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Bipolar Q – Fuzzy H – Ideals over Γ - Hemiring

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Abstract

In this paper, we consider bipolar Q – fuzzy concept of H – ideals extension in Γ - hemiring, introduce the concept of the extension of bipolar Q – fuzzy h – ideals one sided Γ - Hemiring and we have explained the extension of h – quasi – ideal, bipolar Q – fuzzy h – bi and h – interior ideals. Some properties of these concepts are discussed.

Keywords: Γ - Himering; h – bi - Ideal; h – quasi – ideal; Bipolar Q – Fuzzy Sets; Bipolar Q – Fuzzy h - Ideals.

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1. INTRODUCTION

In 1994, Zhang [18,19] introduced the concept of bipolar fuzzy sets as a generalization of fuzzy set. Majumder [15] gave the idea of bipolar fuzzy set in Γ - Semigroups. After the introduction of fuzzy subgroup many researchers discussed on the expansion of bipolar – valued fuzzy set notion. Sahaya Arockiaselvi et al [16] studied bipolar valued Q – fuzzy subgroups. A bipolar – valued multi fuzzy subgroups is discussed by Santhi and Shyamala [17].On the other hand Balasubramanian[1] introduced the notion of homomorphism in bipolar valued fuzzy subgroups. Gunasekaran and Gunaseelan [2,3] investigated operation and interior operator on bipolar intuitionistic M – fuzzy prime group. Hongxing [4] introduces HX group notion also the authors Chengzhong, Honghai and Hongxing [5] introduce fuzzy HX groups notion. Later, several authors such as Muthuraj and Sridharan [12, 14], Muthuraj et al [11, 13], Massa'deh and Fora [9] and Massa'deh [7] studied abipolar fuzzy HX group and its level, bipolar fuzzy cosets of bipolar and bipolar anti fuzzy HX subgroup, bipolar normal HX subgroup , bipolar valued Q – fuzzy HX subgroup and L – fuzzy M – cosets of M – HX - groups respectively. Massa'deh and Al naser

[10] and Massa'deh [8] discussed and introduced homomorphism in bipolar Q – fuzzy subring as new algebraic structure, cosets and ideals of Γ - near rings. Through this research we give a modern notion of bipolar Q – fuzzy h – ideal extension in one sided Γ - Hemiring also study on their related properties.

2. PRELIMINARIES

Definition 2.1 A hemiring is anon empty set R on which operations multiplications & addition have been explained as (R, +) is monoid commutative together identity 0, (R,.) is a semigroup (respectively monoid with identity 1_R) multiplication distributes over addition from either side, $1_R \neq 0$ also $0_R = 0 = x_0\ \forall\ x \in R$.

Definition 2.2 If R and Γ are two additive commutative semigroup with zero. Then R is said to be Γ - hemiring if ∃ a mapping $R \times \Gamma \times R \rightarrow R$ $((a, \propto, b) \rightarrow a{\propto}b)$satisfying the following conditions

1. $(a + b){\propto}c = a{\propto}c + b{\propto}c$
2. $a{\propto}(b +c) = a{\propto}b + a{\propto}c$.
3. $a(\propto + \partial)b = a{\propto}b + a\partial b$.
4. $a{\propto}(b\partial c) = (a{\propto}b)\partial c$.
5. $0_R{\propto}a = 0_R = a{\propto}0_R$.
6. $a_\Gamma b = 0_R = b0_\Gamma a$.

For all a, b & c in R and ∝, ∂ in Γ. For simplification we write 0 instead of 0_R and 0_Γ.

Definition 2.3 [6] Let I be an h – ideal from a Γ- hemiring R, therefore the following conditions are equivalent.

(i) I is a prime h – ideal of R.

(ii) If $a\Gamma R\Gamma b \subseteq I$ then either $a \in I$ or $b \in I$ where $a ,b \in R$.

Definition 2.4 A right ideal of a Γ - hemiring R is said to be a right h – ideal if for any a, c in R and x, y in I, a + x = y + c then a in I. A left h – ideal is defined similarly.

Definition 2.5 [9] If X and Q are non-empty arbitrary sets. A bipolar Q- fuzzy set δ in X×Q is an object having the form $\delta = \{(a, q), \delta^+ (a, q), \delta^-(a, q) \mid a \in X\ \&\ q \in Q\}$, such that $\delta^+ : X{\times}Q \rightarrow[0, 1]$ also $\delta^-: X{\times}Q \rightarrow[-1, 0]$ are mappings. The positive membership degree δ^+ (a) denotes the satisfaction degree of an element a to the property corresponding to a bipolar Q- fuzzy set $\delta = \{(a,q), \delta^+(a, q), \delta^-(a, q) \mid a \in X\ \&\ q \in Q \}$, and the negative membership degree $\delta^-(a, q)$ denotes the satisfaction degree of an element a to some implicit counter property corresponding to a bipolar-valued fuzzy set $\delta = \{(a,q), \delta^{+}(a, q), \delta^{-}(a, q) \mid a \in X \ \& \ q \in Q\}$. If $\delta^{+}(\delta, q) \neq (0, q)$ and $\delta^{-}(a, q) = (0, q)$, then it is the situation that a is regarded as having only positive satisfaction for $\delta = \{(a,q), \delta^{+}(a, q), \delta^{-}(a, q) \mid a \in X \ \& \ q \in Q\}$. If $\delta^{+}(a, q) = (0, q)$ and $\delta^{-}(x, q) \neq (0, q)$, then it is the situation that a does not satisfy the property of $\delta = \{(a ,q), \delta^{+}(x, q), \delta^{-}(x, q) \mid a \in X \ \& \ q \in Q\}$, but somewhat satisfies the counter property of $\delta= \{(a,q), \delta^{+}(a, q), \delta^{-}(a, q) \mid a \in X \ \& \ q \in Q\}$.It is possible for an element a to be such that $\delta^{+}(a, q) \neq (0, q)$ and $\delta^{-}(a, q) \neq (0, q)$, when the membership function of property overlaps with its counter property over some portion of X. For the sake of simplicity, we shall use the symbol $\delta = (\delta^{+}, \delta^{-})$ for the bipolar-valued fuzzy set $\delta= \{(a,q), \delta^{+}(a, q), \delta^{-}(a, q) \mid a \in X \ \& \ q \in Q\}$.

Definition 2.6 If δ is a non empty bipolar Q – fuzzy set of Γ - hemiring R (this means that $\delta^{+}(a,q) \neq (0, q) \neq \delta^{-}(a,q)$ for some a in R and $q \in Q$). Then δ is said to be a bipolar Q – fuzzy right ideal (left ideal) of R if the following conditions are holds.

1. $\delta^{-}(a + b, q) \leq \max\{\delta^{-}(a, q), \delta^{-}(b, q)\}$.
2. $\delta^{+}(a + b, q) \geq \min\{\delta^{+}(a, q), \delta^{+}(b, q)\}$.
3. $\delta^{-}(a\gamma b, q) \leq \delta^{-}(b, q)$.
4. $\delta^{+}(a\gamma b, q) \geq \delta^{+}(b, q)$.

For all $a, b \in R$, $\gamma \in \Gamma$ and $q \in Q$.

Definition 2.7 Let μ and δ be two bipolar Q – fuzzy subset of Γ - hemiring R, then the product of μ and δ which is denoted by $\mu \times \delta$ is defined as:

$(\mu \times \delta)((u, q), (v, q)) = \min\{\mu^{+}(u, q), \delta^{+}(v, q)\}$ and$(\mu \times \delta)((u, q), (v, q)) = \max\{\mu^{-}(u, q), \delta^{-}(v, q)\}$where $u, v \in R$ and $q \in Q$.

Remark2.8(I) A bipolar Q – fuzzy ideal of a Γ -hemiring R is a bipolar Q – fuzzy subset that is non empty of R which is a bipolar Q – fuzzy left ideal as well as a bipolar Q – fuzzy right ideal of R.

(II) Let λ be a bipolar Q – fuzzy right or left ideal of a Γ - hemiring R, then $\lambda^{+}(0, q) \geq \lambda^{+}(u, q)$ and $\lambda^{-}(0, q) \leq \lambda^{-}(u, q)$ for all $u \in R$ and $q \in Q$.

3. BIPOLAR Q – FUZZY h – IDEAL EXTENSION IN Γ- HEMIRINGS

Definition3.1 If $\delta = \langle \delta^{+}, \delta^{-} \rangle$ is a bipolar Q – fuzzy subset of R and $z \in R$, then the bipolar Q – fuzzy subset $\langle (z, q), \mu^{+}, \mu^{-} \rangle$ of R defined by:

$\left\langle (z,q),\delta^{-}\right\rangle (v,q) = \underset{\gamma\in\Gamma}{Sup}\, \mu^{-}(z\gamma v,q)$ $\left\langle (z,q),\delta^{+}\right\rangle (v,q) = \underset{\gamma\in\Gamma}{Inf}\, \delta^{+}(z\gamma v,q),$

For all v ∈R and q ∈Q, is called the extension of δ of (z,q).

Theorem3.2 Let δ = < δ^+, δ^- > be a bipolar Q – fuzzy right h - ideal of a Γ - hemiring R and u in R, then the extension <(u, q), δ^+, δ^- > is a bipolar Q – fuzzy right h – ideal of a Γ - hemiring R.

Proof:

take a, i, c, j, k ∈ R, γ, ∂ ∈ Γ and q ∈Q. Then

$$\left\langle (u,q),\delta^{+}\right\rangle (a+i,q) = \underset{\gamma\in\Gamma}{Inf}\, \delta^{+}(u\gamma(a+i),q)$$

$$= \underset{\gamma\in\Gamma}{Inf}\, \mu^{+}(u\gamma a + u\gamma i, q)$$

$$\geq \underset{\gamma\in\Gamma}{Inf}\, \min\{\delta^{+}(u\gamma a,q), \delta^{+}(u\gamma i,q)\}$$

$$= \min\{\underset{\gamma\in\Gamma}{Inf}\, \delta^{+}(u\gamma a,q), \underset{\gamma\in\Gamma}{Inf}\, \delta^{+}(u\gamma i,q)\}$$

= min{< (x, q), δ^+ > (a, q), <(u, q), δ^+ > (i, q) }.

And

$$\left\langle (u,q),\delta^{-}\right\rangle (a+i,q) = \underset{\gamma\in\Gamma}{Sup}\, \delta^{-}(u\gamma(a+i),q)$$

$$= \underset{\gamma\in\Gamma}{Sup}\, \delta^{-}(u\gamma a + u\gamma i, q)$$

$$\leq \underset{\gamma\in\Gamma}{Sup}\, \max\{\delta^{-}(u\gamma a,q), \delta^{-}(u\gamma i,q)\}$$

$$= \max\{\underset{\gamma\in\Gamma}{Sup}\, \delta^{-}(u\gamma a,q), \underset{\gamma\in\Gamma}{Sup}\, \delta^{-}(u\gamma i,q)\}$$

= max{<(u, q), δ^- > (a, q), <(u, q), δ^- > (i, q)}.

Also

$$\left\langle (u,q),\delta^{+}\right\rangle (a\vartheta i,q) = \underset{\gamma\in\Gamma}{Inf}\, \delta^{+}(u\gamma a\vartheta i,q)$$

$$\geq \underset{\gamma\in\Gamma}{Inf}\, \delta^{+}(u\gamma a,q)$$

= <(u, q), δ^+ > (a, q).

And

$$\left\langle (u,q),\delta^{-}\right\rangle (a\vartheta i,q) = \underset{\gamma\in\Gamma}{Sup}\,\delta^{-}(u\gamma a\vartheta i,q)$$

$$\leq \underset{\gamma\in\Gamma}{Sup}\,\delta^{-}(u\gamma a,q)$$

$$= \langle (u, q), \delta^{-} \rangle (a, q).$$

Now let a + c + k = j + k so uγa + uγc + uγk = uγj +uγk, then

$$\left\langle (u,q),\delta^{+}\right\rangle (a,q) = \underset{\gamma\in\Gamma}{Inf}\,\delta^{+}(u\gamma a,q)$$

$$\geq \underset{\gamma\in\Gamma}{Inf}\,\min\{\delta^{+}(u\gamma c,q),\delta^{+}(u\gamma j,q)\}$$

$$= \min\{\underset{\gamma\in\Gamma}{Inf}\,\delta^{+}(u\gamma c,q),\underset{\gamma\in\Gamma}{Inf}\,\delta^{+}(u\gamma j,q)\}$$

$$= \min\{\langle (u, q), \delta^{+} \rangle (c, q), \langle (u, q), \delta^{+} \rangle (j, q)\}.$$

And

$$\left\langle (u,q),\delta^{-}\right\rangle (i,q) = \underset{\gamma\in\Gamma}{Sup}\,\delta^{-}(u\gamma i,q)$$

$$\leq \underset{\gamma\in\Gamma}{Sup}\,\max\{\delta^{-}(u\gamma c,q),\delta^{-}(u\gamma j,q)\}$$

$$= \max\{\underset{\gamma\in\Gamma}{Sup}\,\delta^{-}(u\gamma c,q),\underset{\gamma\in\Gamma}{Sup}\,\delta^{-}(u\gamma j,q)\}$$

$$= \max\{\langle (u, q), \delta^{-} \rangle (c, q), \langle (u, q), \delta^{-} \rangle (j, q)\}.$$

Therefore <(u, q), δ^+, δ^- > is a bipolar Q – fuzzy right h – ideal of a Γ - hemiring R.

Proposition3.3 Let R be a commutative Γ - hemiring, μ = < μ^+, μ^- > be a bipolar Q – fuzzy h - ideal and u ∈ R, then the extension <(u, q), μ^+, μ^- > is a bipolar Q – fuzzy h – ideal of R.

Proof: Straight forward.

Corollary3.4 If μ = < μ^+_i , μ^-_i >, i = 1, 2, …are arbitrary collection of bipolar Q – fuzzy h - ideal of a Γ - hemiring R , u in R, then the extension <(u, q), $\cap\mu^+$, $\cap\mu^-$ > is a bipolar Q – fuzzy h – ideal of a Γ - hemiring R.

Proof: Straight forward.

Corollary3.5 Let δ = < δ^+, δ^- > be a bipolar Q – fuzzy h - ideal and u in R, then the following statement are holds.

1. A bipolar Q – fuzzy h – ideal μ is subset of its extension.

2. < $((u\gamma)^{n-1}u, q), \delta^+, \delta^-$ > $\subseteq$ <$((u\gamma)^n u, q), \delta^+, \delta^-$ > where $\gamma \in \Gamma$ and $q \in Q$.

3. If $\delta > 0$ then Supp <(u, q), δ^+, δ^- > = R × Q, where Supp μ is defined by Supp δ = {(u, q) ∈ R × Q; $\delta^+(u, q) > \delta^+(0, q)$ and $\delta^-(u, q) < \mu^-(0, q)$}.

Proof:

1. Take z ∈ R, q ∈Q. Therefore

$$\langle (x,q),\mu^+\rangle(z,q) = \underset{\gamma\in\Gamma}{Inf}\ \mu^+(x\gamma z,q)$$

$$\geq \delta^+(z,q)$$

And

$$\langle (u,q),\delta^-\rangle(z,q) = \underset{\gamma\in\Gamma}{Sup}\ \delta^-(u\gamma z,q)$$

$$\leq \delta^-(z,q)$$

Therefore $\delta \subseteq$ <(u, q), δ^+, δ^- >.

2. Let z ∈ R, q ∈Q and n ∈N. Then

$$\langle ((u\gamma)^n u,q),\delta^+\rangle\ (z,q) = \underset{\gamma\in\Gamma}{Inf}\ \delta^+((u\gamma)^n u\gamma z,q)$$

$$\geq \underset{\gamma\in\Gamma}{Inf}\ \delta^+((u\gamma)(u\gamma)^{n-1}u\gamma z,q)$$

$$\geq \underset{\gamma\in\Gamma}{Inf}\ \delta^+((u\gamma)^{n-1}u\gamma z,q)$$

$$= \langle ((u\gamma)^{n-1}u, q), \delta^+ \rangle(z, q).$$

Also

$$\langle ((u\gamma)^n u,q),\delta^-\rangle(z,q) = \underset{\gamma\in\Gamma}{Sup}\ \delta^-((u\gamma)^n u\gamma z,q)$$

$$\leq \underset{\gamma\in\Gamma}{Sup}\ \delta^-((u\gamma)(u\gamma)^{n-1}u\gamma z,q)$$

$$\leq \underset{\gamma\in\Gamma}{Sup}\ \delta^-((u\gamma)^{n-1}u\gamma z,q)$$

= < $((u\gamma)^{n-1}u, q), \delta^-$ >(z, q).

Thus < $((u\gamma)^{n-1}u, q), \delta^+, \delta^-$ > $\subseteq$ <$((u\gamma)^n u, q), \delta^+, \delta^-$ >.

3. Let $\delta > 0$ and (i, q) ∈R × Q, then

$$\langle (u,q),\delta^+\rangle(i,q) = \underset{\gamma\in\Gamma}{Inf}\ \delta^+(u\gamma i,q)$$

$$\geq \delta^+(u, q)$$

And

$$\left\langle (u,q),\delta^{-}\right\rangle (i,q)=\underset{\gamma\in\Gamma}{Sup}\,\delta^{-}(u\gamma i,q)$$

$$\geq \delta^{-}(u, q)$$

Therefore (i, q) ∈ Supp <(x, q), δ^+, δ^-> and consequently R × Q ⊆ Supp <(u, q), δ^+, δ^- >, thus R × Q = Supp <(u, q), δ^+, δ^- >.

Corollary3.6 If R is a commutative Γ - hemiring**,** δ = < δ^+, δ^- > is a bipolar Q – fuzzy h - bi - ideal and u ∈ R, then its extension by (u, q) ∈R×Q is also a bipolar Q – fuzzy h – bi – ideal of a commutative Γ - hemiring R.

Proof:

Since μ is a bipolar Q – fuzzy h – bi – ideal, we need to prove

1. <(u, q), μ^+> (i∝j϶c, q) ≥ min{{<(u, q), μ^+> (i, q), <(u, q), μ^+ > (k, q) }.

For all I, j, k ∈ R, q ∈ Q and ∝, ϑ ∈ Γ we have

$$\left\langle (u,q),\mu^{+}\right\rangle (i\alpha\, j\vartheta k,q)=\underset{\gamma\in\Gamma}{Inf}\;\mu^{+}(u\gamma i\alpha\, j\,\vartheta k,q)$$

$$\geq \underset{\gamma\in\Gamma}{Inf}\;\mu^{+}(u\gamma\, i,q)$$

$$= <(u, q), \mu^{+}> (i, q).$$

$$\left\langle (u,q),\mu^{+}\right\rangle (i\alpha\, j\vartheta k,q)=\underset{\gamma\in\Gamma}{Inf}\;\mu^{+}(u\gamma i\alpha\, j\,\vartheta k,q)$$

$$\geq \underset{\gamma\in\Gamma}{Inf}\;\mu^{+}(u\gamma\, k,q)$$

$$= <(u, q), \mu^{+}> (k, q).$$

Since R is commutative, then<(u, q), μ^+> (i∝jϑk, q) ≥ min {{<(u, q), μ^+> (i, q), <(u, q), μ^+ > (k, q)}.

2. <(u, q), μ^- > (i∝jϑk, q) ≤ max{{<(u, q), μ^- > (i, q), <(u, q), μ^- > (k, q) }.

For all i, j, k ∈ R, q ∈ Q and∝, ϑ ∈ Γ we have

$$\left\langle (u,q),\mu^{-}\right\rangle (i\alpha\, j\vartheta k,q)=\underset{\gamma\in\Gamma}{Sup}\,\mu^{-}(u\gamma i\alpha\, j\,\vartheta k,q)$$

$$\leq \underset{\gamma\in\Gamma}{Sup}\,\mu^{-}(u\gamma\, i,q)$$

$$= <(u, q), \mu^{-}> (i, q).$$

$$\left\langle (u,q),\mu^{-}\right\rangle (i\alpha j\vartheta k,q)=\underset{\gamma\in\Gamma}{Sup}\,\mu^{-}(u\gamma i\alpha j\vartheta k,q)$$

$$\leq \underset{\gamma\in\Gamma}{Sup}\,\mu^{-}(x\gamma c,q)$$

$= <(u, q), \mu^{-} > (k, q)$.

Since R is commutative, then$<(u, q), \mu^{-} > (i\propto j\vartheta k, q) \leq \max \{\{<(u, q), \mu^{-} > (i, q), <(u, q), \mu^{-} > (k, q)\}$.

Thus $<(u, q), \mu^{+}, \mu^{-} >$ is a bipolar Q – fuzzy h – bi – ideal of a commutative Γ - hemiring R .

Theorem3.7 If R a commutative Γ - hemiring and $\delta = < \delta^{+}, \delta^{-} >$ is a bipolar Q – fuzzy h - interior - ideal, then its extension by $(u, q) \in R\times Q$ is also a bipolar Q – fuzzy h – interior – ideal of a commutative Γ - hemiring R.
Proof:

If $<(u, q), \delta^{+}, \delta^{-} >$ be the extension of $\delta = < \delta^{+}, \delta^{-} >$ by $(u, q) \in R\times Q$, we need to show $<(u, q), \delta^{+}> (i\propto j\partial k, q) \geq <(u, q), \delta^{+}> (j, q)$ and$<(u, q), \delta^{-} > (i\propto j\partial k, q) \leq <(u, q), \delta^{-} > (j, q)$ for all $i, j, k \in R$, $q \in Q$ and $\propto, \vartheta \in \Gamma$. Now

$$\left\langle (u,q),\delta^{+}\right\rangle (i\alpha j\vartheta k,q)=\underset{\gamma\in\Gamma}{Inf}\,\delta^{+}(u\gamma i\alpha j\vartheta k,q)$$

$$=\underset{\gamma\in\Gamma}{Inf}\,\delta^{+}(u\gamma j\alpha i\vartheta k,q)$$

$$\geq \underset{\gamma\in\Gamma}{Inf}\,\delta^{+}(u\gamma j,q)$$

$= <(u, q), \delta^{+}> (j, q)$.

And

$$\left\langle (u,q),\delta^{-}\right\rangle (i\alpha j\vartheta k,q)=\underset{\gamma\in\Gamma}{Sup}\,\delta^{-}(u\gamma i\alpha j\vartheta k,q)$$

$$=\underset{\gamma\in\Gamma}{Sup}\,\delta^{-}(u\gamma j\alpha i\vartheta k,q)$$

$$\leq \underset{\gamma\in\Gamma}{Sup}\,\delta^{+}(u\gamma j,q)$$

$= <(u, q), \delta^{-} > (j, q)$.

Therefore $<(u, q), \delta^{+}, \delta^{-} >$ is a bipolar Q – fuzzy h – interior – ideal of a commutative Γ - hemiring R

Theorem3.8 If R is a Γ - hemiring and $\mu = < \mu^{+}, \mu^{-} >$ be a bipolar Q – fuzzy h - quasi

- ideal, then its extension by (x, q) $\in R \times Q$ is a bipolar Q – fuzzy h – quasi – ideal also.
Proof: Straight for ward.

Corollary 3.9 Let μ and δ be any two bipolar Q – fuzzy h – ideals of a Γ - hemiring R, then for any u, v ∈R and q in Q we obtain:

1. $<(u, q), \mu \times \delta >$ is a bipolar Q – fuzzy h –ideal of Γ - hemiring R.
2. $<(u, q), \mu^+, \mu^- > \times <(u, q), \delta^+, \delta^- >$ is bipolar Q – fuzzy h –ideal of R

Proof: Straight for ward.

4. CONCLUSION

In this article author defined the extension of bipolar Q – fuzzy h – ideals in one sided Γ - hemiring and author discussed few important properties of them and we hope to generalize the results we have obtained to other new algebraic systems.

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