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More On fswg closed Set

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Abstract

In this paper a new type of compactness and T_2-space are introduced and studied by using $fswg$-closed set as a basic tool. Again we establish the interrelations of $fswg$-closed set (resp., $fswg$-closed function, $fswg$-continuous function) with the sets (resp., functions) defined earlier. Lastly, the applications of $fswg$-continuous function, $fswg$-irresolute function, $fswg$-strongly continuous function, strongly $fswg$-continuous function on the newly defined spaces are shown here.

AMS Subject Classifications: 54A40, 54C99, 03E72.

Keywords: $fswg$-closed set, $fswg$-closed function, $fswg$-continuous function, $fswg$-irresolute function, $fswg$-strongly continuous function, strongly $fswg$-continuous function, $fswg$-compact space, fsT_g-space.

1. INTRODUCTION

In [2, 3], fuzzy generalized version of closed set is introduced. Afterwards, different types of generalized version of fuzzy closed sets are introduced and studied. In this context, we have to mention [3, 6, 7, 8, 9, 10, 11, 12]. In [10], $fswg$-closed set is introduced. In [18], $fswg$-open function, $fswg$-closed function, $fswg$-continuous function, $fswg$-irresolute function, $fswg$-strongly continuous function, strongly $fswg$-continuous function are introduced and studied.

2. PRELIMINARIES

Throughout this paper (X, τ) or simply by X we shall mean a fuzzy topological space (fts, for short) in the sense of Chang [21]. In [33], L.A. Zadeh introduced fuzzy set as follows: A fuzzy set A is a function from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The support [33] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement of a fuzzy set A in X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$ [33]. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [33] while AqB means A is quasi-coincident (q-coincident, for short) with B, if there exists $x \in X$ such that $A(x) + B(x) > 1$ [31]. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not q B$ respectively. For a fuzzy point x_t and a fuzzy set A, $x_t \in A$ means $A(x) \geq t$, i.e., $x_t \leq A$. For a fuzzy set A, clA and $intA$ will stand for fuzzy closure [21] and fuzzy interior [21] of A respectively. A fuzzy set A is called a fuzzy neighbourhood (fuzzy nbd, for short) of a fuzzy point x_α if there exists a fuzzy open set U in X such that $x_\alpha \in U \leq A$ [31]. If, in addition, A is fuzzy open, then A is called fuzzy open nbd of x_α [31]. A fuzzy set A is called a fuzzy quasi neighbourhood (fuzzy q-nbd, for short) [31] of a fuzzy point x_α in an fts X if there is a fuzzy open set U in X such that $x_\alpha q U \leq A$. If, in addition, A is fuzzy open, then A is called fuzzy open q-nbd [31] of x_α. A fuzzy set A in X is called fuzzy regular open [1] (resp., fuzzy semiopen [1], fuzzy preopen [30], fuzzy α-open [19], fuzzy β-open [24], fuzzy γ-open [5]) if $A = int(clA)$ (resp., $A \leq cl(intA)$, $A \leq int(clA)$, $A \leq int(cl(intA))$, $A \leq cl(int(clA))$, $A \leq cl(intA) \bigvee int(clA)$). A fuzzy set A is called fuzzy π-open if A is the union of finite number of fuzzy regular open sets [9]. The complement of a fuzzy regular open (resp., fuzzy semiopen, fuzzy preopen, fuzzy α-open, fuzzy β-open, fuzzy γ-open) set is called fuzzy regular closed [1] (resp., fuzzy semiclosed [1], fuzzy preclosed [30], fuzzy α-closed [19], fuzzy β-closed [24], fuzzy γ-closed [5]). The intersection of all fuzzy semiclosed (resp., fuzzy preclosed, fuzzy α-closed, fuzzy β-closed, fuzzy γ-closed) sets containing a fuzzy set A is called fuzzy semiclosure [1] (resp., fuzzy preclosure [30], fuzzy α-closure [19], fuzzy β-closure [24], fuzzy γ-closure [5]) of A, to be denoted by $sclA$ (resp., $pclA$, αclA, βclA, γclA). The collection of all fuzzy open (resp., fuzzy regular open, fuzzy semiopen, fuzzy preopen, fuzzy α-open, fuzzy β-open, fuzzy γ-open, fuzzy π-open) sets in an fts (X, τ) is denoted by τ (resp., $FRO(X, \tau)$, $FSO(X, \tau)$, $FPO(X, \tau)$, $F\alpha O(X, \tau)$, $F\beta O(X, \tau)$, $F\gamma O(X, \tau)$, $F\pi O(X, \tau)$). The collection of all fuzzy closed (resp., fuzzy regular closed, fuzzy semiclosed, fuzzy preclosed, fuzzy α-closed, fuzzy β-closed, fuzzy γ-closed, fuzzy π-closed) sets in an fts X is denoted by τ^c (resp., $FRC(X, \tau)$, $FSC(X, \tau)$, $FPC(X, \tau)$, $F\alpha C(X, \tau)$, $F\beta C(X, \tau)$, $F\gamma C(X, \tau)$, $F\pi C(X, \tau)$).

3. $fswg$-CLOSED SET: SOME MUTUAL RELATIONSHIPS

In this section, we first recall the definition of $fswg$-closed set and then establish the mutual relationships of this function with the functions defined in [2, 3, 6, 7, 8, 10, 11, 12].

First we recall the following definition from [10] for ready references.

Definition 3.1 [10]. Let (X, τ) be an fts and $A \in I^X$. Then A is called $fswg$-closed set in X if $clintA \leq U$ whenever $A \leq U \in FSO(X, \tau)$.

The complement of $fswg$-closed set is called $fswg$-open set in X.

Note 3.2. Fuzzy closed set, fuzzy regular closed set, fuzzy preclosed set, fuzzy α-closed set in an fts (X, τ) are $fswg$-closed set in X. But the converses are not necessarily true, in general, follow from the following example.

Example 3.3. $fswg$-closed set $\not\Rightarrow$ fuzzy closed set, fuzzy regular closed set, fuzzy preclosed set, fuzzy α-closed set

Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Consider the fuzzy set B defined by $B(a) = 0.6, B(b) = 0.4$. As $1_X \in FSO(X, \tau)$ only containing B, so B is $fswg$-closed set in (X, τ). But as $clintB = 1_X \setminus A \not\leq B \Rightarrow B \notin FPC(X, \tau)$. Also $B \notin \tau^c$, $B \notin FRC(X)$, $B \notin F\alpha C(X)$ as $clintclB = 1_X \not\leq B$.

Let us now recall the following definitions from [2, 3, 6, 7, 8, 10, 12, 13] for ready references.

Definition 3.4. Let (X, τ) be an fts and $A \in I^X$. Then A is called

(i) fg-closed set [2, 3] if $clA \leq U$ whenever $A \leq U \in \tau$,

the complement of an fg-closed set in X is called an fg-open set in X,

(ii) fgp-closed set [3] if $pclA \leq U$ whenever $A \leq U \in \tau$,

(iii) fpg-closed set [3] if $pclA \leq U$ whenever $A \leq U \in FPO(X, \tau)$,

(iv) $fg\alpha$-closed set [3] if $\alpha clA \leq U$ whenever $A \leq U \in \tau$,

(v) $f\alpha g$-closed set [3] if $\alpha clA \leq U$ whenever $A \leq U \in F\alpha O(X, \tau)$,

(vi) $fg\beta$-closed set [8] if $\beta clA \leq U$ whenever $A \leq U \in \tau$,

(vii) $f\beta g$-closed set [8] if $\beta clA \leq U$ whenever $A \leq U \in F\beta O(X, \tau)$,

(viii) fgs-closed set [3] if $sclA \leq U$ whenever $A \leq U \in \tau$,

(ix) fsg-closed set [3] if $sclA \leq U$ whenever $A \leq U \in FSO(X, \tau)$,

(x) fgs^*-closed set [6] if $clA \leq U$ whenever $A \leq U \in FSO(X, \tau)$,

(xi) fs^*g-closed set [7] if $clA \leq U$ whenever $A \leq U$ where U is fg-open set in X,

(xii) fmg-closed set [10] if $cl(intA) \leq U$ whenever $A \leq U$ and U is fg-open set in X,

(xiii) $frwg$-closed set [10] if $cl(intA) \leq U$ whenever $A \leq U \in FRO(X, \tau)$,

(xiv) $f\pi g$-closed set [10] if $clA \leq U$ whenever $A \leq U$ where $U \in F\pi O(X)$,

(xv) fwg-closed set [10] if $cl(intA) \leq U$ whenever $A \leq U \in \tau$,

(xvi) $fg\gamma$-closed set [12] if $\gamma clA \leq U$ whenever $A \leq U \in \tau$,
(xvii) $fg\gamma^*$-closed set [13] if $\gamma clA \leq U$ whenever $A \leq U \in FSO(X, \tau)$,
(xviii) $fgpr$-closed set [10] if $pclA \leq U$ whenever $A \leq U \in FRO(X)$.

Remark 3.5. It is clear from definitions that
(i) fgs^*-closed set $\Rightarrow$ $fswg$-closed set $\Rightarrow$ fgp-closed set, $fgpr$-closed set, $fg\beta$-closed set, $fg\gamma$-closed set, $fg\gamma^*$-closed set, fwg-closed set, $frwg$-closed set. But the reverse implications are not true, in general, as it seen from the following examples.
(ii) fg-closed set, $f\pi g$-closed set, fpg-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set, $f\beta g$-closed set, fgs-closed set, fsg-closed set, fs^*g-closed set, fmg-closed set are independent concepts of $fswg$-closed set.

Example 3.6. fg-closed set, fgp-closed set, fgs-closed set, fsg-closed set, $fgpr$-closed set, $frwg$-closed set, fwg-closed set, $fg\gamma$-closed set, $fg\gamma^*$-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set, $f\pi g$-closed set $\not\Rightarrow$ $fswg$-closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Here $FSO(X, \tau) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$. Consider the fuzzy set B defined by $B(a) = B(b) = 0.5$. Here $B \in FSO(X, \tau)$ such that $B \leq B$. But $clintB = 1_X \setminus A \not\leq B \Rightarrow B$ is not an $fswg$-closed set in X. As 1_X is the only fuzzy open as well as fuzzy regular open set in X containing B, clearly B is an fg-closed set, fgp-closed set, $fg\alpha$-closed set, fgs-closed set, $frwg$-closed set, $fgpr$-closed set, $fg\gamma$-closed set, $f\pi g$-closed set, fwg-closed set in X. Now $B \in FSC(X, \tau)$ also and so B is fsg-closed set in X. Again as $(clintB) \bigwedge (intclB) = A \leq B, B \in F\gamma C(X, \tau)$ and so B is $fg\gamma^*$-closed set in X. Again $F\alpha O(X, \tau) = \tau$ and so $1_X \in F\alpha O(X, \tau)$ only containing B. Consequently, B is $f\alpha g$-closed set in X.

Example 3.7. $fswg$-closed set $\not\Rightarrow$ fg-closed set, $f\pi g$-closed set, fgs^*-closed set, fs^*g-closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Here $FSO(X, \tau) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$. Consider the fuzzy set B defined by $B(a) = 0.5, B(b) = 0.3$. As $clintB = 0_X$, B is clearly $fswg$-closed set in X. Now $A \in \tau$ (resp., $A \in FSO(X, \tau)$, $A \in F\pi O(X, \tau)$) and $B \leq A$. But $clB = 1_X \setminus A \not\leq A \Rightarrow B$ is not fg-closed set as well as fgs^*-closed set and $f\pi g$-closed set in X. Again B is fg-open set in X and so $B \leq B$. But $clB = 1_X \setminus A \not\leq B \Rightarrow B$ is not fs^*g-closed set in X.

Example 3.8. $fswg$-closed set $\not\Rightarrow$ fsg-closed set, fgs-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.3, B(b) = 0.5$. Then (X, τ) is an fts. Consider the fuzzy set C defined by $C(a) = 0.2, C(b) = 0.55$. As $clintC = 0_X$, C is clearly $fswg$-closed set in X. Now $FSO(X, \tau) = \{0_X, 1_X, U, V\}$ where $U \geq A$, $B \leq V \leq 1_X \setminus B$. Then

$C < A \in FSO(X,\tau)$ (resp., $C < A \in \tau$). But as $sclC = 1_X \not\leq A$, C is not fsg-closed set as well as fgs-closed set in X. Again as $C < A \in F\alpha O(X,\tau)$ (resp., $A \in \tau$) and $\alpha clC = 1_X \not\leq A \Rightarrow C$ is not $fg\alpha$-closed set as well as $f\alpha g$-closed set in X.

Example 3.9. $fswg$-closed set $\not\Rightarrow fpg$-closed set
Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.3, A(b) = 0.5$. Then (X,τ) is an fts. Consider the fuzzy set B defined by $B(a) = 0.5, B(b) = 0.6$. As $1_X \in FSO(X,\tau)$ only containing B, clearly B is $fswg$-closed set in X. Now $intclB = 1_X \geq B \Rightarrow B \in FPO(X,\tau)$ and $B \leq B$. But as $clintB = 1_X \setminus A \not\leq B \Rightarrow B \notin FPC(X,\tau) \Rightarrow pclB \not\leq B \Rightarrow B$ is not an fpg-closed set in X.

Example 3.10. $fswg$-closed set $\not\Rightarrow f\beta g$-closed set, fmg-closed set
Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A, B, C\}$ where $A(a) = 0.3, A(b) = 0.4, B(a) = B(b) = 0.4, C(a) = 0.6, C(b) = 0.5$. Then (X,τ) is an fts. Consider the fuzzy set D defined by $D(a) = 0.35, D(b) = 0.55$. Now $FSO(X,\tau) = \{0_X, 1_X, U, V\}$ where $A \leq U \leq 1_X \setminus C$, $C \leq V \leq 1_X \setminus B$. Then $D \leq V_1 \in FSO(X,\tau)$ where $V_1(a) = 0.6, V_1(b) = 0.55$. Then $clintD = 1_X \setminus C < V_1 \Rightarrow D$ is $fswg$-closed set in X. Now $clintclD = 1_X \setminus B \geq D \Rightarrow D \in F\beta O(X,\tau)$ and so $D \leq D$. But as $intclintD = B \not\leq D$, $D \notin F\beta C(X,\tau) \Rightarrow \beta clD \not\leq D \Rightarrow D$ is not an $f\beta g$-closed set in X. Next consider the fuzzy set E defined by $E(a) = 0.3, E(b) = 0.55$. Then $E < F \in FSO(X,\tau)$ where $F(a) = 0.6, F(b) = 0.55$. Then $clintE = 1_X \setminus C < F \Rightarrow E$ is $fswg$-closed set in X. Clearly E is fg-open set in X and so $E \leq E$. But as $clintE = 1_X \setminus C \not\leq E \Rightarrow E$ is not an fmg-closed set in X.

Now we recall the definitions of some spaces from [3, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18] in which the reverse implications in Remark 3.5 hold.

Definition 3.11. An fts (X,τ) is said to be
(i) $f\beta T_b$-space [9] if every $f\beta g$-closed set in X is fuzzy closed set in X,
(ii) fT_β-space [9] if every $fg\beta$-closed set in X is fuzzy closed set in X,
(iii) fT_α-space [3] if every $fg\alpha$-closed set in X is fuzzy closed set in X,
(iv) $f\alpha T_b$-space [3] if every $f\alpha g$-closed set in X is fuzzy closed set in X,
(v) fT_b-space [3] if every fgs-closed set in X is fuzzy closed set in X,
(vi) fT_{sg}-space [3] if every fsg-closed set in X is fuzzy closed set in X,
(vii) fT_γ-space [12] if every $fg\gamma$-closed set in X is fuzzy closed set in X,
(viii) fT_{γ^*}-space [13] if every $fg\gamma^*$-closed set in X is fuzzy closed set in X,
(ix) frT_g-space [17] if every $frwg$-closed set in X is fuzzy closed set in X,
(x) fmT_g-space [16] if every fmg-closed set in X is fuzzy closed set in X,
(xi) fT_p-space [3] if every fgp-closed set in X is fuzzy closed set in X,
(xii) fpT_b-space [3] if every fpg-closed set in X is fuzzy closed set in X,
(xiii) fT_{pr}-space [11] if every $fgpr$-closed set in X is fuzzy closed set in X,

(xiv) fT_w-space [15] if every fwg-closed set in X is fuzzy closed set in X,
(xv) fT_π-space [14] if every $f\pi g$-closed set in X is fuzzy closed set in X,
(xvi) fT_{s^*}-space [7] if every fs^*g-closed set in X is fuzzy closed set in X,
(xvii) fT_g-space [3] if every fg-closed set in X is fuzzy closed set in X,
(xviii) fsT_g-space [18] if every $fswg$-closed set in X is fuzzy closed set in X.

Note 3.12. (i) In fsT_g-space, $fswg$-closed set is fg-closed set, $f\pi g$-closed set, fs^*g-closed set, fgs^*-closed set, fgs-closed set, fsg-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set, $f\beta g$-closed set, fmg-closed set and fpg-closed set.
(ii) In fT_β-space (resp., frT_g-space, fT_{pr}-space, fT_γ-space, fT_{γ^*}-space, fT_w-space, fT_p-space, fT_g-space, fT_{s^*}-space, fT_b-space, fT_{sg}-space, $f\alpha T_b$-space, fT_α-space, $f\beta T_b$-space, fmT_g-space, fpT_b-space) $fg\beta$-closed set (resp., $frwg$-closed set, $fgpr$-closed set, $fg\gamma$-closed set, $fg\gamma^*$-closed set, fwg-closed set, fgp-closed set, fg-closed set, fs^*g-closed set, fgs-closed set, fsg-closed set, $f\alpha g$-closed set, $fg\alpha$-closed set, $f\beta g$-closed set, fmg-closed set, fpg-closed set) is $fswg$-closed set.

4. $fswg$-CLOSED FUNCTION : MUTUAL RELATIONSHIPS

In this section we first recall the definitions of fuzzy open function, $fswg$-open function [18] and $fswg$-closed function [18] and then establish the mutual relationships of $fswg$-closed function with the functions defined in [3, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17].

Definition 4.1 [32]. A function $f : X \to Y$ is called fuzzy open if $f(U)$ is fuzzy open set in Y for every fuzzy open set U in X.

Definition 4.2 [18]. A function $h : X \to Y$ is called $fswg$-open (resp., $fswg$-closed) function if $h(U)$ is $fswg$-open (resp., $fswg$-closed) set in Y for every fuzzy open (resp., fuzzy closed) set U in X.

Now to establish the mutual relationships of $fswg$-closed function with the functions defined in [3, 6, 7, 8, 11, 12, 13, 14, 15, 16, 17], we have to recall the following definitions first.

Definition 4.3. Let $h : (X, \tau_1) \to (Y, \tau_2)$ be a function. Then h is called an
(i) fg-closed function [3] if $h(A)$ is fg-closed set in Y for every $A \in \tau_1^c$,
(ii) $fg\beta$-closed function [8] if $h(A)$ is $fg\beta$-closed set in Y for every $A \in \tau_1^c$,
(iii) $f\beta g$-closed function [8] if $h(A)$ is $f\beta g$-closed set in Y for every $A \in \tau_1^c$,
(iv) $fg\alpha$-closed function [3] if $h(A)$ is $fg\alpha$-closed set in Y for every $A \in \tau_1^c$,
(v) $f\alpha g$-closed function [3] if $h(A)$ is $f\alpha g$-closed set in Y for every $A \in \tau_1^c$,
(vi) fgp-closed function [3] if $h(A)$ is fgp-closed set in Y for every $A \in \tau_1^c$,
(vii) fpg-closed function [3] if $h(A)$ is fpg-closed set in Y for every $A \in \tau_1^c$,
(viii) fgs-closed function [3] if $h(A)$ is fgs-closed set in Y for every $A \in \tau_1^c$,
(ix) fsg-closed function [3] if $h(A)$ is fsg-closed set in Y for every $A \in \tau_1^c$,

(x) fgs^*-closed function [6] if $h(A)$ is fgs^*-closed set in Y for every $A \in \tau_1^c$,
(xi) fs^*g-closed function [7] if $h(A)$ is fs^*g-closed set in Y for every $A \in \tau_1^c$,
(xii) $fg\gamma$-closed function [12] if $h(A)$ is $fg\gamma$-closed set in Y for every $A \in \tau_1^c$,
(xiii) $fg\gamma^*$-closed function [13] if $h(A)$ is $fg\gamma^*$-closed set in Y for every $A \in \tau_1^c$,
(xiv) fmg-closed function [16] if $h(A)$ is fmg-closed set in Y for every $A \in \tau_1^c$,
(xv) $frwg$-closed function [17] if $h(A)$ is $frwg$-closed set in Y for every $A \in \tau_1^c$,
(xvi) $f\pi g$-closed function [14] if $h(A)$ is $f\pi g$-closed set in Y for every $A \in \tau_1^c$,
(xvii) fwg-closed function [15] if $h(A)$ is fwg-closed set in Y for every $A \in \tau_1^c$,
(xviii) $fgpr$-closed function [11] if $h(A)$ is $fgpr$-closed set in Y for every $A \in \tau_1^c$.

Remark 4.4. (i) fgs^*-closed function is $fswg$-closed function. Again $fswg$-closed function implies $fgpr$-closed function, $fg\beta$-closed function, $fg\gamma$-closed function, $fg\gamma^*$-closed function, fwg-closed function, $frwg$-closed function, fgp-closed function. But the converses are not necessarily true, in general, follow from the following examples.

(ii) fg-closed function, $f\pi g$-closed function, fs^*g-closed function, fgs-closed function, fsg-closed function, $fg\alpha$-closed function, $f\alpha g$-closed function, $f\beta g$-closed function, fmg-closed function, fpg-closed function are independent concepts of $fswg$-closed function follow from the following examples.

Example 4.5. $fswg$-closed function $\not\Rightarrow$ fgs^*-closed function

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = A(b) = 0.5, B(a) = 0.5, B(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $FSO(X, \tau_2) = \{0_X, 1_X, U\}$ where $U \geq B$. Now $A \in \tau_1^c, i(A) = A$. Then $cl_{\tau_2} int_{\tau_2} A = 0_X \Rightarrow A$ is $fswg$-closed set in $(X, \tau_2) \Rightarrow i$ is an $fswg$-closed function. Again $A < B \in FSO(X, \tau_2)$. But $cl_{\tau_2} A = 1_X \not\leq B \Rightarrow A$ is not fgs^*-closed set in $(X, \tau_2) \Rightarrow i$ is not an fgs^*-closed function.

Example 4.6. fg-closed function, fgp-closed function, fgs-closed function, fsg-closed function, $fgpr$-closed function, $frwg$-closed function, fwg-closed function, $fg\gamma$-closed function, $fg\gamma^*$-closed function, $fg\alpha$-closed function, $f\alpha g$-closed function, $f\pi g$-closed function $\not\Rightarrow$ $fswg$-closed function

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = A(b) = 0.5, B(a) = 0.5, B(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $FSO(X, \tau_2) = \{0_X, 1_X, U\}$ where $B \leq U \leq 1_X \setminus B$. Now $A \in \tau_1^c, i(A) = A \in FSO(X, \tau_2)$. But $cl_{\tau_2} int_{\tau_2} A = 1_X \setminus B \not\leq A \Rightarrow A$ is not $fswg$-closed set in $(X, \tau_2) \Rightarrow i$ is not an $fswg$-closed function. Now $F\alpha O(X, \tau_2) = \tau_2$. Here 1_X is the only fuzzy open (resp., fuzzy regular open, fuzzy π-open, fuzzy α-open) set in (X, τ_2) containing A and so A is fg-closed set, fwg-closed set, $fgpr$-closed set, $frwg$-closed set, $f\pi g$-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set, $fg\gamma$-closed set, fgp-closed set, fgs-closed set in $(X, \tau_2) \Rightarrow i$ is an

fg-closed function, fwg-closed function, $fgpr$-closed function, $frwg$-closed function, $f\pi g$-closed function, $fg\alpha$-closed function, $f\alpha g$-closed function, $fg\gamma$-closed function, fgp-closed function, fgs-closed function. Since $(cl_{\tau_2}int_{\tau_2}A)\bigwedge(int_{\tau_2}cl_{\tau_2}A) = B \leq A, A \in F\gamma C(X,\tau_2) \Rightarrow A$ is $fg\gamma^*$-closed set in $(X,\tau_2) \Rightarrow i$ is $fg\gamma^*$-closed function. Again $int_{\tau_2}cl_{\tau_2}A = B \leq A \Rightarrow A \in FSC(X,\tau_2) \Rightarrow i$ is fsg-closed function.

Example 4.7. $fswg$-closed function $\not\Rightarrow$ fg-closed function, $f\pi g$-closed function, fs^*g-closed function

Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.7$, $B(a) = 0.5, B(b) = 0.4$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i : (X,\tau_1) \to (X,\tau_2)$. Now $1_X \setminus A \in \tau_1^c$, $i(1_X \setminus A) = 1_X \setminus A$. So $cl_{\tau_2}int_{\tau_2}(1_X \setminus A) = 0_X \Rightarrow 1_X \setminus A$ is $fswg$-closed set in $(X,\tau_2) \Rightarrow i$ is an $fswg$-closed function. Now $1_X \setminus A < B \in \tau_2$ (resp., $B \in F\pi O(X,\tau_2)$ and also B is fg-open set in (X,τ_2)). But $cl_{\tau_2}(1_X \setminus A) = 1_X \setminus B \not\leq B \Rightarrow 1_X \setminus A$ is not fg-closed set (resp., $f\pi g$-closed set, fs^*g-closed set) in $(X,\tau_2) \Rightarrow i$ is not an fg-closed function, $f\pi g$-closed function, fs^*g-closed function.

Example 4.8. $fswg$-closed function $\not\Rightarrow$ fgs-closed function, fsg-closed function, $fg\alpha$-closed function, $f\alpha g$-closed function

Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, C\}$, $\tau_2 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6$, $B(a) = 0.3, B(b) = 0.5, C(a) = 0.8, C(b) = 0.45$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i : (X,\tau_1) \to (X,\tau_2)$. Here $FSO(X,\tau_2) = \{0_X, 1_X, U, V\}$ where $U \geq A$, $B \leq V \leq 1_X \setminus B$ and $FSC(X,\tau_2) = \{0_X, 1_X, 1_X \setminus U, 1_X \setminus V\}$ where $1_X \setminus U \leq 1_X \setminus A$, $B \leq 1_X \setminus V \leq 1_X \setminus B$. Now $1_X \setminus C \in \tau_1^c, i(1_X \setminus C) = 1_X \setminus C$. Since $cl_{\tau_2}int_{\tau_2}(1_X \setminus C) = 0_X$, clearly $1_X \setminus A$ is $fswg$-closed set in $(X,\tau_2) \Rightarrow i$ is $fswg$-closed function. Now $1_X \setminus C < A \in \tau_2$ (also $A \in FSO(X,\tau_2)$), but $scl_{\tau_2}(1_X \setminus C) = 1_X \not\leq A \Rightarrow 1_X \setminus C$ is not fgs-closed set as well as fsg-closed set in $(X,\tau_2) \Rightarrow i$ is not an fgs-closed function as well as fsg-closed function. Also $A \in \tau_2$ (also $A \in F\alpha O(X,\tau_2)$) such that $1_X \setminus C < A$, but $\alpha cl_{\tau_2}(1_X \setminus C) = 1_X \not\leq A \Rightarrow 1_X \setminus C$ is not an $fg\alpha$-closed set as well as $f\alpha g$-closed set in $(X,\tau_2) \Rightarrow i$ is not an $fg\alpha$-closed function as well as $f\alpha g$-closed function.

Example 4.9. $fswg$-closed function $\not\Rightarrow$ fpg-closed function

Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, B\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.3, A(b) = 0.5, B(a) = 0.5, B(b) = 0.4$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i : (X,\tau_1) \to (X,\tau_2)$. Now $1_X \setminus B \in \tau_1^c, i(1_X \setminus B) = 1_X \setminus B$. Now $FSO(X,\tau_2) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$. Then $1_X \setminus B < 1_X \in FSO(X,\tau_2)$ only and so $1_X \setminus B$ is $fswg$-closed set in $(X,\tau_2) \Rightarrow i$ is an $fswg$-closed function. Clearly $1_X \setminus B \in FPO(X,\tau_2)$ and so $1_X \setminus B \leq 1_X \setminus B$, but as $1_X \setminus B \notin FPC(X,\tau_2)$, $pcl_{\tau_2}(1_X \setminus B) \not\leq 1_X \setminus B \Rightarrow 1_X \setminus B$ is not an fpg-closed set in $(X,\tau_2) \Rightarrow i$ is not an fpg-closed function.

Example 4.10. $fswg$-closed function $\not\Rightarrow$ $f\beta g$-closed function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, D\}$, $\tau_2 = \{0_X, 1_X, A, B, C\}$ where $A(a) = 0.3, A(b) = 0.4$, $B(a) = B(b) = 0.4, C(a) = 0.6, C(b) = 0.5, D(a) = 0.65, D(b) = 0.45$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus D \in \tau_1^c$, $i(1_X \setminus D) = 1_X \setminus D$. Now $FSO(X, \tau_2) = \{0_X, 1_X, U, V\}$ where $A \leq U \leq 1_X \setminus C$, $C \leq V \leq 1_X \setminus B$. Now $1_X \setminus D < V_1 \in FSO(X, \tau_2)$ where $V_1(a) = 0.6, V_1(b) = 0.55$. Then $cl_{\tau_2}int_{\tau_2}(1_x \setminus D) = 1_X \setminus C < V_1 \Rightarrow 1_X \setminus D$ is $fswg$-closed set in $(X, \tau_2) \Rightarrow i$ is an $fswg$-closed function. Again $1_X \setminus D \in F\beta O(X, \tau_2)$ and so $1_X \setminus D \leq 1_X \setminus D \in F\beta O(X, \tau_2)$. But as $1_X \setminus D \notin F\beta C(X, \tau_2)$, $\beta cl_{\tau_2}(1_X \setminus D) \not\leq 1_X \setminus D \Rightarrow 1_X \setminus D$ is not an $f\beta g$-closed set in $(X, \tau_2) \Rightarrow i$ is not an $f\beta g$-closed function.

Example 4.11. $fswg$-closed function $\not\Rightarrow$ fmg-closed function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, E\}$, $\tau_2 = \{0_X, 1_X, A, B, C\}$ where $A(a) = 0.3, A(b) = 0.4$, $B(a) = B(b) = 0.4, C(a) = 0.6, C(b) = 0.5, E(a) = 0.7, E(b) = 0.45$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus E \in \tau_1^c$, $i(1_X \setminus E) = 1_X \setminus E < F \in FSO(X, \tau_2)$ where $F(a) = 0.6, F(b) = 0.55$. Then $cl_{\tau_2}int_{\tau_2}(1_X \setminus E) = 1_X \setminus C < F \Rightarrow 1_X \setminus E$ is $fswg$-closed set in $(X, \tau_2) \Rightarrow i$ is an $fswg$-closed function. Again $1_X \setminus E$ is fg-open set in (X, τ_2) and so $1_X \setminus E \leq 1_X \setminus E$, but $cl_{\tau_2}int_{\tau_2}(1_X \setminus E) = 1_X \setminus C \not\leq 1_X \setminus E \Rightarrow 1_X \setminus E$ is not an fmg-closed set in $(X, \tau_2) \Rightarrow i$ is not an fmg-closed function.

Remark 4.12. (i) Let $h : X \to Y$ be an $fswg$-closed function where Y is an fsT_g-space. Then h is an fg-closed function, $f\pi g$-closed function, fgs^*-closed function, fs^*g-closed function, fgs-closed function, fsg-closed function, $fg\alpha$-closed function, $f\alpha g$-closed function, $f\beta g$-closed function, fmg-closed function, fpg-closed function.
(ii) Let $h : X \to Y$ be a function where Y is an $f\beta T_b$-space (resp., fT_β-space, fT_α-space, $f\alpha T_b$-space, fT_b-space, fT_{sg}-space, fT_γ-space, fT_{γ^*}-space, frT_g-space, fT_π-space, fT_g-space, fT_p-space, fpT_b-space, fmT_g-space, fT_w-space, fT_{s^*}-space). Then if h is an $f\beta g$-closed function (resp., $fg\beta$-closed function, $fg\alpha$-closed function, $f\alpha g$-closed function, fgs-closed function, fsg-closed function, $fg\gamma$-closed function, $fg\gamma^*$-closed function, $frwg$-closed function, $f\pi g$-closed function, fg-closed function, fgp-closed function, fpg-closed function, fmg-closed function, fwg-closed function, fs^*g-closed function), h is $fswg$-closed function.

5. $fswg$-CONTINUOUS FUNCTIONS : MUTUAL RELATIONSHIPS

In this section four different types of functions are recalled from [18]. Afterwards, the mutual relationships of $fswg$-continuous function with the functions defined in

[3, 6, 7, 8, 11, 12, 13, 14, 15, 16, 17] are established.

Definition 5.1 [18]. A function $h : X \to Y$ is said to be $fswg$-continuous function if $h^{-1}(V)$ is $fswg$-closed set in X for every fuzzy closed set V in Y.

Definition 5.2 [18]. A function $h : X \to Y$ is called $fswg$-irresolute function if $h^{-1}(U)$ is an $fswg$-open set in X for every $fswg$-open set U in Y.

Definition 5.3 [18]. A function $h : X \to Y$ is called strongly $fswg$-continuous function if $h^{-1}(V)$ is fuzzy closed set in X for every $fswg$-closed set V in Y.

Definition 5.4 [18]. A function $h : X \to Y$ is called $fswg$-strongly continuous function if $h^{-1}(U)$ is $fswg$-closed set in X for every $U \in FSC(Y)$.

To establish the mutual relationship of $fswg$-continuous function with the functions defined in [3, 6, 7, 8, 11, 12, 13, 14, 15, 16, 17], we first recall the definitions of the functions defined in [3, 6, 7, 8, 11, 12, 13, 14, 15, 16, 17].

Definition 5.5. Let $h : (X, \tau_1) \to (Y, \tau_2)$ be a function. Then h is called

(i) fg-continuous function [3] if $h^{-1}(V)$ is fg-closed set in X for every $V \in \tau_2^c$,

(ii) $fg\beta$-continuous function [8] if $h^{-1}(V)$ is $fg\beta$-closed set in X for every $V \in \tau_2^c$,

(iii) $f\beta g$-continuous function [8] if $h^{-1}(V)$ is $f\beta g$-closed set in X for every $V \in \tau_2^c$,

(iv) fgp-continuous function [3] if $h^{-1}(V)$ is fgp-closed set in X for every $V \in \tau_2^c$,

(v) fpg-continuous function [3] if $h^{-1}(V)$ is fpg-closed set in X for every $V \in \tau_2^c$,

(vi) $fg\alpha$-continuous function [3] if $h^{-1}(V)$ is $fg\alpha$-closed set in X for every $V \in \tau_2^c$,

(vii) $f\alpha g$-continuous function [3] if $h^{-1}(V)$ is $f\alpha g$-closed set in X for every $V \in \tau_2^c$,

(viii) fgs-continuous function [3] if if $h^{-1}(V)$ is fgs-closed set in X for every $V \in \tau_2^c$,

(ix) fsg-continuous function [3] if $h^{-1}(V)$ is fsg-closed set in X for every $V \in \tau_2^c$,

(x) fgs^*-continuous function [6] if $h^{-1}(V)$ is fgs^*-closed set in X for every $V \in \tau_2^c$,

(xi) fs^*g-continuous function [7] if $h^{-1}(V)$ is fs^*g-closed set in X for every $V \in \tau_2^c$,

(xii) $fg\gamma$-continuous function [12] if $h^{-1}(V)$ is $fg\gamma$-closed set in X for every $V \in \tau_2^c$,

(xiii) $fg\gamma^*$-continuous function [13] if $h^{-1}(V)$ is $fg\gamma^*$-closed set in X for every $V \in \tau_2^c$,

(xiv) $frwg$-continuous function [17] if $h^{-1}(V)$ is $frwg$-closed set in X for every $V \in \tau_2^c$,

(xv) fmg-continuous function [16] if $h^{-1}(V)$ is fmg-closed set in X for every $V \in \tau_2^c$,

(xvi) $fgpr$-continuous function [11] if $h^{-1}(V)$ is $fgpr$-closed set in X for every $V \in \tau_2^c$,

(xvii) fwg-continuous function [15] if $h^{-1}(V)$ is fwg-closed set in X for every $V \in \tau_2^c$,

(xviii) $f\pi g$-continuous function [14] if $h^{-1}(V)$ is $f\pi g$-closed set in X for every $V \in \tau_2^c$.

Remark 5.6. It is clear from definitions that

(i) fgs^*-continuity $\Rightarrow$ $fswg$-continuity$\Rightarrow$ $fgpr$-continuity, $fg\beta$-continuity, $fg\gamma$-continuity, $fg\gamma^*$-continuity, fwg-continuity, $frwg$-continuity, fgp-continuity. But the reverse implications are not necessarily true, follow from the following examples.

(ii) $fswg$-continuity is independent concept of fg-continuity, $f\pi g$-continuity, $fg\alpha$-continuity, $f\alpha g$-continuity, $f\beta g$-continuity, fgs-continuity, fsg-continuity, fs^*g-continuity, fmg-continuity, fpg-continuity, follow from the following examples.

Example 5.7. fg-continuity, fgs-continuity, fsg-continuity, fgp-continuity, $fgpr$-continuity, $frwg$-continuity, fwg-continuity, $fg\gamma$-continuity, $fg\gamma^*$-continuity, $fg\alpha$-continuity, $f\alpha g$-continuity, $f\pi g$-continuity $\not\Rightarrow$ $fswg$-continuity

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, B\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = A(b) = 0.5, B(a) = 0.5, B(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Here $A \in \tau_2^c$, $i^{-1}(A) = A \leq A \in FSO(X, \tau_1)$. But $cl_{\tau_1} int_{\tau_1} A = 1_X \setminus B \not\leq A \Rightarrow A$ is not $fswg$-closed set in $(X, \tau_1) \Rightarrow i$ is not an $fswg$-continuous function. Since 1_X is the only fuzzy open (resp., fuzzy regular open, fuzzy α-open, fuzzy π-open) set containing A, it is clear that A is fg-closed set, fgp-closed set, $fgpr$-closed set, fgs-closed set, $fg\gamma$-closed set, fwg-closed set, $frwg$-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set, $f\pi g$-closed set in $(X, \tau_1) \Rightarrow i$ is fg-continuous function, fgp-continuous function, $fgpr$-continuous function, fgs-continuous function, $fg\gamma$-continuous function, fwg-continuous function, $frwg$-continuous function, $fg\alpha$-continuous function, $f\alpha g$-continuous function, $f\pi g$-continuous function. Also $A \in FSC(X, \tau_1)$ and $A \in F\gamma C(X, \tau_1)$ and so A is fsg-closed set as well as $fg\gamma^*$-closed set in $(X, \tau_1) \Rightarrow i$ is fsg-continuous function, $fg\gamma^*$-continuous function.

Example 5.8. $fswg$-continuity $\not\Rightarrow$ $f\pi g$-continuity, fg-continuity, fgs^*-continuity, fs^*g-continuity

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, B\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.7, B(a) = 0.5, B(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus A \in \tau_2^c$, $i^{-1}(1_X \setminus A) = 1_X \setminus A$. As $cl_{\tau_1} int_{\tau_1}(1_X \setminus A) = 0_X, 1_X \setminus A$ is $fswg$-closed set in $(X, \tau_1) \Rightarrow i$ is $fswg$-continuous function. Now $1_X \setminus A < B \in \tau_1$ (resp., $B \in FSO(X, \tau_1)$, $B \in F\pi O(X, \tau_1)$ and B is fg-open set in (X, τ_1)). Then $cl_{\tau_1}(1_X \setminus A) = 1_X \setminus B \not\leq B \Rightarrow 1_X \setminus A$ is not fg-closed set, fgs^*-closed set, $f\pi g$-closed set, fs^*g-closed set in $(X, \tau_1) \Rightarrow i$ is not an fg-continuous function, fgs^*-continuous function, $f\pi g$-continuous function, fs^*g-continuous function.

Example 5.9. $fswg$-continuity $\not\Rightarrow$ fgs-continuity, fsg-continuity, $fg\alpha$-continuity, $f\alpha g$-continuity

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X, C\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.3, B(b) = 0.5, C(a) = 0.8, C(b) = 0.45$. Then (X, τ_1) and $(X.\tau_2)$ are fts's. Consider the identity function $i : (X.\tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau_2^c$, $i^{-1}(1_X \setminus C) = 1_X \setminus C$. Here $cl_{\tau_1} int_{\tau_1}(1_X \setminus C) = 0_X \Rightarrow 1_X \setminus C$ is $fswg$-closed set in $(X, \tau_1) \Rightarrow i$

is $fswg$-continuous function. Now $FSO(X,\tau_1) = \{0_X, 1_X, U, V\}$ where $U \geq A$ and $B \leq V \leq 1_X \setminus B$ and $FSC(X,\tau_1) = \{0_X, 1_X, 1_X \setminus U, 1_X \setminus V\}$ where $1_X \setminus U \leq 1_X \setminus A$ and $B \leq 1_X \setminus V \leq 1_X \setminus B$. Now $1_X \setminus C < A \in \tau_1$ (also, $A \in FSO(X,\tau_1)$, $A \in F\alpha O(X,\tau_1)$). But $scl_{\tau_1}(1_X \setminus C) = \alpha cl_{\tau_1}(1_X \setminus C) = 1_X \not\leq A \Rightarrow 1_X \setminus C$ is not fgs-closed set, fsg-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set in $(X,\tau_1) \Rightarrow i$ is not an fgs-continuous function, fsg-continuous function, $fg\alpha$-continuous function, $f\alpha g$-continuous function.

Example 5.10. $fswg$-continuity $\not\Rightarrow$ fpg-continuity
Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.3, A(b) = 0.5, B(a) = 0.5, B(b) = 0.4$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i : (X,\tau_1) \to (X,\tau_2)$. Now $1_X \setminus B \in \tau_2^c$, $i^{-1}(1_X \setminus B) = 1_X \setminus B < 1_X \in FSO(X,\tau_1)$ only (as $FSO(X,\tau_1) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$) and so $1_X \setminus B$ is $fswg$-closed set in $(X,\tau_1) \Rightarrow i$ is $fswg$-continuous function. Now $1_X \setminus B \leq 1_X \setminus B \in FPO(X,\tau_1)$, but as $1_X \setminus B \notin FPC(X,\tau_1)$, $pcl_{\tau_1}(1_X \setminus B) \not\leq 1_X \setminus B \Rightarrow 1_X \setminus B$ is not an fpg-closed set in $(X,\tau_1) \Rightarrow i$ is not an fpg-continuous function.

Example 5.11. $fswg$-continuity $\not\Rightarrow$ $f\beta g$-continuity
Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A, B, C\}$, $\tau_2 = \{0_X, 1_X, D\}$ where $A(a) = 0.3, A(b) = 0.4, B(a) = B(b) = 0.4, C(a) = 0.6, C(b) = 0.5, D(a) = 0.65, D(b) = 0.45$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i : (X,\tau_1) \to (X,\tau_2)$. Now $1_X \setminus D \in \tau_2^c$, $i^{-1}(1_X \setminus D) = 1_X \setminus D$. Here $FSO(X,\tau_1) = \{0_X, 1_X, U, V\}$ where $A \leq U \leq 1_X \setminus C, C \leq V \leq 1_X \setminus B$. Now $1_X \setminus D < V_1 \in FSO(X,\tau_1)$ where $V_1(a) = 0.6, V_1(b) = 0.55$. Now $cl_{\tau_1} int_{\tau_1}(1_X \setminus D) = 1_X \setminus C < V_1 \Rightarrow 1_X \setminus D$ is $fswg$-closed set in $(X,\tau_1) \Rightarrow i$ is $fswg$-continuous function. Again $1_X \setminus D \in F\beta O(X,\tau_1)$ and $1_X \setminus D \leq 1_X \setminus D$. But as $1_X \setminus D \notin F\beta C(X,\tau_1)$, $\beta cl_{\tau_1}(1_X \setminus D) \not\leq 1_X \setminus D \Rightarrow 1_X \setminus D$ is not an $f\beta g$-closed set in $(X,\tau_1) \Rightarrow i$ is not an $f\beta g$-continuous function.

Example 5.12. $fswg$-continuity $\not\Rightarrow$ fmg-continuity
Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A, B, C\}$, $\tau_2 = \{0_X, 1_X, E\}$ where $A(a) = 0.3, A(b) = 0.4, B(a) = B(b) = 0.4, C(a) = 0.6, C(b) = 0.5, D(a) = 0.7, D(b) = 0.45$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i : (X,\tau_1) \to (X,\tau_2)$. Here $FSO(X,\tau_1) = \{0_X, 1_X, U, V\}$ where $A \leq U \leq 1_X \setminus C, C \leq V \leq 1_X \setminus B$. Now $1_X \setminus E \in \tau_2^c, i^{-1}(1_X \setminus E) = 1_X \setminus E < F_1 \in FSO(X,\tau_1)$ where $F_1(a) = 0.6, F_1(b) = 0.55$. Now $cl_{\tau_1} int_{\tau_1}(1_X \setminus E) = 1_X \setminus C < F_1 \Rightarrow 1_X \setminus E$ is an $fswg$-closed set in $(X,\tau_1) \Rightarrow i$ is an $fswg$-continuous function. Again $1_X \setminus E$ is fg-open set in (X,τ_1) and so $1_X \setminus E \leq 1_X \setminus E$, but $cl_{\tau_1} int_{\tau_1}(1_X \setminus E) = 1_X \setminus C \not\leq 1_X \setminus E \Rightarrow 1_X \setminus E$ is not an fmg-closed set in $(X,\tau_1) \Rightarrow i$ is not an fmg-continuous function.

Remark 5.13. (i) Let $h : X \to Y$ be a function where X is an fsT_g-space. If h is $fswg$-continuous function, then h is an fg-continuous function, $f\pi g$-continuous function, fgs^*-continuous function, fs^*g-continuous function, fgs-continuous

function, fsg-continuous function, $fg\alpha$-continuous function, $f\alpha g$-continuous function, $f\beta g$-continuous function, fmg-continuous function, fpg-continuous function.
(ii) Let $h : X \to Y$ be a function where X is an fT_β-space (resp., frT_g-space, fT_{pr}-space, fT_γ-space, fT_{γ^*}-space, fT_w-space, fT_p-space, fT_g-space, fT_{s^*}-space, fT_π-space, fT_{sg}-space, $f\alpha T_b$-space, fT_α-space, $f\beta T_b$-space, fmT_g-space, fpT_b-space). If h is $fg\beta$-continuous function (resp., $frwg$-continuous function, $fgpr$-continuous function, $fg\gamma$-continuous function, $fg\gamma^*$-continuous function, fwg-continuous function, fgp-continuous function, fg-continuous function, fs^*g-continuous function, $f\pi g$-continuous function, fsg-continuous function, $f\alpha g$-continuous function, $fg\alpha$-continuous function, $f\beta g$-continuous function, fmg-continuous function, fpg-continuous function), then h is an $fswg$-continuous function.

6. $fswg$-COMPACT SPACE

In this section we first introduce a new form of fuzzy compactness. Afterwards, the applications of functions defined in [18] on this space are shown here.
Let us now recall the following definitions from [21, 20, 22, 28, 23, 27, 25] for ready references.
Definition 6.1. Let (X, τ) be an fts and $A \in I^X$. A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $\bigcup \mathcal{U} \geq A$ [25]. If each member of $\mathcal{U}$ is fuzzy open (resp., fuzzy semiopen, fuzzy regular open, $fswg$-open) in X, then $\mathcal{U}$ is called a fuzzy open [25] (resp., fuzzy semiopen [20], fuzzy regular open [23], $fswg$-open) cover of A. If, in particular, $A = 1_X$, we get the definition of fuzzy cover of X as $\bigcup \mathcal{U} = 1_X$ [21].
Definition 6.2. Let (X, τ) be an fts and $A \in I^X$. Then a fuzzy cover $\mathcal{U}$ of A (resp., of X) is said to have a finite subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$ [25]. If, in particular $A = 1_X$, we get $\bigcup \mathcal{U}_0 = 1_X$ [21].
Definition 6.3. Let (X, τ) be an fts and $A \in I^X$. Then A is called fuzzy compact [21] (resp., fuzzy almost compact [22], fuzzy semicompact [20], fuzzy nearly compact [28]) set if every fuzzy open (resp., fuzzy open, fuzzy semiopen, fuzzy regular open) cover $\mathcal{U}$ of A has a finite subcollection $\mathcal{U}_0$ such that $\bigcup \mathcal{U}_0 \geq A$ (resp., $\bigcup_{U \in \mathcal{U}_0} clU \geq A$, $\bigcup \mathcal{U}_0 \geq A$, $\bigcup \mathcal{U}_0 \geq A$). If, in particular, $A = 1_X$, we get the definition of fuzzy compact [21] (resp., fuzzy almost compact [22], fuzzy semicompact [27], fuzzy nearly compact [23]) space as $\bigcup \mathcal{U}_0 = 1_X$ (resp., $\bigcup_{U \in \mathcal{U}_0} clU = 1_X$, $\bigcup \mathcal{U}_0 = 1_X$, $\bigcup \mathcal{U}_0 = 1_X$).
Let us now introduce the following concept.
Definition 6.4. Let (X, τ) be an fts and $A \in I^X$. Then A is called $fswg$-compact if every fuzzy cover $\mathcal{U}$ of A by $fswg$-open sets of X has a finite subcover. If, in particular,

$A = 1_X$, we get the definition of $fswg$-compact space X.

Theorem 6.5. Every $fswg$-closed set in an $fswg$-compact space X is $fswg$-compact.

Proof. Let $A(\in I^X)$ be an $fswg$-closed set in an $fswg$-compact space X. Let $\mathcal{U}$ be a fuzzy cover of A by $fswg$-open sets of X. Then $\mathcal{V} = \mathcal{U} \bigcup (1_X \setminus A)$ is a fuzzy cover of X by $fswg$-open sets of X. As X is $fswg$-compact space, $\mathcal{V}$ has a finite subcollection $\mathcal{V}_0$ which also covers X. If $\mathcal{V}_0$ contains $1_X \setminus A$, we omit it and get a finite subcover of A. Hence A is $fswg$-compact set.

Theorem 6.6. Let $h : X \to Y$ be an $fswg$-continuous function from X onto an fts Y and $A(\in I^X)$ be an $fswg$-compact set in X. Then $h(A)$ is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Proof. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of $h(A)$ by fuzzy open (resp., fuzzy open, fuzzy regular open) sets of Y. Then $h(A) \leq \bigcup\limits_{\alpha \in \Lambda} U_\alpha \Rightarrow A \leq h^{-1}(\bigcup\limits_{\alpha \in \Lambda} U_\alpha) = \bigcup\limits_{\alpha \in \Lambda} h^{-1}(U_\alpha)$. Then $\mathcal{V} = \{h^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a fuzzy cover of A by $fswg$-open sets of X as h is an $fswg$-continuous function. As A is $fswg$-compact set in X, there exists a finite subcollection Λ_0 of Λ such that $A \leq \bigcup\limits_{\alpha \in \Lambda_0} h^{-1}(U_\alpha) \Rightarrow h(A) \leq h(\bigcup\limits_{\alpha \in \Lambda_0} h^{-1}(U_\alpha) \leq \bigcup\limits_{\alpha \in \Lambda_0} U_\alpha \Rightarrow h(A)$ is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Since fuzzy open set is $fswg$-open, we can state the following theorems easily the proofs of which are same as that of Theorem 6.6.

Theorem 6.7. Let $h : X \to Y$ be an $fswg$-irresolute function from X onto an fts Y and $A(\in I^X)$ be an $fswg$-compact set in X. Then $h(A)$ is $fswg$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) set in Y.

Theorem 6.8. Let $h : X \to Y$ be an $fswg$-continuous function from an $fswg$-compact space X onto an fts Y . Then Y is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.9. Let $h : X \to Y$ be an $fswg$-irresolute function from an $fswg$-compact space X onto an fts Y . Then Y is $fswg$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.10. Let $h : X \to Y$ be an $fswg$-continuous function from a fuzzy compact, fsT_g-space X onto an fts Y . Then Y is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.11. Let $h : X \to Y$ be an $fswg$-irresolute function from a fuzzy compact, fsT_g-space X onto an fts Y . Then Y is $fswg$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.12. Let $h : X \to Y$ be a strongly $fswg$-continuous function from X onto an fts Y and $A(\in I^X)$ be a fuzzy compact set in X. Then $h(A)$ is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact, $fswg$-compact) set in Y.

Theorem 6.13. Let $h : X \to Y$ be a strongly $fswg$-continuous function from a fuzzy compact space X onto an fts Y. Then Y is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact, $fswg$-compact) set in Y.

Theorem 6.14. Let $h : X \to Y$ be an $fswg$-strongly continuous function from X onto an fts Y and $A(\in I^X)$ be an $fswg$-compact set in X. Then $h(A)$ is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact, fuzzy semicompact) set in Y.

Theorem 6.15. Let $h : X \to Y$ be an $fswg$-strongly continuous function from an $fswg$-compact space X onto an fts Y. Then Y is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact, fuzzy semicompact) space.

Theorem 6.16. Let $h : X \to Y$ be an $fswg$-strongly continuous function from a fuzzy compact, fsT_g-space X onto an fts Y. Then Y is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact, fuzzy semicompact) space.

7. $fswg$-T_2-SPACE

In this section a new type of T_2 axiom is introduced and studied. The applications of the functions defined in [18] on this space are established.

We recall the following two definitions from [26, 28] for ready references.

Definition 7.1 [26]. An fts (X, τ) is called fuzzy T_2-space if for any two distinct fuzzy points x_α and y_β; when $x \neq y$, there exist fuzzy open sets U_1, U_2, V_1, V_2 such that $x_\alpha \in U_1, y_\beta q V_1, U_1 \not{q} V_1$ and $x_\alpha q U_2, y_\beta \in V_2, U_2 \not{q} V_2$; when $x = y$ and $\alpha < \beta$ (say), there exist fuzzy open sets U and V in X such that $x_\alpha \in U, y_\beta q V$ and $U \not{q} V$.

Theorem 7.2 [28]. An fts (X, τ) is fuzzy T_2-space if and only if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist fuzzy open sets U, V in X such that $x_\alpha q U$, $y_\beta q V$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has a fuzzy open nbd U and y_β has a fuzzy open q-nbd V such that $U \not{q} V$.

Now we introduce the following concept.

Definition 7.3. An fts (X, τ) is called $fswg$-T_2-space if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist $fswg$-open sets U, V in X such that $x_\alpha q U$, $y_\beta q V$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has an $fswg$-open nbd U and y_β has an $fswg$-open q-nbd V such that $U \not{q} V$.

Theorem 7.4. If an injective function $h : X \to Y$ is $fswg$-continuous function from an fts X onto a fuzzy T_2-space Y, then X is $fswg$-T_2-space.

Proof. Let x_α and y_β be two distinct fuzzy points in X. Then $h(x_\alpha)$ $(= z_\alpha$, say) and $h(y_\beta)$ $(= w_\beta$, say) are two distinct fuzzy points in Y.

Case I. Suppose $x \neq y$. Then $z \neq w$. Since Y is fuzzy T_2-space, there exist fuzzy

open sets U, V in Y such that $z_\alpha qU, w_\beta qV$ and $U \not{q} V$. As h is $fswg$-continuous function, $h^{-1}(U)$ and $h^{-1}(V)$ are $fswg$-open sets in X with $x_\alpha qh^{-1}(U), y_\beta qh^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ [Indeed, $z_\alpha qU \Rightarrow U(z) + \alpha > 1 \Rightarrow U(h(x)) + \alpha > 1 \Rightarrow [h^{-1}(U)](x) + \alpha > 1 \Rightarrow x_\alpha qh^{-1}(U)$. Again, $h^{-1}(U)qh^{-1}(V) \Rightarrow$ there exists $t \in X$ such that $[h^{-1}(U)](t) + [h^{-1}(V)](t) > 1 \Rightarrow U(h(t)) + V(h(t)) > 1 \Rightarrow UqV$, a contradiction].
Case II. Suppose $x = y$ and $\alpha < \beta$ (say). Then $z = w$ and $\alpha < \beta$. Since Y is fuzzy T_2-space, there exist a fuzzy open nbd U of z_α and a fuzzy open q-nbd V of w_β such that $U \not{q} V$. Then $U(z) \geq \alpha \Rightarrow [h^{-1}(U)](x) \geq \alpha \Rightarrow x_\alpha \in h^{-1}(U), y_\beta qh^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ where $h^{-1}(U)$ and $h^{-1}(V)$ are $fswg$-open sets in X as h is $fswg$-continuous function. Consequently, X is $fswg$-T_2-space.
Similarly we can state the following theorems easily the proofs of which are similar to that of Theorem 7.4.
Theorem 7.5. If a bijective function $h : X \to Y$ is $fswg$-irresolute function from an fts X onto an $fswg$-T_2-space (resp., fuzzy T_2-space) Y, then X is $fswg$-T_2-space.
Theorem 7.6. If a bijective function $h : X \to Y$ is $fswg$-continuous function from an fsT_g-space X onto a fuzzy T_2-space Y, then X is fuzzy T_2-space.
Theorem 7.7. If a bijective function $h : X \to Y$ is $fswg$-irresolute function from an fsT_g-space X onto an $fswg$-T_2-space (resp., fuzzy T_2-space) Y, then X is fuzzy T_2-space.
Theorem 7.8. If a bijective function $h : X \to Y$ is $fswg$-open function from a fuzzy T_2-space X onto an fts Y, then Y is $fswg$-T_2-space.
Theorem 7.9. If a bijective function $h : X \to Y$ is $fswg$-open function from a fuzzy T_2-space X onto an fsT_g-space Y, then Y is fuzzy T_2-space.
Theorem 7.10. If a bijective function $h : X \to Y$ is strongly $fswg$-continuous function from an fts X onto an $fswg$-T_2-space (resp., fuzzy T_2-space) Y, then X is fuzzy T_2-space.
Theorem 7.11. If a bijective function $h : X \to Y$ is $fswg$-strongly continuous function from an fts X onto a fuzzy T_2-space Y, then X is $fswg$-T_2-space.
Theorem 7.12. If a bijective function $h : X \to Y$ is $fswg$-strongly continuous function from an fsT_g-space X onto a fuzzy T_2-space Y, then X is fuzzy T_2-space.
Remark 7.13. It is clear from the fact that fuzzy open set is $fswg$-open set, every fuzzy T_2-space is $fswg$-T_2-space, but the converse is not necessarily true, follows from the following example.
Example 7.14. Let $X = \{a\}$, $\tau = \{0_X, 1_X\}$. Then (X, τ) is an fts. Clearly (X, τ) is not a fuzzy T_2-space. Here every fuzzy set in (X, τ) is $fswg$-open set in (X, τ). Consider two fuzzy points $a_{0.1}$ and $a_{0.4}$. Then there exist two $fswg$-open sets U, V in X where $U(a) = 0.2, V(a) = 0.61$ such that $a_{0.1} \in U$, $a_{0.4}qV$ and $U \not{q} V$ and this is

true for every pair of distinct fuzzy points in X. So (X, τ) is an $fswg$-T_2-space.

REFERENCES

[1] Azad, K.K.; On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity, *J.Math. Anal. Appl.*, 82 (1981), 14-32.

[2] Balasubramanian, G. and Sundaram, P.; On some generalizations of fuzzy continuous functions, *Fuzzy Sets and Systems*, 86 (1997), 93-100.

[3] Bhattacharyya, Anjana; $fg^*\alpha$-continuous functions in fuzzy topological spaces, *International Journal of Scientific and Engineering Research*, Vol. 4, Issue 8 (2013), 973-979.

[4] Bhattacharyya, Anjana; s^*-closure operator and s^*-regularity in fuzzy setting, *International Journal of Pure and Applied Mathematics*, Vol. 96 (2) (2014), 279-288.

[5] Bhattacharyya, Anjana; Fuzzy γ-continuous multifunction, *International Journal of Advance Research in Science and Engineering*, Vol. 4(2) (2015), 195-209.

[6] Bhattacharyya, Anjana; fgs^*-closed sets and fgs^*-continuous functions in fuzzy topological spaces, *"Vasile Alecsandri" University of Bacău, Faculty of Sciences, Scientific Studies and Research, Series Mathematics and Informatics*, Vol. 26, No. 2 (2016), 95-114.

[7] Bhattacharyya, Anjana; Fuzzy generalized continuity, *Annals of Fuzzy Mathematics and Informatics*, Vol. 11 (4) (2016), 645-659.

[8] Bhattacharyya, Anjana; Several concepts of fuzzy generalized continuity in fuzzy topological spaces, *Analele Universităţii Oradea Fasc. Matematica*, Tom XXIII, Issue No. 2 (2016), 57-68.

[9] Bhattacharyya, Anjana; Fuzzy generalized closed sets in a fuzzy topological space, *Jour. Fuzzy Math.*, Vol. 25, No. 2 (2017), 285-301.

[10] Bhattacharyya, Anjana; Fuzzy regular generalized α-closed sets and fuzzy regular generalized α-continuous functions, *Advances in Fuzzy Mathematics*, Vol. 12, No. 4 (2017), 1047-1066.

[11] Bhattacharyya, Anjana; More on $fgpr$-closed set in a fuzzy topological space, *The 13th International Conference MSAST 2019, December 21-23, Kolkata, India*, vol. 8 (2019), 99-110.

[12] Bhattacharyya, Anjana; Generalized γ-closed set in fuzzy setting, *Jour. Fuzzy Math.*, Vol. 28, No. 3 (2020), 681-708.

[13] Bhattacharyya, Anjana; On $fg\gamma^*$-closed sets in fuzzy topological spaces, *"Vasile Alecsandri" University of Bacău, Faculty of Sciences, Scientific Studies and Research, Series Mathematics and Informatics*, Vol. 30, No. 1 (2020), 17-44.

[14] Bhattacharyya, Anjana; Concerning $f\pi g$-closed set (Communicated).

[15] Bhattacharyya, Anjana; fwg-closed set and its applications, *Bull. Cal. Math. Soc.*, Vol. 114, No. 4 (2022), 519-554.

[16] Bhattacharyya, Anjana; fmg-closed sets in fuzzy topological spaces, *"Vasile Alecsandri" University of Bacău, Faculty of Sciences, Scientific Studies and Research, Series Mathematics and Informatics*, Vol. 30, No. 2 (2020), 21-56.

[17] Bhattacharyya, Anjana; Fuzzy topological properties of spaces and functions with respect to the $frwg$-closure operator, *"Vasile Alecsandri" University of Bacău, Faculty of Sciences, Scientific Studies and Research, Series Mathematics and Informatics*, Vol. 31, No. 1 (2021), 29-74 .

[18] Bhattacharyya, Anjana; Concerning $fswg$-closed set and its applications *Bull. Cal. Math. Sco.* (Accepted for publication)

[19] Bin Shahna, A.S.; On fuzzy strong semicontinuity and fuzzy precontinuity, *Fuzzy Sets and Systems*, Vol. 44 (1991), 303-308.

[20] Chakraborty, R.P., Mukherjee, M.N. and Bhattacharyya, Anjana; Semicompactness in fuzzy topological spaces, *Bull. Malays. Math. Sci. Soc.*, Vol. 28 (2) (2005), 205-213.

[21] Chang, C.L.; Fuzzy topological spaces, *J. Math. Anal. Appl.*, Vol. 24 (1968), 182-190.

[22] DiConcillio, A. and Gerla, G.; Almost compactness in fuzzy topological spaces, *Fuzzy Sets and Systems*, Vol. 13 (1984), 187-192.

[23] ES, A. Hayder; Almost compactness and near compactness in topological spaces, *Fuzzy Sets and Systems*, Vol. 22 (1987), 289-295.

[24] Fath Alla, M.A.; On fuzzy topological spaces, *Ph.D. Thesis, Assiut Univ., Sohag, Egypt*, (1984).

[25] Ganguly, S. and Saha, S.; A note on compactness in fuzzy setting, *Fuzzy Sets and Systems*, Vol. 34 (1990), 117-124.

[26] Hutton, B. and Reilly, I.; Separation axioms in fuzzy topological spaces, *Fuzzy Sets and Systems*, Vol. 31 (1980), 93-104.

[27] Malakar, S.; On fuzzy semi-irresolute and strongly irresolute functions, *Fuzzy Sets and Systems*, Vol. 45 (1992), 239-244.

[28] Mukherjee, M.N. and Ghosh, B.; On nearly compact and θ-rigid fuzzy sets in fuzzy topological spaces, *Fuzzy Sets and Systems*, Vol. 43 (1991), 57-68.

[29] Mukherjee, M.N. and Sinha, S.P.; Almost compact fuzzy sets in fuzzy topological spaces, *Fuzzy Sets and Systems*, Vol. 38 (1990), 389-396.

[30] Nanda, S.; Strongly compact fuzzy topological spaces, *Fussy Sets and Systems*, Vol. 42 (1991), 259-262.

[31] Pu, Pao Ming and Liu, Ying Ming; Fuzzy topology I. Neighbourhood structure of a fuzzy point and Moore-Smith Convergence, *J. Math Anal. Appl.*, Vol. 76 (1980), 571-599.

[32] Wong, C.K.; Fuzzy points and local properties of fuzzy topology, *J. Math. Anal. Appl.*, Vol. 46 (1974), 316-328.

[33] Zadeh, L.A.; Fuzzy Sets, *Inform. Control*, Vol. 8 (1965), 338-353.