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Intuitionistic Fuzzy Approach to Multi Person Decision Making

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Abstract

Ranking of Intuitionistic Fuzzy Sets (IFSs) and Interval Valued Intuitionistic Fuzzy Sets (IVIFSs) are very often required in decision making. Various decision making models are already in literature. Shiny Jose and Sunny Kuriakose in 2013 introduced a score function for ranking interval valued intuitionistic fuzzy numbers. In this paper I have proposed a decision making model in mutiperson decision making problems in IVIF environment.

AMS subject classification:
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1. Introduction

Following the introduction of Fuzzy set (FS for short) by L. A. Zadeh in 1965, Krassimir Atanassov introduced the notion of IFS which has been found a better tool to model decision problems [1]. Multicriteria decision making models based on IFS theoretical tools were introduced in the decision theory by Z. S. Xu [2,3]. This was extended to IVIFS [4]. Later many researchers studied the problem of ranking IFSs. Shiny Jose and Sunny Kuriakose introduced a score function in [5] to rank the alternatives in IVIF context. In this paper I consider the situation of multiperson multicriteria decision making in IVIF context, and solve the problem using the accuracy function given in [5]. Section 2 contains basic definitions and results. Section 3 contains the required score function. New decision making model and its illustration are given in section 4.

2. Preliminaries

Definition 2.1. [1] Intuitionistic Fuzzy Sets. Let X be a given set. An Intuitionistic fuzzy set A in X is given by,

$$A = \{(x,\ \mu_A(x),\ \nu_A(x)) \,/\, x \in X\}$$

where $\mu_A, \nu_A : X \to [0,1]$, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, $\mu_A(x)$ is the degree of membership of the element x in A and $\nu_A(x)$ is the degree of non membership of x in A. For each $x \in X, \pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the degree of hesitation.

Definition 2.2. [3] Interval valued intuitionistic fuzzy sets. Let D [0,1] be the set of all closed subintervals of the interval [0,1]. Let X $\neq \phi$ be a given set. An interval valued intuitionistic fuzzy set A in X is given by

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\},$$

where $\mu_A : X \to D[0,1]$, $\nu_A : X \to D[0,1]$ with the condition

$$0 \leq sup_x \mu_A(x) + sup_x \nu_A(x) \leq 1$$

The intervals $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the degree of belongingness and the degree of nonbelongingness of the element x to the set A. Thus, for each $x \in$ X, $\mu_A(x)$ and $\nu_A(x)$ are closed intervals whose lower and upper end points are respectively, denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$ and $\nu_{AL}(x)$, $\nu_{AU}(x)$.

A can also be denoted by,

$$A = \{(x, [\mu_{AL}(x), \mu_{AU}(x)], [\nu_{AL}(x), \nu_{AU}(x)]) : x \in X\},$$

where $0 \leq \mu_{AU}(x) + \nu_{AU}(x) \leq 1$ $\mu_{AL}(x) \geq 0$ and $\nu_{AL}(x) \geq 0$. For every element $x \in$ X, the hesitancy degree of an intuitionistic fuzzy interval of $x \in$ X in A is defined as

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) = [1 - \mu_{AU}(x) - \nu_{AU}(x), 1 - \mu_{AL}(x) - \nu_{AL}(x)].$$

We will denote the set of all the IVIFS in X by IVIFS(X).

Definition 2.3. [4] Weighted geometric average operator for IVIFSs. Let $A_j (j = 1, 2, \ldots, n) \in$ IVIFS(X). The weighted geometric average operator is defined by

$$G_w(A_1, A_2, A_3, \ldots, A_n) = \prod {A_j}^{w_j} = ([\prod {\mu_{AjL}}^{w_j}(x), \prod {\mu_{AjU}}^{w_j}(x)],$$
$$[1 - \prod (1 - \nu_{AjL}(x))^{w_j}, 1 - \prod (1 - \nu_{AjU}(x))^{w_j}])$$

where w_j is the weight of A_j (j = 1,2, …, n), $w_j \in [0,1]$ and $\Sigma_{i=1}^n w_j = 1$. If we assume $w_j = 1/n$ $(j = 1, 2, \ldots, n)$ then G_w is called an geometric average operator for $A_1, A_2, \ldots, A_n$. Clearly G_w is an IVIFS.

Definition 2.4. [4] Weighted arithmetic average operator for IVIFSs. Let $A_j(j = 1, 2, \ldots, n) \in$ IVIFS(X). The weighted arithmetic average operator is defined by

$$F_w(A_1, A_2, A_3, \ldots, A_n) = \sum_{j=1}^{n} w_j A_j = ([1 - \Pi(1 - \mu_{AjL}(x))^{w_j},$$
$$1 - \Pi(1 - \mu_{AjU}(x))^{w_j}], [\Pi\nu_{AjL}^{w_j}(x), \Pi\nu_{AjU}^{w_j}(x)])$$

where w_j is the weight of $A_j(j = 1, 2, \ldots, n)$, $w_j \in [0, 1]$ and $\Sigma_{i=1}^{n} w_j = 1$. If we assume $w_j = 1/n$ $(j = 1, 2, \ldots, n)$ then F_w is called an arithmetic average operator for $A_1, A_2, \ldots, A_n$. Clearly F_w is an IVIFS.

3. Score function [5]

For an interval valued intuitionistic fuzzy number A_1 obtained by the weighted geometric average operator, say $A_1 = ([a_1, b_1], [c_1, d_1])$, the score function S at A_1 based on the hesitancy degree denoted by $S(A_1)$ is defined by,

$$S(A_1) = \frac{(a_1 + b_1 - c_1 d_1)}{2}$$

Result 3.1. [5] For any interval-valued intuitionistic fuzzy subset $A = ([a, b], [c, d])$, the new proposed score S(A) $\in$ [-1, 1].

Result 3.2. [5] For any two comparable interval valued intuitionistic fuzzy sets A and B, if A $\subseteq$ B then S(A) $\leq$ S(B).

Result 3.3. [5] If $A = [a_1, b_1], [c_1, d_1]$ and $B = [a_2, b_2], [c_2, d_2]$ be two interval valued intuitionistic fuzzy sets such that $S(A) = S(B)$, then $A = B$.

4. Multi-person Decision Making model in IVIF environment

Xu in [6] elicit a method for group decision making, where any piece of information provided by the decision makers is expressed as intuitionistic fuzzy decision matrices and the information about attribute weights is partially known, or may be constructed by various methods. They first use the intuitionistic fuzzy hybrid geometric (IFHG) operator to aggregate all individual intuitionistic fuzzy decision matrix provided by the decision makers into the collective intuitionistic fuzzy decision matrix. Then they utilize the score function to calculate the score value of each attribute and construct the score matrix of the collective intuitionistic fuzzy decision matrix. Based on the score matrix and the given information about attribute weight, they establish an optimization model to determine the weights of attributes. After that they use the obtained attribute weights and the intuitionistic fuzzy weighted geometric (IFWG) operator to aggregate the intuitionistic fuzzy information in the collective intuitionistic fuzzy decision matrix to get overall intuitionistic fuzzy values of alternatives by which the ranking of all the given alternatives can be obtained.

I apply this method to interval valued intuitionistic fuzzy decision making model, but in a different way as follows.

4.1. Computational procedure

Step 1. First we calculate intuitionistic fuzzy weighted geometric average value for each alternatives using definition 2.3 Here we assume that criteria weights are known.

Step 2. Find the score for each alternatives, by the score function in section 3, i.e., Obtain the collective score matrix as follows.

$$\begin{array}{lcccc} & A_1 & A_2 & \ldots & A_m \\ D_1 & S_{11} & S_{12} & \ldots & S_{1m} \\ D_2 & S_{21} & S_{22} & \ldots & S_{2m} \\ . & & & & \\ . & & & & \\ . & & & & \\ D_n & S_{m1} & S_{m2} & \ldots & S_{nm} \end{array}$$

Step 3. Let W_{D_i} be the weight for the decision maker $D_i, i = 1, 2, \ldots, n.$ Then find the collective score for each alternative $A_j (j = 1, 2, \ldots, m)$ by the formula

$$S(A_j) = \sum_{i=1}^{n} W_{D_i} S_{ij}$$

Step 4. Rank the alternatives according to their collective score. Now we illustrate this with an example.

4.2. Illustrative example

In a youth festival, three decision makers have to rank four dancers (A_1, A_2, A_3, A_4) based on five criteria (C_1, C_2, C_3, C_4, C_5). The decision of the decision makers are given respectively in the following decision matrices.

Decision Matrix -1

	A_1	A_2	A_3	A_4
C_1	[.2, .3], [.4, .5]	[.7, .8], [.1, .15]	[.6, .7], [.1, .2]	[.2, .3], [.4, .5]
C_2	[.6, .7], [.2, .3]	[.5, .6], [.2, .3]	[.6, .7], [.1, .3]	[.3, .4], [.5, .6]
C_3	[.5, .6], [.2, .3]	[.4, .5], [.3, .4]	[.5, .6], [.1, .2]	[.5, .6], [.1, .2]
C_4	[.4, .6], [.2, .3]	[.6, .7], [.1, .2]	[.6, .7], [.1, .2]	[.6, .65], [.1, .2]
C_5	[.5, .6], [.1, .2]	[.4, .5], [.1, .2]	[.5, .6], [.1, .2]	[.4, .5], [.2, .3]

Decision Matrix -2

	A_1	A_2	A_3	A_4
C_1	[.1, .3], [.4, .5]	[.6, .7], [.1, .2]	[.7, .8], [.1, .2]	[.2, .3], [.5, .6]
C_2	[.6, .7], [.1, .2]	[.7, .8], [.2, .3]	[.6, .7], [.1, .2]	[.3, .4], [.5, .6]
C_3	[.4, .5], [.2, .3]	[.4, .5], [.3, .4]	[.6, .7], [.1, .3]	[.5, .6], [.1, .2]
C_4	[.4, .6], [.2, .3]	[.5, .6], [.2, .3]	[.5, .6], [.1, .2]	[.3, .4], [.1, .2]
C_5	[.4, .6], [.1, .2]	[.4, .5], [.1, .2]	[.4, .5], [.1, .2]	[.4, .5], [.1, .2]

Decision Matrix -3

	A_1	A_2	A_3	A_4
C_1	[.2, .3], [.4, .5]	[.6, .7], [.1, .2]	[.3, .4], [.4, .5]	[.1, .2], [.4, .5]
C_2	[.7, .8], [.1, .2]	[.5, .6], [.2, .3]	[.3, .4], [.4, .5]	[.3, .4], [.5, .6]
C_3	[.4, .5], [.2, .3]	[.4, .5], [.3, .4]	[.6, .7], [.1, .2]	[.4, .5], [.1, .2]
C_4	[.4, .6], [.2, .3]	[.5, .7], [.1, .2]	[.3, .4], [.1, .2]	[.6, .7], [.1, .2]
C_5	[.4, .6], [.1, .2]	[.3, .4], [.4, .5]	[.4, .5], [.2, .3]	[.4, .5], [.2, .3]

Assuming weights for criteria C_1, C_2, C_3, C_4 and C_5 as 0.3, 0.2, 0.15, 0.2 and 0.15 respectively, we obtain weighted geometric average value for A_i as follows.

Aggregated Matrix

	A_1	A_2	A_3	A_4
D_1	[.38, .50], [.25, .35]	[.54, .64][.15, .24]	[.57, .67][.1, .23]	[.34, .44][.31, .41]
D_2	[.29, .49], [.24, .34]	[.53, .63][.17, .28]	[.57, .67][.1, .22]	[.29, .40][.33, .43]
D_3	[.36, .50], [.24, .34]	[.47, .59][.20, .30]	[.35, .45][.28, .38]	[.27, .39][.30, .41]

We know that each weighted geometric average value is an IVIFN. So we can find the collective score for each alternative as given in step-3. It is elicited in the table given below.

Collective Score Matrix

	A_1	A_2	A_3	A_4
D_1	.396	.572	.609	.326
D_2	.349	.556	.609	.274
D_3	.389	.5	.347	.268

Let the weight for each decision maker be as follows.
Weight for decision maker-1, $W_{D_1} = 0.4$
Weight for decision maker-2, $W_{D_2} = 0.3$
Weight for decision maker-3, $W_{D_3} = 0.3$
Thus the collective score for each alternative be,

$$S(A_1) = (0.4 \times 0.396) + (0.3 \times 0.349) + (0.3 \times 0.389),$$

Similarly we can find the score for other alternatives also.
i.e., $S(A_1) = 0.3798$, $S(A_2) = 0.5456$, $S(A_3) = 0.5304$ and $S(A_4) = 0.293$.
Thus we rank the alternatives according to their score as follows,

$$A_2 \succ A_3 \succ A_1 \succ A_4$$

5. Conclusion

In this paper I have introduced a decision making model which is elegant than the existing models in literature. This can be used in Multi person multi criteria decision making problems in intervalvalued intuitionistic fuzzy environment.

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