

Mathematical Modelling of affected diseases due to HIV Infection using Fuzzy Cognitive Maps

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Abstract

This paper highlights how different deadly diseases affect the human body suffering from HIV (Human Immunodeficiency Virus) Infection. The mathematical modeling of the interpretation of the effects and results of these diseases are explained using Fuzzy Cognitive Maps.

Keywords: Fuzzy Cognitive Maps, HIV, Antiretroviral therapy, diseases affected due to HIV infection.

I. INTRODUCTION

HIV infection in a human body is a disease caused by the virus known as Human Immunodeficiency Virus which spread through certain body fluids (seimen, vaginal fluid, blood, breast milk, saliva) that largely attacks the body's immune system. HIV can be transmitted in many ways, such as unprotected sexual contact ((including anal and also Oral sex)), blood transfusion, and contaminated hypodermic needles, and from mother to child during pregnancy, birth, or breast-feeding. The virus specially attacks the CD4 cells, often called T cells and destroys them hugely and weakens the human immune system to a great extent. This damage to the immune system makes it harder and harder for the body to fight off infections making people much more vulnerable to infections and diseases. Opportunistic infections such as Tuberculosis, cancers, etc. take advantage of a very weak immune system and signal that the person has AIDS.

It is widely believed that HIV originated in Kinshasa, in the Democratic Republic of Congo around 1920 when HIV crossed species from chimpanzees to humans. Scientists identified a type of chimpanzee in Central Africa as the source of HIV infection in

humans. They believe that the chimpanzee version of the immunodeficiency virus (called simian immunodeficiency virus, or SIV) most likely was transmitted to humans and mutated into HIV when humans hunted these chimpanzees for meat and came into contact with their infected blood [1]. Studies show that HIV may have jumped from apes to humans as far back as the late 1800s. Over decades, the virus slowly spread across Africa and later into other parts of the world. We know that the virus has existed in the United States since at least the mid- to late 1970s. AIDS has caused almost 30 million dead (as of 2009). From 2010 approximately 34 million people are living with HIV worldwide. No effective cure for HIV currently exists, but with proper treatment and medical care, HIV can be controlled. The medicine used to treat HIV is called antiretroviral therapy or ART. If taken the right way, every day, this medicine can dramatically prolong the lives of many people with HIV, keep them healthy, and greatly lower their chance of transmitting the virus to others. Today, a person who is diagnosed with HIV, treated before the disease is far advanced, and stays on treatment can live a nearly as long as someone who does not have HIV.

In recent years Fuzzy Cognitive Maps (FCM) has become a useful Soft Computing technique for modeling and simulation. They are connectionist and recurrent structures involving concepts describing the system behavior, and causal connections. A simple Cognitive Map is a directed graph composed by two components: nodes and connections [2]. Nodes or concepts represent variables describing the system; while links among concepts are used to regulate the causality and take values either 0, -1 or +1. If increase (or decrease) in one concept, a lead to increase (or decrease) in another, then it gives the value 1. If no relation exists between two concepts, then the value 0 is given. If increase (or decrease) in one causalities decreases (or increases) another, then give the value -1. When the nodes of the FCM are fuzzy sets then they are called as fuzzy nodes. The Cognitive Mapping theory was originally proposed by R. Axelrod [2] who focused on policy domain studies, that is, for modeling social scientific knowledge.

FCM theory has gained a lot of attention among researchers. For example, in [3] the authors propose a FCM for studying travel behavior in modern societies allowing policy-makers better understanding of these issues. As another example, [4] describes a model based on FCM for analyzing the dynamics of HIV protease protein; enabling to discover relevant knowledge when a mutation takes place. Also, FCM theory has been widely used in other applications fields including: engineering, risk analysis, business, decision making tasks, management, system control, medicine, game theory, and also telecommunications [5, 6], etc.

II. FORMULATION OF MATHEMATICAL MODEL

The model formulation to interpret the effects of different diseases resulting through HIV is done via the following algorithmic steps [7]:

Step 1:

Identify the factors characterized by nodes. Let V_1, V_2, \dots, V_n be the nodes or concepts of the FCM. A directed graph is drawn using edge weight say $a_{ij} = (0, 1, -1)$.

Step 2:

Construct an adjacency matrix $A = [a_{ij}]$ of the FMC. This matrix is anti-symmetric in nature with diagonal entries as zero.

Step 3:

A row vector called the instantaneous state vector $p = (a_1, a_2, \dots, a_n)$ is formed which denotes the on-off position of the node at an instant. It is characterized as:

$a_i = 1$, if a_i is on and

$a_i = 0$, if a_i is off

Step 4:

Consider the state vector p where a_1 is in the on position. Find $p \times A$. The value is calculated by assigning 1 for the value greater than 1 and 0 for the values less than one.

Step 5:

Every component in the state vector p is considered in the 'on' position one by one and their product with the matrix A is calculated.

Step 6:

The product with the highest number of 1's is considered as the optimal result p^* .

III. INTERPRETATION AND ANALYSIS OF THE MATHEMATICAL MODEL

Now, a model for HIV infected people is explained to find out most probable diseases that attack these persons.

HIV infection weakens the human immune system, making highly susceptible to numerous infections and certain types of cancers.

Infections common to HIV are:

- **Tuberculosis (TB).** TB is the most common opportunistic infection associated with HIV and a leading cause of death among people with AIDS.
- **Lung Disorder:** As a result of HIV a virus named Cytomegalovirus enters the immune system and cause Lung disorder.
- **Neurological disorder:** One of the most common neurological complications is HIV dementia complex, which leads to behavioral changes and diminished mental functioning.
- **Candidiasis:** This infection causes inflammation and a thick, white coating on the mucous membranes of your mouth, tongue, esophagus or vagina.
- **Wasting syndrome.** The main symptoms of HIV infected person is weight loss often accompanied by diarrhea, weakness and fever.
- **Kaposi's sarcoma:** This is a type of Cancer resulting in tumor of the blood vessel walls. Kaposi's sarcoma can also affect the internal organs, including the digestive tract and lungs.
- **Lymphomas:** This type of cancer originates in the white blood cells and usually first appears in lymph nodes.
- **Toxoplasmosis:** This potentially deadly infection is caused by *Toxoplasma gondii*, a parasite spread primarily by cats.

- **Cryptosporidiosis:** This infection is caused by an intestinal parasite that is commonly found in animals. The parasite grows in intestines and bile ducts, leading to severe, chronic diarrhea in people infected with HIV/ AIDS.
- **Meningitis:** Meningitis is an inflammation of the membranes and fluid surrounding the brain and spinal cord.

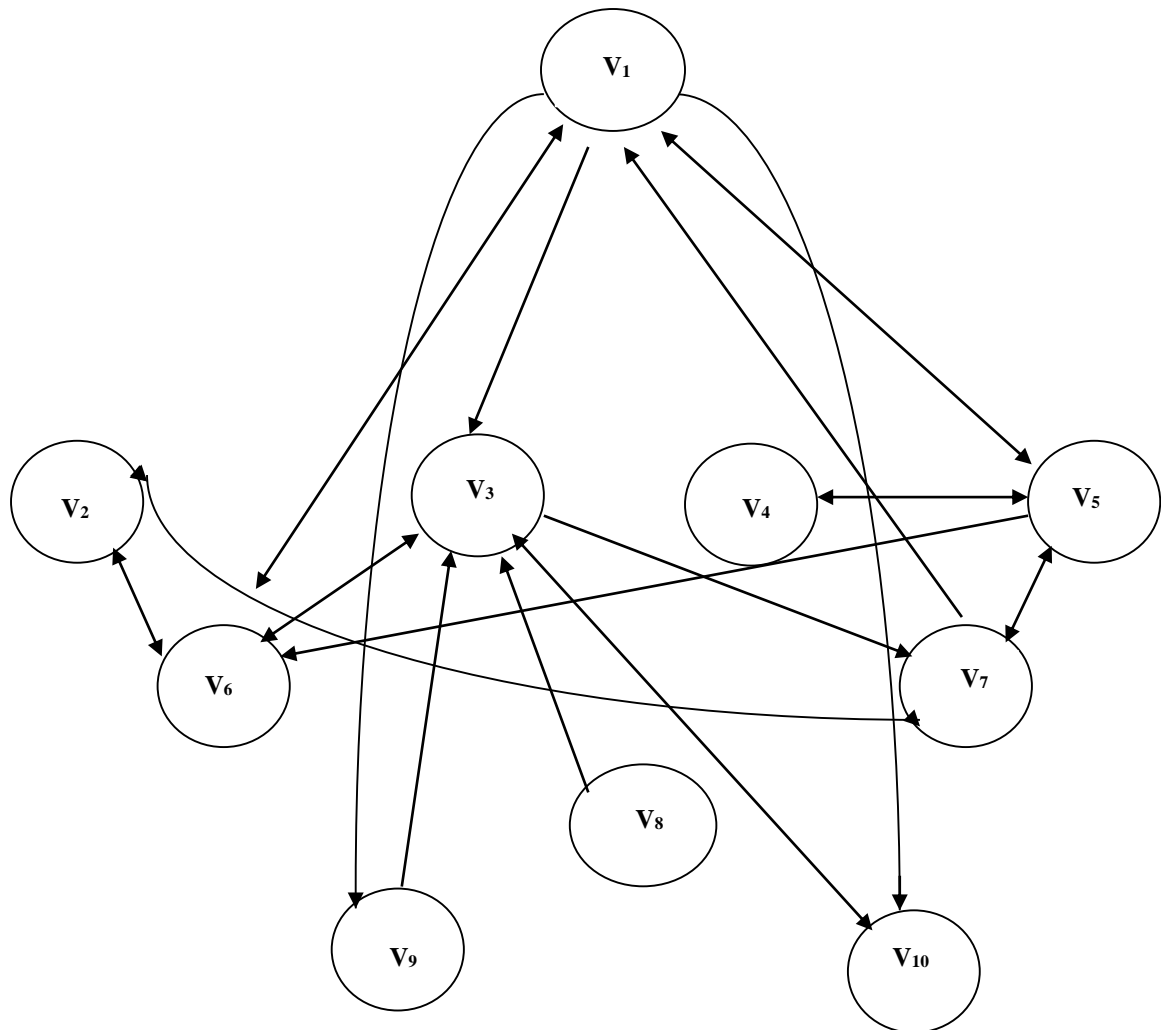
The following diseases affected by HIV Infection are interpreted as nodes:

V_1 = Tuberculosis, V_2 = Lung Disorder, V_3 = Neurological disorder, V_4 = Candidiasis

V_5 = Wasting syndrome, V_6 = Kaposi's sarcoma, V_7 = Lymphomas, V_8 = Toxoplasmosis

V_9 = Cryptosporidiosis, V_{10} = Meningitis

Here a Directed Graph is mentioned as follows:



IV. IMPLEMENTATION OF MATHEMATICAL MODEL

According to the study of attack of different diseases related to HIV infected people, the model is implemented. An adjacency matrix ‘A’ for the directed graph is formed as below:

$$A = [a_{ij}] = \begin{bmatrix} & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 & V_8 & V_9 & V_{10} \\ V_1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ V_2 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ V_3 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ V_4 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ V_5 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ V_6 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ V_7 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ V_8 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ V_9 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ V_{10} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The person suffering from HIV/AIDS, is prone to many infections/diseases. Initially, the person infected from HIV has high chance of getting affected by TB. To interpret the problem here it is assumed that a₁ is ON and others are OFF in the state vector.

Then $p = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$

Multiply, p with A

$$pA = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)A \rightarrow (0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0)$$

In this process, all a_i's supposed to be ON one by one, then all the products are shown below.

$$pA = (0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)A \rightarrow (0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0)$$

$$pA = (0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)A \rightarrow (0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0)$$

$$pA = (0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)A \rightarrow (0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

$$pA = (0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)A \rightarrow (1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0)$$

$$pA = (0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0)A \rightarrow (1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

$$pA = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0)A \rightarrow (1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

$$pA = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0)A \rightarrow (0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

$$pA = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0)A \rightarrow (0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

$$pA = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0)A \rightarrow (0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

The product with the highest number of 1's is considered as the optimal result. The optimal result is

$$p^* = (1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0)$$

In the calculated value zero (0) in fifth place is replaced by one (1) because of the hypothesis that only a5 state is in the ON position. So, the updated optimal result is

$$p^* = (1 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0).$$

V. CONCLUSION

The FCM theory is a Soft Computing tool for modeling and solving real world decision making problems of complex and dynamical systems. As the nodes and parameters of FMC are imprecise in nature, so it's perfect formulation in terms of discrete and continuous parameters is a challenging task for decision makers.

Here, the results obtained as an outcome of the FMC can be analyzed as follows:

- When a₅ (wasting Syndrome) is ON, the optimal result is obtained.
- a₁ (TB) is a major disease that attacks a person suffering from HIV infection for a long time.
- It is found that a₁, a₄, a₅, a₆, a₇ are major diseases affected to humans suffering from HIV infection.
- Thus, TB, Candidiasis, wasting Syndrome, Kaposi's sarcoma and Lymphomas are the diseases which have the highest chance to attack a person affected with HIV virus. These people are more prone to such diseases.
- Lung disorder, Neurological disorder, Meningitis may not always affect the HIV infected person. They may be common to any human with weak immune system.
- Toxoplasmosis and Cryptosporidiosis are diseases primarily influenced by parasites spread from animals. These diseases may affect any human, may or may not be the HIV infected person.

The FMC theory can be extended to complex directed graph having large number of nodes and edges. The analysis of the diseases can be done by iterative process using other soft computing techniques such as genetic algorithms, artificial intelligence, neural network, etc. which remains as a scope for future study.

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