 0.6 + 0.6 = 1.5$

Therefore, $d(\sigma(u), \sigma(v)) = \min [0.8, 1.5]$
 $= 0.8$

III REGULAR ANTI FUZZY GRAPH

Definition 3.1

$G_A(\sigma, \mu)$ is an anti fuzzy graph on $G^*(V, E)$. If $d_{G_A}(\sigma(v)) = k$ for all $v \in V$, i.e., if each vertex has the same degree k , then G_A is said to be a regular anti fuzzy graph of degree k or a k -regular anti fuzzy graph.

Example 3.2

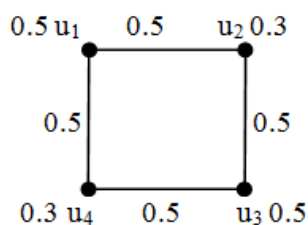


Fig.10 Anti Fuzzy graph G_A

In Fig. 10, $d(\sigma(u_1)) = 1$, $d(\sigma(u_2)) = 1$, $d(\sigma(u_3)) = 1$, $d(\sigma(u_4)) = 1$.

So G_A is 1-regular anti fuzzy graph.

Definition 3.3

Let $G_A(\sigma, \mu)$ be an anti fuzzy graph on G^* . The total degree of a vertex $u \in V$ is defined by $td(u) = td_{G_A}(u) = \sum_{u \neq v} \mu(u, v) + \sigma(u) = \sum_{uv \in E} \mu(u, v) + \sigma(u) = d_{G_A}(\sigma(u)) + \sigma(u)$.

If each vertex of G_A has the same total degree k then G_A is said to be a totally regular anti fuzzy graph of total degree k or a k -totally regular anti fuzzy graph.

In fig.10, $td(\sigma(u_1)) = 1.5$, $td(\sigma(u_2)) = 1.3$, $td(\sigma(u_3)) = 1.5$, $td(\sigma(u_4)) = 1.3$. Here G_A is not totally regular anti fuzzy graph.

Remark

1. Any connected anti fuzzy graph with two vertices is regular.
2. G_A is a k -regular anti fuzzy graph if and only if $\delta = \Delta = k$.
3. A complete anti fuzzy graph G_A need not be regular.
4. If G_A is k -regular then $k = l(n-1)$ where $l = \sigma(u_i) \forall i=1, 2, \dots, n$ and $u_i \in V$, n is the number of vertices in G_A and all arcs of G_A are strong arcs.

Theorem 3.4

For any strong anti fuzzy graph G_A with $\sigma(u_i) = l \forall i=1, 2, \dots, n$, G_A is regular iff G_A is totally regular.

Proof:

Let us consider that G_A is strong anti fuzzy graph with $\sigma(u_i)=l \forall i=1,2,..n$. we know that the degrees of all vertices in G_A is $k=l(n-1)$. Therefore, Consider for any $\sigma(u_1)$, $td(\sigma(u_1)) = d(\sigma(u_1)) + \sigma(u_1) = k + l$. Similarly, $td(\sigma(u_i)) = d(\sigma(u_i)) + \sigma(u_i) = k + l$. for all $u_i \in V$. Hence G_A is totally regular anti fuzzy graph.

Conversely. Suppose G_A is k_1 - totally anti fuzzy graph. Then for u_1 ,

$$td(\sigma(u_1)) = d(\sigma(u_1)) + \sigma(u_1)$$

$$k_1 = d(\sigma(u_1)) + l.$$

$$\therefore d(\sigma(u_1)) = k_1 - l.$$

Similarly, $d(\sigma(u_i)) = k_1 - l$ for all $u_i \in V$. since $\sigma(u_i)=l$. Hence G_A is regular anti fuzzy graph.

Theorem 3.5

If an anti fuzzy graph G_A is both regular and totally regular anti fuzzy graph then $\sigma(u_i)$ is a constant for all i .

Proof:

Let us consider that G_A is regular and totally regular anti fuzzy graph with n vertices.

For regular anti fuzzy graph G_A , $d(\sigma(u_1))=k$ and for totally regular anti fuzzy graph G_A , $td(\sigma(u_1))=k_1$.

By the definition of totally regular anti fuzzy graph, for any u_1 ,

$$td(\sigma(u_1)) = d(\sigma(u_1)) + \sigma(u_1)$$

$$k_1 = k + \sigma(u_1)$$

$$\sigma(u_1) = k_1 - k \text{ for all } u_1 \in V$$

$$= \text{a constant.}$$

Therefore, $\sigma(u_i)$ is a constant, for all i .

Theorem 3.6

G_A is strong anti fuzzy graph with n vertices. Every vertices in G_A has a constant degree values then one of the following condition holds

1. If G_A is cycle then G_A is k -regular anti fuzzy graph
2. If G_A is k -regular anti fuzzy graph then the size of G_A is $kn/2$.

Proof:

Let us consider that G_A is strong anti fuzzy graph with n vertices and all vertices in G_A has same membership values then which have constant degree values. ie., $d(\sigma(u_i)) = l \forall i=1,2,\dots,n$ and $u_i \in V$. G_A is strong anti fuzzy graph on a cycle G_A^* . So all edges in G_A^* are strong. Then $\sigma(u_i) = l \forall u_i \in C$. ie., all edges in cycle assign same membership value as l . Every vertex in cycle are incident with only two edges. Therefore, $d(\sigma(u_i)) = 2l$ which is a constant say k , then C is k -regular anti fuzzy graph.

Suppose G_A is not strong anti fuzzy graph and k -regular anti fuzzy graph. If C is a cycle with odd length then $\mu(u,v) = k/2$. If C is a cycle with even length then alternate edges assign same membership values. Therefore, G_A is cycle then G_A is k -regular anti fuzzy graph.

By the definition of k -regular anti fuzzy graph of G_A , $d(\sigma(u_i)) = k \forall u_i \in V$. Let us consider that the size of G_A is $q = \sum_i \mu(u_i, u_{i+1})$.

We know that sum of the degree of vertices in G_A is twice the sum of the degree of membership of (u_i, u_{i+1}) .

$$\text{ie., } \sum_{i=1}^n d(\sigma(v_i)) = 2 \sum_{i=1}^n \mu(v_i, v_{i+1}).$$

$$\sum_{i=1}^n d(\sigma(v_i)) = 2 q$$

$$k + k + \dots + k(n \text{ times}) = 2 q$$

$$nk = 2 q$$

$$q = nk/2.$$

Hence proved.

Theorem 3.7

If G_A is k -regular and k_1 - totally regular anti fuzzy graph then $p = n (k_1 - k)$ where n is the number of vertices.

Proof:

By the definition of k_1 - totally regular anti fuzzy graph G_A , $td(\sigma(u)) = d(\sigma(u)) + \sigma(u)$

$$\Rightarrow k_1 = \frac{p}{n} + k$$

$$\Rightarrow n k_1 = p + n k$$

$$\Rightarrow n k_1 - k = p$$

$$\Rightarrow p = n (k_1 - k).$$

IV. IRREGULAR ANTI FUZZY GRAPHS

Definition 4.1

An anti fuzzy graph $G_A(\sigma, \mu)$ is called irregular, if there is a vertex which is adjacent only to vertices with distinct degrees.

For example, Consider Fig.9, $d(\sigma(u_1)) = 1$, $d(\sigma(u_2)) = 1.2$, $d(\sigma(u_3)) = 1.1$, $d(\sigma(u_4)) = 1.4$, $d(\sigma(u_5)) = 0.7$, so the adjacent vertices have distinct values in G_A . Hence G_A is an irregular anti fuzzy graph.

Definition 4.2

An anti fuzzy graph $G_A(\sigma, \mu)$ is called a neighbourly irregular anti fuzzy graph if every two adjacent vertices of G have distinct degree.

Example 4.3

Define $G_A(\sigma, \mu)$ by $\sigma(u) = 0.4$, $\sigma(v) = 0.2$, $\sigma(w) = 0.3$, $\sigma(x) = 0.8$, and $\mu(u, v) = 0.4$, $\mu(v, w) = 0.4$, $\mu(x, u) = 0.8$. Then $d(\sigma(u)) = 1.2$, $d(\sigma(v)) = 0.8$, $d(\sigma(w)) = 0.4$, $d(\sigma(x)) = 0.8$. Here every pair of adjacent vertices have distinct degree values. Hence G_A is neighbourly irregular anti fuzzy graph. We observe that neighbourhood of a vertex may assign same degree values.

Definition 4.4

An anti fuzzy graph $G_A(\sigma, \mu)$ is said to be totally irregular anti fuzzy graph, if there is a vertex which is adjacent to vertices with distinct total degrees.

For example, Consider Fig.8, $td(\sigma(u_1)) = 2.6$, $td(\sigma(u_2)) = 2.8$, $td(\sigma(u_3)) = 2.3$, $td(\sigma(u_4)) = 2.7$, $td(\sigma(u_5)) = 1.6$, here every pair of adjacent vertices have distinct total degree values in G_A . Hence G_A is totally irregular anti fuzzy graph.

Definition 4.5

If every two adjacent vertices of an anti fuzzy graph $G_A(\sigma, \mu)$ have distinct total degree, then G is said to be a neighbourly total irregular fuzzy graph.

For example, Consider Fig.9, $td(\sigma(u_1)) = 1.2$, $td(\sigma(u_2)) = 1.8$, $td(\sigma(u_3)) = 1.5$, $td(\sigma(u_4)) = 1.7$, $td(\sigma(u_5)) = 1.2$, here every pair of adjacent vertices have distinct total degree values in G_A . Hence G_A is neighbourly totally irregular anti fuzzy graph.

Definition 4.6

An anti fuzzy graph $G_A(\sigma, \mu)$ is said to be a highly irregular fuzzy graph if every vertex of G is adjacent to vertices with distinct degrees.

For example, Consider Fig.8, $d(\sigma(u_1)) = 2.1$, $d(\sigma(u_2)) = 2.2$, $d(\sigma(u_3)) = 1.6$, $d(\sigma(u_4)) = 1.8$, $d(\sigma(u_5)) = 0.8$. Here in G_A , every vertex has distinct degrees. Hence G_A is a highly irregular anti fuzzy graph.

Proposition 4.7

- i) A highly irregular anti fuzzy graph need not be a neighbourly irregular anti fuzzy graph.
- ii) A neighbourly irregular anti fuzzy graph need not be a highly irregular anti fuzzy graph.

Theorem 4.8

Let $G_A(\sigma, \mu)$ be an anti fuzzy graph. G_A is highly irregular anti fuzzy graph and neighbourly irregular anti fuzzy graph if and only if the degrees of all vertices of G_A are distinct.

Proof:

Let us consider an neighbourly irregular and highly irregular anti fuzzy graph G_A with n vertices say $v_1, v_2, v_3, \dots, v_n$. By the definition of highly irregular, the vertices are adjacent to distinct degrees and neighbourly irregular fuzzy graphs, no two adjacent vertices have same degree. Therefore, all the vertices in G_A assign different degrees such as $k_1, k_2, k_3, \dots, k_n$ respectively. Hence the degrees of all vertices of G are distinct.

Conversely assume that the degrees of all vertices of G_A are distinct. This means that every two adjacent vertices have distinct degrees and every vertex of the adjacent vertices have distinct degrees. Hence G is neighbourly irregular and highly irregular fuzzy graphs.

Theorem 4.9

$G_A(\sigma, \mu)$ is cyclic anti fuzzy graph with vertices $n (> 3)$. G_A is neighbourly irregular and highly irregular anti fuzzy graph if and only if each $\mu(u, v) \rightarrow [0, 1]$, one of the following two conditions holds.

1. The weights of the edges between every pair of vertices are all distinct.
2. Every edge in G_A is strong edge.

Proof:

Let us assume that, G_A is neighbourly irregular and highly irregular anti fuzzy graph. So, all the vertices in G_A assign distinct degrees. Suppose any two edges assign same weights then the vertices of this incident edges may assign same degrees which is contradict to our assumption.

In cyclic anti fuzzy graph, the adjacent vertices assign distinct values. Suppose, edges in G_A are not strong edges which implies the vertices may assign same values. This is contracting to neighbourly irregular and highly irregular anti fuzzy graph. So, all edges in G_A are strong.

V. CONCLUSION

The concept of anti fuzzy graph is introduced. We have done a complete analysis in types of anti fuzzy graph. This analysis implies to identify the degrees and distance in anti fuzzy graph.

Further we introduced regular anti fuzzy graph, irregular anti fuzzy graph and types of them. We derived some theorems and results.

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