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Some Properties of Fuzzy Soft B-Open Sets and Fuzzy Soft B-Continuous Functions in Fuzzy Soft Topological Spaces

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Abstract

In this paper we introduce fuzzy soft semi pre-open set and studied its relation with fuzzy soft b-open set and also b-continuous functions in fuzzy soft topological spaces and investigate some of their properties. Further generalized b-closed set in fuzzy soft topology, fuzzy soft generalized b-continuous functions and fuzzy soft generalized b-irresolute functions and fuzzy soft b^* - continuous functions have been introduced and studied few of their properties.

Keywords: Fuzzy soft Topology, Fuzzy soft semi-pre-open set, Fuzzy soft b-continuous functions, Fuzzy soft generalized b-continuous functions, Fuzzy soft generalized b-irresolute functions, fuzzy soft b* - continuous functions.

1. INTRODUCTION

In recent times, the problems in the field of Engineering, Physics, Social sciences and Medical sciences etc.. involving uncertainities cannot be dealt with crisp data. The general mathematical tool for dealing with uncertainities known as Fuzzy set was introduced by zadeh [15] in 1965. Fuzzy topology was introduced by Chang [4] in 1968. To overcome the existing difficulties in Fuzzy set theory, Molodtsov[10] introduced Soft sets in 1999. The theory of soft sets can be successfully applied in several directions such as Game theory, Riemann integration, Smoothness of functions, Probability theory etc…The hybridisation of Fuzzy set and soft set called Fuzzy soft set was introduced by Maji et..al [9]. The notion of topological structure of Fuzzy soft sets was introduced by Tanay and Kandemir [13] in 2011. And studied further by Varol and Aygun [14] , S Roy and Samanta [11] and many more authors [6],[5]. Mappings of fuzzy soft classes are studied by A.Kharal and B.Ahmad [8]. The concept of fuzzy soft semi open set was introduced by A.Kandil et.al [8] in 2014.

The concept of fuzzy soft pre-open and regular open sets was introduced by Sabir Hussain in 2016[12]. A new form of fuzzy subset called fuzzy b-open set was introduced by S.S Benchalli and K.Jenifer[2]. S. S.Benchalli and Jenifer J.Karnel also introduced Fuzzy gb-Continuous Maps in Fuzzy Topological Spaces[3]. The concept of fuzzy soft b-open sets was introduced by Anil P.N [1] in 2016.

In this paper a new form of set known as fuzzy soft semi-pre-open set and a new class of continuous functions known as b-continuous functions, generalised b-continuous , generalised b-irresolute functions and b*- continuous functions in Fuzzy soft topology are introduced and few of their properties are studied.

2. PRELIMINARIES

Definition 2.1[9] : Let X be the initial universe and E be the set of parameters. I^x be the set of all fuzzy sets on X. Let $A \subseteq E$ and $f : A \to I^x$. A pair (f, A) is called fuzzy soft set over X. It is also denoted by f_A. i.e for every $a \in A, f(a) = f_a : X \to I$ is a fuzzy set on X.

Definition 2.2[13] : Let τ be a collection of all fuzzy soft sets over a universe X with a fixed set of parameter set E, then a triplet (X, τ, E) is called fuzzy soft topological space[FSTS] if it satisfies the following axioms.

(i) $\tilde{0}_E, \tilde{1}_E \in \tau$

(ii)Arbitrary union of members of τ is a member of τ.

(iii) Finite intersection of members of τ is a member of τ.

Every member of τ is called fuzzy soft open set. i.e a fuzzy soft set $f_A \in \tau$ is called fuzzy soft open set in X and its complement $1 - f_A$ is called fuzzy soft closed set.

Definition 2.3[14] : The intersection of all fuzzy soft closed super sets of f_A is called fuzzy soft closure of f_A , denoted by,

$Fscl(f_A) = \bigcap \{ h_D, h_D \textit{ is fuzzy soft closed set and } f_A \subseteq h_D \}$

Definition 2.4[14] : The union of all fuzzy soft open subsets of g_B is called fuzzy soft interior of g_B, denoted by,

$Fs\,\mathrm{int}(g_B) = \bigcup \{ h_D, h_D \textit{ is fuzzy soft open set and } h_D \subseteq g_B \}$

Definition 2.4[7] : A Fuzzy soft set f_A in Fuzzy soft topological space (X, τ, E), is called Fuzzy soft semi-open if $f_A \leq Fscl\, Fs\mathrm{int}\,(f_A)$, Fuzzy soft semi-closed if $Fs\mathrm{int} Fscl\,(f_A) \leq f_A$

Definition 2.4[12] : A Fuzzy soft set f_A in Fuzzy soft topological space (X, τ, E), is called Fuzzy soft pre-open if $f_A \leq Fs\mathrm{int}\; Fscl\,(f_A)$, Fuzzy soft pre-closed if

$Fscl\,Fs\text{int}\,(f_A) \leq f_A$

Definition 2.4[12] : A Fuzzy soft set f_A in Fuzzy soft topological space (X,τ,E), is called Fuzzy soft regular -open if $f_A = Fs\text{int}\ Fscl\ (f_A)$, Fuzzy soft regular –closed if, $Fscl\,Fs\text{int}\,(f_A) = f_A$

Definition 2.5[1] : A fuzzy soft set f_A in a fuzzy soft topological space (X,τ,E) is called

--Fuzzy soft b-open set iff $f_A \leq ((Fs\ \text{int}\ Fscl\ (f_A)) \vee (Fscl\ Fs\,\text{int}\,(f_A)))$

--Fuzzy soft b-closed set
iff $f_A \geq ((Fs\ \text{int}\ Fscl\ (f_A)) \vee (Fscl\ Fs\,\text{int}\ (f_A)))$

Definition 2.6[1] : Let f_A be a fuzzy soft set in a fuzzy soft topological space (X,τ,E) then

-Fuzzy soft b-closure of f_A (fsb-cl (f_A)) is defined as ,

$$fsbcl\,(f_A) = \bigcap \{ g_B : g_B\ is\ a\ fsb-closed\ set\ of\ X\ \&\ g_B \geq f_A \}$$

-Fuzzy soft b-interior of f_A (fsb-int (f_A)) is defined as ,

$$fsb\ \text{int}\,(f_A) = \bigcup \{ h_c : h_c\ is\ a\ fsb-open\ set\ of\ X\ \&\ h_C \leq f_A \}$$

3. FUZZY SOFT B-OPEN SETS

Definition 3.1: A Fuzzy soft set f_A in Fuzzy soft topological space (X,τ,E), is called Fuzzy soft semi pre-open if $f_A \leq Fscl\,Fs\text{int}\ Fscl\ (f_A)$, Fuzzy soft semi pre-closed if $F\sin t\,Fscl\,Fs\text{int}\,(f_A) \leq f_A$

Theorem 3.1: *In FSTS X, every Fuzzy soft pre-open set is Fuzzy soft b-open set.*

Proof: Let f_A be Fuzzy soft pre-open set in X.

$\Rightarrow f_A \leq Fs\text{int}\ Fscl\ (f_A)$

$\Rightarrow f_A \leq Fs\text{int}\ Fscl\ (f_A) \vee Fscl\,Fs\text{int}\,(f_A)$

$\Rightarrow f_A$ is fuzzy soft b-open set in X.

But converse is not true as seen from the below example.

Let $\tau = \{\tilde{0},\tilde{1},(F_1,E),(F_2,E)\}$

$$(F_1,E) = \left\{\left\{\frac{1}{5},\frac{1}{4}\right\},\left\{0,\frac{1}{3}\right\}\right\}, \quad (F_2,E) = \left\{\left\{\frac{1}{3},\frac{1}{2}\right\},\left\{\frac{1}{4},\frac{1}{2}\right\}\right\}$$

Let $f_A = \left\{\left\{\frac{1}{5},\frac{1}{3}\right\},\left\{\frac{3}{4},\frac{1}{3}\right\}\right\}$

f_A is fuzzy soft b-open set but not fuzzy soft pre-open as $f_A \geq F\sin tFscl(f_A)$.

Remarks:

(i) If f_A is Fuzzy soft subset of FSTS (X,τ,E), then

$Fsbcl(f_A)$ is the smallest fuzzy soft b-closed set containing f_A,

$\Rightarrow Fsbcl(f_A) = f_A \vee [F\sin t\, Fscl(f_A) \wedge Fscl\, F\sin t(f_A)$

(ii) $Fsb\text{int}(f_A)$ is the largest fuzzy soft b-closed set contained in f_A,

$\Rightarrow Fsb\text{int}(f_A) = f_A \wedge [F\sin t\, Fscl(f_A) \vee Fscl\, F\sin t(f_A)]$

Theorem 3.2 *: In FSTS X, every Fuzzy soft b-open set is Fuzzy soft semi pre-open.*

Proof: Let f_A be Fuzzy soft b-open set in X.

$\Rightarrow f_A \leq ((Fs\text{ int }Fscl\,(f_A)) \vee (Fscl\ Fs\text{int}\,(f_A)))$

Since $Fscl(f_A)$ is the smallest fuzzy soft closed set containing f_A, $f_A \leq Fscl(f_A)$

$f_A \leq ((Fs\text{ int }Fscl\,(f_A)) \vee (Fscl\ Fs\text{int}\,Fscl(f_A)))$

$\Rightarrow f_A \leq (Fscl\ Fs\text{int}\,Fscl(f_A))$

$\Rightarrow f_A$ is Fuzzy soft semi-pre open set in X.

But converse is not true as seen from the below example.

Let $\tau = \{\tilde{0},\tilde{1},(F_1,E),(F_2,E)\}$

$(F_1,E) = \left\{\left\{\frac{1}{2},\frac{1}{3}\right\},\left\{\frac{1}{3},\frac{1}{5}\right\}\right\}$ $\quad(F_2,E) = \left\{\left\{\frac{1}{2},\frac{2}{3}\right\},\left\{\frac{1}{3},\frac{1}{4}\right\}\right\}$ Let

$k = \left\{\left\{\frac{1}{3},\frac{1}{5}\right\},\left\{\frac{1}{3},\frac{1}{4}\right\}\right\}$

k is fuzzy soft semi pre-open but not fuzzy soft b-open set.

Theorem 3.3 : ***(i)*** *In a FSTS, a fuzzy soft set* f_A *is fuzzy soft b-closed iff* $f_A = Fsbcl(f_A)$

(ii) *In a FSTS, a fuzzy soft set* f_A *is fuzzy soft b-open iff* $f_A = Fsb\text{int}(f_A)$

Proof: (i) Let $f_A = Fsbcl(f_A)$

$\Rightarrow f_A = \wedge\{g_B ; g_B \textit{ is } Fsb-\textit{closed set and } f_A \leq g_B\}$

$\Rightarrow f_A \in \wedge\{g_B ; g_B \text{ is } Fsb-closed\ set\ and\ f_A \le g_B\}$

$\Rightarrow f_A\ is\ Fsb-closed\ set.$

Conversely, let f_A be fuzzy soft b-closed set.

$\Rightarrow f_A \in \wedge\{g_B ; g_B\ is\ Fsb-closed\ set\}\ and\ f_A \le f_A$

$\Rightarrow f_A$ is the smallest closed set containing f_A.

$\Rightarrow f_A = \wedge\{g_B ; g_B\ is\ Fsb-closed\ set\ and\ f_A \le g_B\}$

Thus $f_A = Fsbcl(f_A)$

(ii) Let $f_A = Fsb\text{int}(f_A)$

Then f_A is fuzzy soft b-open set.

If f_A is fuzzy soft b-open set, then $f_A \in \{g_B ; g_B\ is\ Fsb-open\ set\}\ and\ f_A \le f_A$

$f_A \in \{g_B ; g_B\ is\ Fsb-open\ and\ g_B \le f_A\}$

$\Rightarrow f_A = Fsb\text{int}(f_A)$

Theorem3.4: *In a FSTS,*

(i) $Fsbcl(A \vee B) \ge Fsbcl(A) \vee Fsbcl(B)$

(ii) $Fsbcl(A \wedge B) \ge Fsbcl(A) \wedge Fsbcl(B)$

(iii) $Fsb\text{int}(A \vee B) \ge Fsb\text{int}(A) \vee Fsb\text{int}(B)$

(iv) $Fsb\text{int}(A \wedge B) \le Fsb\text{int}(A) \wedge Fsb\text{int}(B)$

Proof: Let A and B be any two fuzzy soft sets in FSTS (X, τ, E)

(i) $A \le A \vee B \Rightarrow Fsbcl(A) \le Fsbcl(A \vee B)$

$B \le A \vee B \Rightarrow Fsbcl(B) \le Fsbcl(A \vee B)$

$Fsbcl(A) \vee Fsbcl(B) \le Fsbcl(A \vee B)$

(ii) $A \wedge B \le A$ and $A \wedge B \le B \Rightarrow Fsbcl(A \wedge B) \le Fsbcl(A) \wedge Fsbcl(B)$

(iii) Let A and B be any two fuzzy soft sets in FSTS (X, τ, E)

$A \le A \vee B \Rightarrow Fsb\text{int}(A) \le Fsb\text{int}(A \vee B)$

$B \le A \vee B \Rightarrow Fsb\text{int}(B) \le Fsb\text{int}(A \vee B)$

$Fsb\text{int}(A) \vee Fsb\text{int}(B) \le Fsb\text{int}(A \vee B)$

(iv) $A \wedge B \le A$ and $A \wedge B \le B \Rightarrow Fsb\text{int}(A \wedge B) \le Fsb\text{int}(A) \wedge Fsb\text{int}(B)$

Theorem 3.5: *Let f_A be a fuzzy soft b-open set in X,*

(i) *If f_A is fuzzy soft regularly closed set then A is fuzzy soft pre-open set.*

(ii) *If f_A is fuzzy soft regularly open set then A is fuzzy soft semi-open set.*

Proof: Let f_A be fuzzy soft b-open set in X.

$f_A \leq ((Fs\text{ int } Fscl\ (f_A)) \vee (Fscl\ Fs\text{int } Fscl(f_A)))$

(i) If f_A is fuzzy soft regularly closed set then $f_A = Fscl\, F\sin t(f_A)$

$\Rightarrow f_A \leq ((Fs\text{ int } Fscl\ (f_A)) \vee (f_A)))$

$\Rightarrow f_A \leq (Fs\text{ int } Fscl\ (f_A))$

$\Rightarrow f_A$ is fuzzy soft pre-open set.

(ii) If f_A is fuzzy soft regularly open set then $f_A = F\sin t\, Fsicl(f_A)$

$\Rightarrow f_A \leq FsclFs\text{ int } (f_A)) \vee (f_A)$

$\Rightarrow f_A \leq FsclFs\text{ int } (f_A)$

$\Rightarrow f_A$ is fuzzy soft semi-open set.

Theorem 3.6: *Let f_A be a fuzzy soft b-open set in X,*

(i) *If f_A is fuzzy soft regularly closed set then A is fuzzy soft semi-closed set.*

(ii) *If f_A is fuzzy soft regularly open set then A is fuzzy soft pre-closed set.*

Proof: Let f_A be fuzzy soft b-open set in X.

$f_A \leq ((Fs\text{ int } Fscl\ (f_A)) \vee (Fscl\ Fs\text{int } Fscl(f_A)))$

(i) If f_A is fuzzy soft regularly closed set then $f_A = Fscl\, F\sin t(f_A)$

$\Rightarrow f_A = ((Fs\text{ int } Fscl\ (f_A)) \vee (f_A)))$

$\Rightarrow f_A = (Fs\text{ int } Fscl\ (f_A))$

$\Rightarrow (Fs\text{ int } Fscl\ (f_A)) \leq f_A$

$\Rightarrow f_A$ is fuzzy soft semi-closed set.

(ii) If f_A is fuzzy soft regularly open set then $f_A = F\sin t\, Fsicl(f_A)$

$\Rightarrow f_A = FsclFs\text{ int } (f_A)) \vee (f_A)$

$\Rightarrow FsclFs\text{ int } (f_A) \leq f_A$ and $f_A \leq f_A$

$\Rightarrow f_A$ is fuzzy soft pre-closed set.

***Theorem 3.7:* (i)** *For any fuzzy soft b-closed set* f_A *in FSTS X,* $Fs\ \mathrm{int}\ (f_A)$ *is fuzzy soft regular open set.*

(ii) *For any fuzzy soft b-open set* f_A *in FSTS X,* $Fscl\ (f_A)$ *is fuzzy soft regular closed set.*

Proof: (i) Let f_A be fuzzy soft b-closed set in X. $\Rightarrow (Fs\ \mathrm{int}\ Fscl\ (f_A) \wedge (Fscl\ Fs\mathrm{int}\ (f_A)) \le f_A$

$\Rightarrow Fs\ \mathrm{int}\ (Fs\ \mathrm{int}\ Fscl\ (f_A)) \wedge Fs\ \mathrm{int}(Fscl\ Fs\mathrm{int}\ (f_A)) \le Fs\ \mathrm{int}(f_A)$

$\Rightarrow Fs\ \mathrm{int}(Fscl\ Fs\mathrm{int}\ (f_A)) \le Fs\ \mathrm{int}(f_A)$ Also

$Fs\ \mathrm{int}(f_A) \le Fs\ \mathrm{int}(Fscl\ Fs\mathrm{int}\ (f_A))\ \therefore Fs\ \mathrm{int}(f_A) = Fs\ \mathrm{int}(Fscl\ Fs\mathrm{int}\ (f_A))$,

$Fs\ \mathrm{int}\ (f_A)$ is fuzzy soft regular open set.

(ii) Let f_A be fuzzy soft b-open set in X.

$\Rightarrow (Fs\ \mathrm{int}\ Fscl\ (f_A) \vee (Fscl\ Fs\mathrm{int}\ (f_A)) \ge f_A$

$\Rightarrow Fscl\ (Fs\ \mathrm{int}\ Fscl\ (f_A)) \vee Fscl(Fscl\ Fs\mathrm{int}\ (f_A)) \ge Fscl(f_A)$

$\Rightarrow Fscl(Fs\mathrm{int}\ Fscl\ (f_A)) \ge Fscl(f_A)$ Also

$Fscl(f_A) \ge Fscl(Fs\mathrm{int}\ Fscl\ \ (f_A))\ \therefore Fscl(f_A) = Fscl(Fs\mathrm{int}\ Fscl\ (f_A))$, $Fscl(f_A)$ is fuzzy soft regular closed set.

***Theorem 3.8 :* (i)** *Let* f_A *be fuzzy soft b-open set in FSTS X, such that* $F\sin t\ (f_A) = 0$ *then* f_A *is fuzzy soft pre-open set.*

(ii) *Let* f_A *be fuzzy soft b-open set in FSTS X, such that* $Fscl\ (f_A) = 0$ *then* f_A *is fuzzy soft semi-open set.*

Proof: (i) Let f_A be fuzzy soft b-open set in X.

$\Rightarrow (Fs\ \mathrm{int}\ Fscl\ (f_A) \vee (Fscl\ Fs\mathrm{int}\ (f_A)) \ge f_A$ and $F\sin t\ (f_A) = 0$, therefore $Fscl\, F\sin t\ (f_A) = 0$

$\Rightarrow Fs\ \mathrm{int}\ Fscl\ (f_A) \ge f_A$. Thus f_A is fuzzy soft pre-open set.

(ii) Let f_A be fuzzy soft b-open set in X.

$\Rightarrow (Fs\ \mathrm{int}\ Fscl\ (f_A) \vee (Fscl\ Fs\mathrm{int}\ (f_A)) \ge f_A$ and $Fscl\ (f_A) = 0$, therefore $F\sin t\, Fscl\ \ (f_A) = 0$

$\Rightarrow Fscl\ Fs\ \mathrm{int}\ (f_A) \ge f_A$, Thus f_A is fuzzy soft semi-open set.

Theorem 3.9 : *If* f_A *is fuzzy soft open set and* g_B *is fuzzy soft b-open set in FSTS X,* *then* $f_A \wedge g_B$ *is fuzzy soft b-open set.*

Proof: Consider $f_A \wedge g_B \leq f_A \wedge FsclF\sin t(g_B) \vee F\sin tFscl(g_B)$

$f_A \wedge g_B = f_A \wedge FsclF\sin t(g_B) \vee f_A \wedge F\sin tFscl(g_B)$

$f_A \wedge g_B = [FsclF\sin t(f_A) \wedge FsclF\sin t(g_B)] \vee [F\sin tFscl(f_A) \wedge FsclF\sin t(g_B)]$

$f_A \wedge g_B = FsclF\sin t(f_A \wedge g_B) \vee F\sin tFscl(f_A \wedge g_B)$

$f_A \wedge g_B$ is fuzzy soft b-open set in X.

4. FUZZY SOFT B-CONTINUOUS FUNCTIONS

Definition 4.1: A function $f:(X,\tau) \to (Y,\tau')$ is said to be fuzzy soft b - continuous (FSb- continuous) if for every fuzzy soft open set (G , E) $\in \tau'$, $f^{-1}(G,E)$ is fuzzy soft b-open set in τ.

Theorem 4.1: *Every fuzzy soft continuous function is Fuzzy soft b-continuous function.*

Proof: Let $f: X \to Y$ be Fuzzy soft continuous function. i.e Inverse of every fuzzy soft open set is open in X. Since every fuzzy soft open set of X is fuzzy soft b-open, f is fuzzy soft b-continuous. But converse need not be true. Consider the following example.

Let $f:(X,\tau) \to (Y,\tau')$ defined by $f(x_1) = y_2$ and $f(x_2) = y_1$

$\tau = \{\tilde{0},\tilde{1},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E)\}$

$\tau' = \{\tilde{0},\tilde{1},(G_1,E),(G_2,E)\}$ be fuzzy soft topological spaces.

Let $X = \{x_1, x_2\}$ $Y = \{y_1, y_2\}$ $E = \{e_1, e_2\}$ $(F_1, E) = \left\{\left\{\frac{1}{2},\frac{1}{3}\right\},\left\{\frac{1}{4},\frac{2}{3}\right\}\right\}$

$(F_2,E) = \left\{\left\{\frac{1}{3},\frac{1}{4}\right\},\left\{0,\frac{1}{6}\right\}\right\}$ $(F_3,E) = \left\{\left\{\frac{1}{2},1\right\},\left\{\frac{2}{3},\frac{1}{6}\right\}\right\}$ $(F_4,E) = \left\{\left\{\frac{1}{5},\frac{1}{3}\right\},\left\{\frac{1}{4},\frac{1}{6}\right\}\right\}$

$(F_5,E) = \left\{\left\{\frac{1}{5},\frac{1}{4}\right\},\left\{0,\frac{1}{6}\right\}\right\}$ $(F_6,E) = \left\{\left\{\frac{1}{2},1\right\},\left\{\frac{2}{3},\frac{2}{3}\right\}\right\}$ $(F_7,E) = \left\{\left\{\frac{1}{3},\frac{1}{3}\right\},\left\{\frac{1}{4},\frac{1}{6}\right\}\right\}$

$(G_1,E) = \left\{\left\{\frac{1}{2},\frac{1}{4}\right\},\left\{\frac{1}{5},0\right\}\right\}$ $(G_2,E) = \left\{\left\{\frac{1}{4},\frac{1}{5}\right\},\left\{\frac{1}{6},0\right\}\right\}$

$f^{-1}(G_1,E)=\left\{\left\{\frac{1}{4},\frac{1}{2}\right\}\left\{0,\frac{1}{5}\right\}\right\}$ $f^{-1}(G_2,E)=\left\{\left\{\frac{1}{5},\frac{1}{4}\right\}\left\{0,\frac{1}{6}\right\}\right\}$

$f^{-1}(G_1,E)$ and $f^{-1}(G_2,E)$ are Fuzzy soft b-open sets in X. But $f^{-1}(G_1,E)$ is not fuzzy soft open set in X.

Therefore f is fuzzy soft b-continuous function but not fuzzy soft continuous.

Definition 4.2: A fuzzy soft set A of a FSTS (X,τ) is called Fuzzy soft generalized b –closed set if $Fsbcl(A)\le B$ whenever $A\le B$ and B is fuzzy soft open set in (X,τ).

Theorem 4.2: Every fuzzy soft b-open (b-closed) set is fuzzy soft generalized b-open (generalized b-closed) .

Proof: By definition every fuzzy soft b-open set is generalized b-open.

But converse is not true.

Example: $\tau=\{\tilde{0},\tilde{1},A=\{1,0.6\}\}$

$fsbO(X)=\{\tilde{0},\tilde{1},A=\{1,0.6\},\{\alpha,\beta\}\}$ where $\alpha\succ 0$ or $\beta\succ 0.4$

$fsbc(X)=\{\tilde{0},\tilde{1},B=\{0,0.4\},\{\alpha,\beta\}\}$ where $\alpha=0$ and $\beta\prec 0.4$

Consider $H=\{0,0.3\}$ which is fuzzy soft generalized b-open set but not fuzzy soft b-open.

Theorem 4.3: *If f_A is Fsg- b closed and fuzzy soft b-open set then it is fuzzy soft b-closed.*

Proof: Let f_A is Fsg- b closed and fuzzy soft b-open set. Then $f_A\le f_A$ and $Fsbcl(f_A)\le f_A$

But $f_A\le Fsbcl(f_A)$, therefore $f_A=Fsbcl(f_A)$, f_A is fuzzy soft b-closed.

Definition 4.3: A function $f:(X,\tau)\to(Y,\tau')$ is said to be fuzzy soft generalized-b continuous if inverse image of every fuzzy soft closed set in Y is fuzzy soft generalized b-closed set in X.

Theorem 4.4: *A function $f:(X,\tau)\to(Y,\tau')$ is Fuzzy soft generalized b - continuous iff the inverse image of each fuzzy soft open set of Y is fuzzy soft generalized b - open set of X.*

Proof: Let B be fuzzy soft open set in Y. Then 1 - B is fuzzy sof closed in Y.

Since f is fuzzy soft gb – continuous, $f^{-1}(1-B)=1-f^{-1}(B)$ is fuzzy soft g-b closed in X.

$\Rightarrow f^{-1}(B)$ is fuzzy soft gb- open set of X. Hence the theorem and converse is obvious.

Remarks

(i) Every fuzzy soft open (closed) set is fuzzy soft b - open (closed) set.

(ii) Every fuzzy soft b - closed (open) set is Fuzzy soft generalized b - closed (open) set.

(iii) Every fuzzy soft open (closed) set is fuzzy soft gb-open (closed) set.

Theorem 4.5: *Every fuzzy soft continuous function is Fuzzy soft generalized b - continuous (FSgb – continuous) function.*

Proof: Let $f:(X,\tau)\rightarrow(Y,\tau')$ be fuzzy soft continuous function. Then inverse of fuzzy soft open set is fuzzy soft open in X. Since every fuzzy soft open set is fuzzy soft gb-open , the function f is FSgb-continuous. But converse need not be true as seen from the following example.

Let $f:(X,\tau)\rightarrow(Y,\tau')$ defined by $f(x_1)=y_1$ and $f(x_2)=y_2$

$\tau=\{\tilde{0},\tilde{1},A=\{1,0.9\}\}$, $\tau'=\{\tilde{0},\tilde{1},B=\{0,0.1\}\}$

$fsbo(X)=\{\tilde{0},\tilde{1},\{1,0.9\},\{\alpha,\beta\}\}\quad where\ \alpha\succ 0\ or\ \beta\succ 0.1$

$fsbc(X)=\{\tilde{0},\tilde{1},\{0,0.1\},\{\alpha,\beta\}\}\quad where\ \alpha=0\ or\ \beta\prec 0.1$

$f^{-1}(B)=\{0,0.05\}$ is not fuzzy soft open set in X.

Thus f is not fuzzy soft continuous, But { 0, 0.05 } is FSgb-open set in X. Thus f is FSgb-continuous function.

Theorem 4.6: *Every fuzzy soft b-continuous function is fuzzy soft generalized b - continuous function.*

Proof: Let $f:(X,\tau)\rightarrow(Y,\tau')$ be fuzzy soft b-continuous function. Let A be fuzzy soft open set in Y. $f^{-1}(A)$ is fuzzy soft b-open in X, since f is FS b - continuous.

$1-f^{-1}(A)=f^{-1}(1-A)$ is FS b-closed set in X for every 1-A closed set in Y. Hence $f^{-1}(1-A)$ is fsgb - closed in X and f is FS gb-continuous. Converse need not be true.

consider the following example:-

Let $f:(X,\tau)\rightarrow(Y,\tau')$ be an identity mapping.

$\tau=\{\tilde{0},\tilde{1},A=\{1,0.6\}\}$

$\tau'=\{\tilde{0},\tilde{1},B=\{0,0.3\}\}$

$fsb0(X) = \{\tilde{0}, \tilde{1}, A = \{1, 0.6\}, \{\alpha, \beta\}\}$ where $\alpha \succ 0$ or $\beta \succ 0.4$

$fsbc(X) = \{\tilde{0}, \tilde{1}, B = \{0, 0.3\}, \{\alpha, \beta\}\}$ where $\alpha = 0$ and $\beta \prec 0.4$

$f^{-1}(B) = \{0, 0.3\}$ is not FS b-open set in X. But it is FS gb-open in X.

Definition 4.4: *A function $f : (X, \tau) \to (Y, \tau')$ is said to be FS generalised b - Irresolute if $f^{-1}(A)$ is FS gb- closed set in X for every FS gb-closed set A in Y.*

Theorem 4.7: *A mapping $f : (X, \tau) \to (Y, \tau')$ is FSgb-irresolute if and only if inverse image of every gb-open FS set in Y is gb-open FS set in X.*

proof is obvious**.**

Theorem 4.8: *Every fuzzy soft gb-irresolute mapping is Fuzzy soft gb-continuous mapping.*

Proof: Let $f : X \to Y$ is fsgb-irresolute. Let V be a closed fuzzy soft set in Y, Then V is

FSgb-closed set in Y. Since f is fsgb-irresolute, $f^{-1}(V)$ is gb-closed fuzzy soft set in X.

Hence f is fsgb-continuous mapping. Converse need not be true as seen from the following example.

Let $f : (X, \tau) \to (Y, \tau')$ be an identity mapping.

Where $\tau = \{\tilde{0}, \tilde{1}, \{0.6, 0.7\}, \{0.6, 0.2\}\}$, $\tau' = \{\tilde{0}, \tilde{1}, \{0.7, 0.7\}\}$

Consider a fsgb-closed set $A = \{0.1, 0.1\}$ in Y.

$FS\,bo(X) = \{\tilde{0}, \tilde{1}, \{0.6, 0.7\}, \{0.6, 0.2\}, \{\alpha, \beta\}\}$ where $\alpha \succ 0$ *or* $\beta \succ 0.3$

$FS\,bc(X) = \{\tilde{0}, \tilde{1}, \{0.4, 0.3\}, \{\alpha, \beta\}\}$ where $\alpha = 0$ *and* $\beta \prec 0.3$

$f^{-1}(A) = \{0.1, 0.1\}$ is not fsgb-closed set in X , hence f is not FSgb- irresolute mapping.

Since inverse image of every FS open set in Y is FS gb-open set in X, f is fsgb-continuous mapping.

Theorem 4.9: *Let $f : (X, \tau) \to (Y, \tau')$ and $g : (Y, \tau') \to (Z, \sigma)$ be two functions. Then*

(i) *$g \bullet f : X \to Z$ is fsgb-continuous, if f is fsgb- continuous and g is fuzzy soft continuous.*

(ii) *$g \bullet f : X \to Z$ is fsgb-irresolute, if f and g are fsgb-irresolute functions.*

(iii) *$g \bullet f : X \to Z$ is fsgb-continuous if f is fsgb-irresolute and g is fsgb-continuous.*

Proof: **(i)** Let A be fuzzy soft closed subset of Z. Then $g^{-1}(A)$ is fuzzy soft closed set of Y, since $g:(Y,\tau') \to (Z,\sigma)$ is fuzzy soft continuous and also $f^{-1}(g^{-1}(A)) = (g \bullet f)^{-1}(A)$ is fsgb-closed in X, Since $f:(X,\tau) \to (Y,\tau')$ is fsgb-continuous. Hence $g \bullet f : X \to Z$ is fsgb-continuous.

(ii) Let $g:(Y,\tau') \to (Z,\sigma)$ be fsgb-irresolute and let A be fsgb-closed subset of Z.

$g^{-1}(A)$ is fsgb-closed set of Y, Since g is fsgb-irresolute function. Also $f:(X,\tau) \to (Y,\tau')$ is fsgb-irresolute, $f^{-1}(g^{-1}(A)) = (g \bullet f)^{-1}(A)$ is fsgb-closed set. Thus $g \bullet f : X \to Z$ is fsgb-irresolute function.

(iii) Let A be fsb-closed subset of, $g^{-1}(A)$ is fuzzy soft gb-closed subset of Y. Since $g:(Y,\tau') \to (Z,\sigma)$ is fsgb-continuous. Also, f is fsgb-irresolute, so every fsgb-closed set of Y is fsgb-closed in X. Thus $f^{-1}(g^{-1}(A)) = (g \bullet f)^{-1}(A)$ is fsgb-closed set of X. Hence $g \bullet f : X \to Z$ is fsgb-continuous.

Definition 4.5: A function $f:(X,\tau) \to (Y,\tau^*)$ is Fs b^*- continuous if inverse of fuzzy soft b-open set is Fsb-open in X.

Theorem 5: *Every Fs b^*- continuous function is b-continuous, but converse is not true.*

Proof: Since every Fuzzy soft open set is Fuzzy soft b-open set theorem is obvious but converse is not true .

Consider a function $f : X \to Y$ defined by $f(x_1) = y_2$ & $f(x_2) = y_2$

$\tau = \{\tilde{0}, \tilde{1}, (F_1,E), (F_2,E)\}$

$\tau^* = \{\tilde{0}, \tilde{1}, (G_1,E), (G_2,E)\}$

Where $(F_1,E) = \left\{\left\{\frac{1}{2},\frac{1}{3}\right\},\left\{\frac{1}{3},\frac{1}{5}\right\}\right\}$ $(F_2,E) = \left\{\left\{\frac{1}{2},\frac{2}{3}\right\},\left\{\frac{1}{3},\frac{1}{4}\right\}\right\}$

$(G_1,E) = \left\{\left\{\frac{1}{4},\frac{1}{3}\right\},\left\{0,\frac{1}{3}\right\}\right\}$ $(G_2,E) = \left\{\left\{\frac{1}{3},\frac{1}{2}\right\},\left\{\frac{1}{4},\frac{1}{2}\right\}\right\}$

$f^{-1}(G_1) = \left\{\left\{\frac{1}{3},\frac{1}{4}\right\},\left\{\frac{1}{3},0\right\}\right\} = A$ $f^{-1}(G_2) = \left\{\left\{\frac{1}{2},\frac{1}{3}\right\},\left\{\frac{1}{2},\frac{1}{4}\right\}\right\} = B$

A and B are fuzzy soft b-open sets in X . Therefore f is fuzzy soft b- continuous.

Consider a fuzzy soft b-open set $H = \left\{ \left\{ \frac{1}{5}, \frac{1}{3} \right\}, \left\{ \frac{1}{4}, \frac{1}{3} \right\} \right\}$ in Y.

$f^{-1}(H) = \left\{ \left\{ \frac{1}{3}, \frac{1}{5} \right\}, \left\{ \frac{1}{3}, \frac{1}{4} \right\} \right\} = K$

K is not Fuzzy soft b-open in X. Therefore f is not Fuzzy soft b^* - continuous.

5. CONCLUSION

In this paper we have introduced fuzzy soft semi pre-open set and its relationship with fuzzy soft b-open set is studied. And we introduced fuzzy soft b-continuous functions, generalized b-continuous functions, generalized b-irresolute functions, b^* - continuous functions and then studied some properties of these functions.

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