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A Note on Fuzzy Perfectly Disconnected Spaces

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Abstract

In this paper, several characterizations of fuzzy perfectly disconnected spaces, are established. The conditions under which fuzzy perfectly disconnected spaces become fuzzy Baire spaces, fuzzy σ-Baire spaces and fuzzy almost irresolvable spaces , are also obtained.

Keywords: Fuzzy dense set, fuzzy residual set, fuzzy simply open set, fuzzy G_δ -set , fuzzy regular closed set, fuzzy Baire space, fuzzy submaximal space, fuzzy σ -Baire space , Fuzzy almost irresolvable space.

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1. INTRODUCTION

In 1965, **L.A. ZADEH** [17] introduced the concept of fuzzy sets as a new approach for modelling uncertainties. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. Topology provided the most natural framework for the concepts of fuzzy sets to flourish. In 1968, the concept of fuzzy topological space was introduced by **C. L . CHANG** [4] . The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

The concept of perfectly disconnected spaces in classical topology was defined and studied by **ERIC K. VAN DOUWEN** [16]. In the recent years, there has been a growing trend among many fuzzy topologists to introduce and study different types fuzzy topological spaces . In [13] the notion of fuzzy perfectly disconnected spaces , was introduced and the fuzzy perfectly disconnectedness had been studied along with fuzzy weakly Baireness, fuzzy extremally disconnectedness and fuzzy hyper-connectedness of fuzzy topological spaces. In continuation of this work, several characterizations of fuzzy perfectly disconnected spaces are established in this paper. It is established that in fuzzy perfectly disconnected spaces , 0_X and 1_X are the only two fuzzy simply open sets and the fuzzy interiors of fuzzy pre-closed sets are fuzzy regular closed sets . The conditions under which fuzzy perfectly disconnected spaces become fuzzy Baire spaces, fuzzy σ-Baire spaces and fuzzy almost irresolvable spaces , are also established.

2 . PRELIMINARIES

In order to make the exposition self-contained, some basic notions and results used in the sequel, are given . In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 [4] : Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior and the closure of λ , are defined respectively as follows :

(i). $\text{Int}(\lambda) = \vee \{ \mu / \mu \leq \lambda , \mu \in T\}$ and (ii). $\text{Cl}(\lambda) = \wedge \{ \mu / \lambda \leq \mu , 1-\mu \in T \}$.

Lemma 2.1 [1] : For a fuzzy set λ of a fuzzy topological space X,

(i). $1 - \text{Int}(\lambda) = \text{Cl}(1-\lambda)$ and (ii). $1 - \text{Cl}(\lambda) = \text{Int}(1-\lambda)$.

Definition 2.2 : A fuzzy set λ in a fuzzy topological space (X,T) is called

(1) fuzzy pre-open if $\lambda \leq \text{int cl}(\lambda)$ and fuzzy pre-closed if $\text{cl int}(\lambda) \leq \lambda$. **[7]**

(2) fuzzy semi-open if $\lambda \leq \text{cl int}(\lambda)$ and fuzzy semi-closed if $\text{int cl}(\lambda) \leq \lambda$. **[1]**

(3) fuzzy β-open if $\lambda \leq \text{cl int cl}(\lambda)$ and fuzzy β -closed if $\text{int cl int}(\lambda) \leq \lambda$. **[3]**

(4) fuzzy regular-open if $\lambda = \text{int cl}(\lambda)$ and fuzzy regular-closed if $\lambda = \text{cl int}(\lambda)$. **[1]**

Definition 2.3 : A fuzzy set λ in a fuzzy topological space (X,T), is called

(i). ***a fuzzy dense set*** if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $\text{cl}(\lambda) = 1$, in (X,T) [**8**] .

(ii). ***a fuzzy nowhere dense set*** if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < \text{cl}(\lambda)$. That is, $\text{int cl}(\lambda) = 0$, in (X,T) [**8**].

(iii). ***a fuzzy first category set*** if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'$s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of ***fuzzy second category*** [**8**] .

(iv). ***a fuzzy σ-nowhere dense set*** if λ is a fuzzy F_σ-set in (X, T) such that Int (λ) = 0 [**6**].

(v). ***a fuzzy simply open set*** if Bd (λ) is a fuzzy nowhere dense set in (X,T). That is, λ is a fuzzy simply open set in (X,T) if [cl (λ) ∧ cl (1− λ)] , is a fuzzy nowhere dense set in (X,T) [12] .

(vi). ***a fuzzy somewhere dense set*** if int cl (λ) ≠ 0 in (X,T) [**9**].

(vii). ***a fuzzy σ-boundary set*** if $\lambda = \vee_{i-1}^{\infty} (\mu_i)$, where $\mu_i = \text{cl}(\lambda_i) \wedge (1 - \lambda_i)$ and (λ_i)'s are fuzzy regular open sets in (X, T) [**5**] .

Definition 2.4 [11]: Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Then, 1 − λ is called a fuzzy residual set in (X,T).

Definition 2.5 : A fuzzy topological space (X,T) is called

(i). ***a fuzzy Baire space*** if $\text{int}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'$s are fuzzy nowhere dense sets in (X,T) [**11**].

(ii). ***a fuzzy submaximal space*** if for each fuzzy set λ in (X, T) such that cl (λ) = 1, λ ∈ T [**2**].

(iii). ***a fuzzy open hereditarily irresolvable space*** if int cl (λ) = 0, then int (λ) = 0, for any non-zero fuzzy set λ in (X, T) [**10**].

(iv). ***a fuzzy σ-Baire space*** if $\text{int}[\vee_{i=1}^{\infty}(\lambda_i)] = 0$, where (λ_i) 's are fuzzy σ-nowhere dense sets in (X,T) [**6**].

Lemma 2.2 [1] : In a fuzzy topological space

(a). The closure of a fuzzy open set is a fuzzy regular closed set.

(b). The interior of a fuzzy closed set is a fuzzy regular open set.

Theorem 2.1 [13] : : If (X, T) is a fuzzy perfectly disconnected space, then

(i). The closure of a fuzzy semi-open set in (X,T) is a fuzzy open set in (X,T).

(ii). The closure of a fuzzy pre-open set in (X,T) is a fuzzy open set in (X,T).

Theorem 2.2 [11] : If $\text{cl}[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$, where (λ_i) 's are fuzzy dense and open sets in a fuzzy topological space (X,T), then (X, T) is a fuzzy Baire space.

Theorem 2.3 [5] : If λ is a fuzzy residual set in a fuzzy topological space (X,T), then there exists a fuzzy G_δ –set η in (X,T) such that $\eta \leq \lambda$.

Theorem 2.4 [13] : If λ is a fuzzy nowhere dense set in a fuzzy perfectly disconnected space (X, T), then $\lambda = 0$, in (X, T).

Theorem 2.5 [14]: If λ is a fuzzy somewhere dense set in a fuzzy topological space (X, T), then there exist a fuzzy regular closed set η in (X, T) such that $\eta \leq$ cl (λ).

Theorem 2.6 [13] : If λ is a fuzzy regular closed set in a fuzzy perfectly disconnected space (X, T), then λ is a fuzzy open set in (X, T).

Theorem 2.7 [14] : If λ is a non-zero fuzzy β -open set in a fuzzy topological space (X, T), then λ is a fuzzy somewhere dense set in (X, T).

Theorem 2.8 [5] : : If λ is a fuzzy σ –nowhere dense set in a fuzzy topological space (X, T) , then $1 - \lambda$ is a fuzzy residual set in (X, T).

Theorem 2.9 [5 **] :** If λ is a fuzzy σ-boundary set in a fuzzy topological space (X,T), then $\lambda = \bigvee_{i=1}^{\infty} (\mu_i)$, where (μ_i)' s are fuzzy pre-closed sets in (X,T).

Theorem 2.10 [5**] :** If λ is a fuzzy residual set in a fuzzy second category but not a fuzzy Baire space (X, T), then there exists a fuzzy closed and fuzzy residual set η in (X, T) such that cl (λ) $\leq \eta$.

Theorem 2.11 [**5**]: If λ is a fuzzy co- σ-boundary set in (X, T), then $1 - \lambda$ is a fuzzy σ-boundary set in (X, T).

Theorem 2.12 [**6**]: In a fuzzy topological space (X, T), a fuzzy set λ is a fuzzy σ-nowhere dense set in (X,T) if and only if $1 - \lambda$ is a fuzzy dense and fuzzy G_δ-set in (X, T).

Theorem 2.13 [**6**] : If the fuzzy topological space (X, T) is a fuzzy σ-Baire and fuzzy open hereditarily irresolvable space, then (X, T) is a fuzzy Baire space.

Theorem 2.14 [15]: If a fuzzy topological space (X, T) is a fuzzy submaximal space, then (X ,T) is a fuzzy open hereditarily irresolvable space.

Theorem 2.15 [15]: If the fuzzy topological space (X, T) is a fuzzy Baire space, then (X, T) is a fuzzy almost irresolvable space.

3. FUZZY PERFECTLY DISCONNECTED SPACES

Definition 3.1 [13] : A fuzzy topological space (X, T) is called a fuzzy perfectly disconnected space if for any two non – zero fuzzy sets λ and μ

defined on X with $\lambda \leq 1 - \mu$, cl $(\lambda) \leq 1 -$ cl (μ), in (X, T).

Proposition 3.1 : If λ is a fuzzy semi-closed set in a fuzzy perfectly disconnected space (X,T), then λ is a fuzzy pre-closed set in (X,T).

Proof : Let λ be a fuzzy semi-closed set in (X,T). Then , int cl $(\lambda) \leq \lambda$, in (X,T). This implies that $1 - \lambda \leq 1 -$ [int cl (λ)] and then $1 - \lambda \leq$ cl int $(1 - \lambda)$. Since (X,T) is a fuzzy perfectly disconnected space, $1 - \lambda \leq 1 - [1 -$ cl int $(1 - \lambda)]$, implies that cl $(1 - \lambda) \leq 1 -$ cl[$1 -$ cl int $(1 - \lambda)$], in (X,T). This implies that cl $(1 - \lambda) \leq$ int cl int $(1 - \lambda)$, in (X,T) and then $1 - \lambda \leq$ cl $(1 - \lambda) \leq$ int cl int $(1 - \lambda)$, in (X,T). This implies that , $1 - \lambda \leq 1 -$ cl int cl (λ) and thus, cl int cl$(\lambda) \leq \lambda$, in (X,T). Since int cl$(\lambda) \leq$ cl [int cl(λ)], int cl$(\lambda) \leq \lambda$ and hence λ is a fuzzy pre-closed set in (X,T).

Proposition 3.2 : If λ is a fuzzy pre-closed set in a fuzzy perfectly disconnected space (X,T), then int (λ) is a fuzzy regular closed set in (X,T).

Proof : Let λ be a fuzzy pre-closed set in (X,T). Then, $1 - \lambda$ is a fuzzy pre-open set in (X,T). Since (X,T) is a fuzzy perfectly disconnected space, by theorem 2.1 , cl $(1 - \lambda)$ is a fuzzy open set in (X, T) and then , by lemma 2.1, $1 -$ int (λ) is a fuzzy open set in (X, T) . Hence int (λ) is a fuzzy closed set in (X,T). Then, cl int (λ) = int (λ) in (X,T). Now cl int [int (λ)] = cl int (λ) = int (λ), implies that int (λ) is a fuzzy regular closed set in (X,T).

Proposition 3.3 : If λ is a fuzzy residual set in a fuzzy perfectly disconnected space (X,T), then there exists a fuzzy G_δ –set η in (X,T) such that cl $(\eta) \leq \lambda$.

Proof : Let λ be a fuzzy residual set in (X,T). Then, by theorem 2.3 , there exists a fuzzy G_δ –set η in (X,T) such that $\eta \leq \lambda$. Since (X,T) is a fuzzy perfectly disconnected space, $1 - \lambda \leq 1 - \eta$, implies that cl $(1 - \lambda) \leq 1 -$ cl (η), in (X,T). Then , by lemma 2.1 , $1 -$ int $(\lambda) \leq 1 -$ cl (η). This implies that cl $(\eta) \leq$ int (λ) and then cl $(\eta) \leq$ int $(\lambda) \leq \lambda$. in (X,T). Thus there exists a fuzzy G_δ –set η in (X,T) such that cl $(\eta) \leq \lambda$.

Proposition 3.4 : If each fuzzy G_δ -set is a fuzzy dense set in a fuzzy perfectly disconnected space (X,T), then 1_X is the only fuzzy residual set in (X,T).

Proof : Let λ be a fuzzy residual set in (X,T). Then, by proposition 3.3, there exists a fuzzy G_δ -set η in (X,T) such that cl $(\eta) \leq \lambda$. By hypothesis, the fuzzy G_δ - set is a fuzzy dense set in (X,T). Then, cl (η) = 1, in (X,T). This implies that $1 \leq \lambda$. That is, $\lambda = 1$, in (X,T). Thus, 1_X is the only fuzzy residual set in (X,T).

Proposition 3.5 : In a fuzzy perfectly disconnected space (X,T), 0_X and 1_X are the only two fuzzy simply open sets in (X,T).

Proof : Let λ be a fuzzy simply open set in (X, T). Then, Bd (λ) is a fuzzy nowhere dense set in (X,T). Since (X,T) is a fuzzy perfectly disconnected space, by theorem 2.4 , $Bd(\lambda) = 0$, in (X,T). That is, $[\, cl(\lambda) \wedge cl(1-\lambda)] = 0$, in (X,T). This implies that $cl\ \lambda\,) \wedge [\, 1 - int(\lambda)\,] = 0$ and then $cl(\lambda) \le (1 - [1 - int(\lambda)\,]\,)$ in (X, T). That is, $cl(\lambda) \le int(\lambda)$. But $int(\lambda) \le cl(\lambda)$, in (X, T) and hence $cl(\lambda) = int(\lambda)$. This is possible if either $\lambda = 0$ or 1. Thus, 0_X and 1_X are the only fuzzy simply open sets in a fuzzy perfectly disconnected space.

Proposition 3.6 : If (λ_i)'s (i = 1 to ∞) are fuzzy somewhere dense sets in a fuzzy perfectly disconnected space (X,T), then there exists a fuzzy G_δ -set η in (X,T) such that $\eta \le \wedge_{i=1}^{\infty}[\, cl(\lambda_i)\,]$.

Proof : Let (λ_i)'s (i = 1 to ∞) be fuzzy somewhere dense sets in (X, T). Then , by theorem 2.5, there exists fuzzy regular closed sets η_i in (X, T) such that $\eta_i \le cl(\lambda_i)$. Since (X,T) is a fuzzy perfectly disconnected space, by theorem 2.6, the fuzzy regular closed sets (η_i)'s are fuzzy open sets in (X,T). Let $\eta = \wedge_{i=1}^{\infty}(\eta_i)$. Then, η is a fuzzy G_δ –set in (X,T). Now $\wedge_{i=1}^{\infty}(\eta_i) \le \eta_i \le cl(\lambda_i)$, implies that $\eta \le cl(\lambda_i)$. Then, $\eta \le \wedge_{i=1}^{\infty}[\, cl(\lambda_i)\,]$.

Proposition 3.7 : If (λ_i)'s (i = 1 to ∞) are fuzzy β -open sets in a fuzzy perfectly disconnected space (X,T), then there exists a fuzzy G_δ -set η in (X,T) such that $\eta \le \wedge_{i=1}^{\infty}[cl(\lambda_i)]$.

Proof : The proof follows from the theorem 2.7 and proposition 3.6.

Proposition 3.8 : If λ is a fuzzy σ - nowhere dense set in a fuzzy perfectly disconnected space (X, T) , then there exists a fuzzy G_δ –set η in (X,T) such that $\lambda \le int(1 - \eta)$.

Proof : Let λ be a fuzzy σ -nowhere dense set in (X,T). Then, by theorem 2.8, $1-\lambda$ is a fuzzy residual set in (X, T). Since (X,T) is a perfectly disconnected space, by proposition 3.3 , there exists a fuzzy G_δ –set η in (X,T) such that $cl(\eta) \le 1 - \lambda$ and then $\lambda \le 1 - cl(\eta)$. This implies that $\lambda \le int(1 - \eta)$.

Proposition 3.9 : If λ is a fuzzy σ-boundary set in a fuzzy perfectly disconnected space (X,T), $\lambda \ge \vee_{i-1}^{\infty}[\, int(\mu_i)\,]$, where $[\, int(\mu_i)]'s$ are fuzzy regular closed sets in (X,T).

Proof : Let λ be a fuzzy σ-boundary in (X,T). Then, by theorem 2.9, $\lambda = \vee_{i=1}^{\infty}(\mu_i)$, where (μ_i)’ s are fuzzy pre-closed sets in (X,T). Since (X,T) is a fuzzy perfectly disconnected space, by proposition 3.2 , for the fuzzy pre-closed closed sets (μ_i))'s in (X,T), $[\, int(\mu_i)]'s$ are fuzzy regular closed sets in (X,T). Now $\vee_{i-1}^{\infty}(\mu_i) \ge \vee_{i-1}^{\infty}[\, int(\mu_i)]$, implies that $\lambda \ge \vee_{i-1}^{\infty}[\, int(\mu_i)\,]$, where $[\, int(\mu_i)]\,'s$ are fuzzy regular closed sets in (X,T).

Proposition 3.10 : If λ is a fuzzy σ-boundary set in a fuzzy perfectly disconnected space (X,T), then there exists a fuzzy F_σ -set η in (X,T) such that $\eta \leq \lambda$.

Proof : Let λ be a fuzzy σ-boundary in (X,T). Then, by proposition 3.9 , $\lambda \geq \vee_{i-1}^{\infty}$ [int (μ_i)], where [int (μ_i)]'s are fuzzy regular closed sets in (X,T). Since fuzzy regular closed sets are fuzzy closed sets in a fuzzy topological space, [int (μ_i)]'s are fuzzy closed sets in (X,T) and then $\vee_{i-1}^{\infty}$ [int (μ_i)] is a fuzzy F_σ -set η in (X,T). Thus, if λ is a fuzzy σ-boundary set in a fuzzy perfectly disconnected space (X,T), then there exists a fuzzy F_σ -set η in (X,T) such that $\eta \leq \lambda$.

Proposition 3.11 : If λ is a fuzzy co- σ-boundary set in a fuzzy perfectly disconnected space (X,T), then there exists a fuzzy G_δ -set δ in (X,T) such that $\lambda \leq \delta$.

Proof : Let λ be a fuzzy co- σ-boundary set in (X, T). Then, $1 - \lambda$ is a fuzzy σ-boundary set in (X, T). Then, by proposition 3.10 , there exists a fuzzy F_σ -set η in (X,T) such that $\eta \leq 1 - \lambda$. This implies that $\lambda \leq 1 - \eta$. Since η is a fuzzy F_σ -set in (X,T), $1 - \eta$ is a fuzzy G_δ –set in (X,T). Let $\delta = 1 - \eta$. Thus, if λ is a fuzzy co- σ-boundary set in (X,T), then there exists a fuzzy G_δ -set δ in (X,T) such that $\lambda \leq \delta$.

Proposition 3.12 : If λ is a fuzzy co- σ-boundary set in a fuzzy perfectly disconnected space (X,T) such that cl (λ) = 1, then there exists a fuzzy σ-nowhere dense set η in (X,T) such that $\eta \leq 1 - \lambda$.

Proof : Let λ be a fuzzy co- σ-boundary set in (X, T) such that cl (λ) = 1. Since the topological space (X,T) is fuzzy perfectly disconnected, by proposition 3.11, there exists a fuzzy G_δ -set δ in (X,T) such that $\lambda \leq \delta$. This implies that cl (λ) $\leq$ cl (δ), in (X, T). But cl (λ) = 1, implies that cl (δ) = 1. Thus δ is a fuzzy dense and fuzzy G_δ -set in (X,T). Then, by theorem 2.12, $1 - \delta$ is a fuzzy σ-nowhere dense set in (X,T). Let $\eta = 1 - \delta$. Then η is a fuzzy σ-nowhere dense set in (X,T). Now $\lambda \leq \delta$, implies that $1 - \delta \leq 1 - \lambda$ and then $\eta \leq 1 - \lambda$, in (X,T).

Proposition 3.13 : If λ is a fuzzy residual set in a fuzzy second category (but not fuzzy Baire) and fuzzy perfectly disconnected space (X,T), then there exists a fuzzy closed and fuzzy residual set η in (X,T) such that cl (λ) $\leq$ int (η), in (X,T).

Proof : Let λ be a fuzzy residual set in a fuzzy second category but not a fuzzy Baire space (X, T). Then, by theorem 2.10 , there exists a fuzzy closed and fuzzy residual set η in (X,T) such that cl (λ) $\leq \eta$. Then, $1 - \eta \leq 1 -$ cl (λ), in (X,T) . Since the fuzzy topological space (X, T) is fuzzy perfectly disconnected , $1 - \eta \leq 1 -$ cl (λ), implies that cl [$1 - \eta$] $\leq 1 -$ cl [cl (λ)], in (X,T) and then $1 -$ int (η) $\leq 1 -$ cl (λ) . This implies that cl (λ) $\leq$ int (η), in (X,T).

Proposition 3.14 : If λ is a fuzzy semi-closed set in a fuzzy perfectly disconnected space (X,T), then int (λ) is a fuzzy regular closed set in (X,T).

Proof : The proof follows from the proposition 3.1 and proposition 3.2.

4 . FUZZY PERFECTLY DISCONNECTED SPACES , FUZZY σ -BAIRE SPACES and FUZZY BAIRE SPACES

Proposition 4.1 : If λ is a fuzzy σ-boundary set in a fuzzy perfectly disconnected space (X,T) such that int(λ) = 0, then there exists a fuzzy σ-nowhere dense η in (X,T) such that η ≤ λ .

Proof : Let λ be a fuzzy σ-boundary set in (X,T). Then, by proposition 3.10, then there exists a fuzzy F_σ-set η in (X,T) such that η ≤ λ . Now η ≤ λ , implies that int (η) ≤ int (λ) , in (X,T) . Since int(λ) = 0, int (η) ≤ 0. That is, int (η) = 0, in (X,T) . Thus, η is a fuzzy F_σ-set in (X,T) such that int (η) = 0 , implies that η is a fuzzy σ-nowhere dense set in (X,T) such that η ≤ λ .

The following propositions give conditions for fuzzy perfectly disconnected spaces to become fuzzy σ -Baire spaces.

Proposition 4.2 : If int [$\bigvee_{i=1}^{\infty}$ (λ_i)] = 0, where (λ_i)'s are fuzzy σ-boundary sets in a fuzzy perfectly disconnected space (X,T) such that int (λ_i) = 0, then (X,T) is a fuzzy σ -Baire space.

Proof : Let (λ_i) 's (i = 1 to ∞) be fuzzy σ-boundary sets in a fuzzy perfectly disconnected space (X,T) such that int (λ_i) = 0. Then, by proposition 4.1, there exist fuzzy σ-nowhere dense sets η_i in (X,T) such that $\eta_i \leq \lambda_i$. This implies that $\bigvee_{i=1}^{\infty}$ (η_i) ≤ $\bigvee_{i=1}^{\infty}$ (λ_i) in (X,T) and then int [$\bigvee_{i=1}^{\infty}$ (η_i)] ≤ int [$\bigvee_{i=1}^{\infty}$ (λ_i)] . By hypothesis, int [$\bigvee_{i=1}^{\infty}$ (λ_i)] = 0. This implies that int [$\bigvee_{i=1}^{\infty}$ (η_i)] = 0 , where (η_i) 's are fuzzy σ-nowhere dense sets in (X,T). Hence (X,T) is a fuzzy σ -Baire space.

Proposition 4.3 : If cl [$\bigwedge_{i=1}^{\infty}$ (λ_i)] = 1, where (λ_i)'s are fuzzy co- σ-boundary sets in a fuzzy perfectly disconnected space (X,T) such that cl(λ_i) = 1, then (X,T) is a fuzzy σ -Baire space.

Proof : Let (λ_i) 's (i = 1 to ∞) be fuzzy co- σ-boundary sets in a fuzzy perfectly disconnected space (X,T) such that cl (λ_i) = 1. Then, by proposition 3.12, there exist fuzzy σ-nowhere dense sets η_i in (X,T) such that $\eta_i \leq 1 -$

$\lambda_{i.}$ This implies that $\bigvee_{i=1}^{\infty} (\eta_i) \leq \bigvee_{i=1}^{\infty} (1-\lambda_i)$ in (X,T) and then $\text{int}\,[\bigvee_{i=1}^{\infty} (\eta_i)] \leq$ int $[\bigvee_{i=1}^{\infty} (1-\lambda_i)]$. By hypothesis, cl $[\bigwedge_{i=1}^{\infty} (\lambda_i)] = 1$ in (X,T) and then $1 -$ cl $[\bigwedge_{i=1}^{\infty} (\lambda_i)] = 0$. This implies that int $[1 - [\bigwedge_{i=1}^{\infty} (\lambda_i)]$ $= 0$ and thus int $[\bigvee_{i=1}^{\infty} (1-\lambda_i)] = 0$. This implies that $\bigvee_{i=1}^{\infty} (\eta_i) \leq 0$. That is, int $[\bigvee_{i=1}^{\infty} (\eta_i)] = 0$, where (η_i) 's are fuzzy σ-nowhere dense sets in (X,T). Hence (X,T) is a fuzzy σ -Baire space.

Proposition 4.4 : If int $[\bigvee_{i=1}^{\infty} \text{int}(\delta_i)] = 0$, where (δ_i)'s are fuzzy F_σ -sets in a fuzzy perfectly disconnected space (X,T) , then (X,T) is a fuzzy σ - Baire space.

Proof : Let (λ_i) 's (i = 1 to ∞) be fuzzy σ - nowhere dense sets in (X,T).Since (X, T) is a fuzzy perfectly disconnected space, by proposition 3.8 , there exist fuzzy G_δ -sets η_i in (X,T) such that $\lambda_i \leq$ int $(1 - \eta_i)$. Let $\delta_i = 1 - \eta_i$. Then, δ_i is a fuzzy F_σ –set in (X,T). Now $\lambda_i \leq$ int (δ_i), implies that $\bigvee_{i=1}^{\infty} (\lambda_i) \leq \bigvee_{i=1}^{\infty}[\text{int}(\delta_i)]$ and then, $\text{int}\,[\bigvee_{i=1}^{\infty} (\lambda_i)] \leq$ int $[\bigvee_{i=1}^{\infty} (\delta_i)]$, in (X,T). By hypothesis, int $[\bigvee_{i=1}^{\infty} (\delta_i)] = 0$. This implies that int $[\bigvee_{i=1}^{\infty} (\lambda_i)]$ $= 0$, where (λ_i) 's are fuzzy σ-nowhere dense sets in (X,T). Hence (X,T) is a fuzzy σ -Baire space.

The following propositions give conditions for fuzzy perfectly disconnected spaces to become fuzzy Baire spaces.

Proposition 4.5 : If int $[\bigvee_{i=1}^{\infty} (\lambda_i)] = 0$, where (λ_i)'s are fuzzy σ-boundary sets such that int $(\lambda_i) = 0$, in a fuzzy perfectly disconnected and fuzzy open hereditarily irresolvable space (X,T) , then (X,T) is a fuzzy Baire space.

Proof : Suppose that int $[\bigvee_{i=1}^{\infty} (\lambda_i)] = 0$, where (λ_i)'s are fuzzy σ-boundary sets such that int $(\lambda_i) = 0$, in a fuzzy perfectly disconnected space (X,T). Then, by proposition 4.2, (X,T) is a fuzzy σ -Baire space. Since (X,T) is a fuzzy open hereditarily irresolvable space, by theorem 2.13, (X, T) is a fuzzy Baire space.

Proposition 4.6 : If cl $[\bigwedge_{i=1}^{\infty} (\lambda_i)] = 1$, where (λ_i)'s are fuzzy co- σ-boundary sets such that cl$(\lambda_i) = 1$, in a fuzzy perfectly disconnected and fuzzy open hereditarily irresolvable space (X,T) , then (X,T) is a fuzzy Baire space.

Proof : *The proof follows from the proposition 4.3 and theorem 2.13.*

Proposition 4.7 : If int $[\bigvee_{i=1}^{\infty} \text{int}(\delta_i)] = 0$, where (δ_i)'s are fuzzy F_σ -sets in a fuzzy perfectly disconnected and fuzzy open hereditarily irresolvable space (X,T) , then (X,T) is a fuzzy Baire space.

Proof : *The proof follows from the proposition 4.4 and theorem 2.13.*

Proposition 4.8 : If int [$\vee_{i=1}^{\infty}$ (λ_i)] = 0, where (λ_i)'s are fuzzy σ-boundary sets such that int (λ_i) = 0, in a fuzzy perfectly disconnected and fuzzy submaximal space (X,T) , then (X,T) is a fuzzy Baire space.

Proof : Suppose that int [$\vee_{i=1}^{\infty}$ (λ_i)] = 0, where (λ_i)'s are fuzzy σ-boundary sets such that int (λ_i) = 0, in a fuzzy perfectly disconnected space (X,T). Since (X,T) is a fuzzy submaximal space , by theorem 2.14, (X,T) is a fuzzy fuzzy open hereditarily irresolvable space . Then , by proposition 4.5 , (X,T) is a fuzzy Baire space.

Proposition 4.9 : If cl [$\wedge_{i=1}^{\infty}$ (λ_i)] = 1, where (λ_i)'s are fuzzy co- σ-boundary sets such that cl(λ_i) = 1, in a fuzzy perfectly disconnected and fuzzy submaximal space (X,T) , then (X,T) is a fuzzy Baire space.

Proof : *The proof follows from the proposition 4.6 and the theorem 2.14.*

Proposition 4.10 : If int [$\vee_{i=1}^{\infty}$ int(δ_i)] = 0, where (δ_i)'s are fuzzy F_σ -sets in a fuzzy perfectly disconnected and fuzzy submaximal space (X,T) , then (X,T) is a fuzzy Baire space.

Proof : *The proof follows from the proposition 4.7 and the theorem 2.14.*

The following propositions give conditions for fuzzy perfectly disconnected spaces to become fuzzy* almost irresolvable *spaces.

Proposition 4.11 : If int [$\vee_{i=1}^{\infty}$ (λ_i)] = 0, where (λ_i)'s are fuzzy σ-boundary sets such that int (λ_i) = 0, in a fuzzy perfectly disconnected and fuzzy open hereditarily irresolvable space (X,T) , then (X,T) is a fuzzy almost irresolvable space .

Proof : Suppose that int [$\vee_{i=1}^{\infty}$ (λ_i)] = 0, where (λ_i)'s are fuzzy σ-boundary sets such that int (λ_i) = 0, in a fuzzy perfectly disconnected and fuzzy open hereditarily irresolvable space (X,T).Then, by proposition 4.5 , (X,T) is a fuzzy Baire space . By theorem 2.13, (X,T) is a fuzzy almost irresolvable space .

Proposition 4.12 : If cl [$\wedge_{i=1}^{\infty}$ (λ_i)] = 1, where (λ_i)'s are fuzzy co- σ-boundary sets such that cl(λ_i) = 1, in a fuzzy perfectly disconnected and fuzzy open hereditarily irresolvable space (X,T) , then (X,T) is a fuzzy almost irresolvable space.

Proof : *The proof follows from the proposition 4.6 and the theorem 2.15.*

Proposition 4.13 : If int [$\vee_{i=1}^{\infty}$ int(δ_i)] = 0, where (δ_i)'s are fuzzy F_σ -sets in a fuzzy perfectly disconnected and fuzzy open hereditarily irresolvable space (X,T) , then (X,T) is a fuzzy almost irresolvable space.

Proof : *The proof follows from the proposition 4.7 and the theorem 2.15.*

Proposition 4.14 : If int [$\vee_{i=1}^{\infty}$ (λ_i)] = 0, where (λ_i)'s are fuzzy σ-boundary sets such that int (λ_i) = 0, in a fuzzy perfectly disconnected and fuzzy submaximal space (X,T) , then (X,T) is a fuzzy almost irresolvable space.

Proof : *The proof follows from the proposition 4.8 and the theorem 2.15.*

Proposition 4.15 : If cl [$\wedge_{i=1}^{\infty}$ (λ_i)] = 1, where (λ_i)'s are fuzzy co- σ-boundary sets such that cl(λ_i) = 1, in a fuzzy perfectly disconnected and fuzzy submaximal space (X,T) , then (X,T) is a fuzzy almost irresolvable space.

Proof : *The proof follows from the proposition 4.9 and the theorem 2.14.*

Proposition 4.16 : If int [$\vee_{i=1}^{\infty}$ int(δ_i)] = 0, where (δ_i)'s are fuzzy F_σ -sets in a fuzzy perfectly disconnected and fuzzy submaximal space (X,T) , then (X,T) is a fuzzy almost irresolvable space.

Proof : *The proof follows from the proposition 4.10 and the theorem 2.14.*

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