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On Generalized Fuzzy Graph Structures II

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Abstract

In an earlier paper, we introduced the concept of a fuzzy graph structure analogous to the concept of graph structure given by E. Sampathkumar, in Bull. Kerala Math. Assoc., Vol 3, No.2 (2006 December), 65–123. In this paper, new concepts like ρ_i-bridge and ρ_i-cutvertex are introduced. Some results are obtained. We continue the study in two other papers which have been communicated.

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1. Introduction

The notion of fuzzy sets introduced by L.A. Zadeh in 1965, involves the concept of a membership function defined on a universal set. The value of the membership function lies in [0,1]. Using this concept, the idea of fuzzy graph was introduced by A. Rosenfeld in 1975 (cf. [4]).

A new concept, namely, graph structure has been introduced by E. Sampathkumar in [5] which, in particular, is a generalisation of the notions like graphs, signed graphs and edge - coloured graphs with colourings. According to him, $G = (V, R_1, R_2, \ldots, R_k)$ is a graph structure if V is a nonempty set and $R_1, R_2, \ldots, R_k$ are relations on V which are mutually disjoint such that each R_i, $i = 1, 2, 3, \ldots, k$, is symmetric and irreflexive.

This is the motivation for the study of fuzzy graph structures. In an earlier paper [1], we introduced new concepts like ρ_i-edge, ρ_i-path, ρ_i-cycle, ρ_i-tree, ρ_i-forest, fuzzy ρ_i-cycle, fuzzy ρ_i-tree, fuzzy ρ_i-forest and ρ_i-connectedness. Here some other concepts like ρ_i-bridges and ρ_i-cut vertices are introduced and studied.

Essential preliminaries are given in section 2. For more details in Graph Theory, reference may be made to [2], for Fuzzy Graph Theory, to [3] and for Graph Structures, to [5].

2. Preliminaries

We recall some definitions on fuzzy graph structure as given in [1].

Definition 2.1. Let $G = (V, R_1, R_2, \ldots, R_k)$ be a graph structure and $\mu, \rho_1, \rho_2, \ldots, \rho_k$ be fuzzy subsets of $(V, R_1, R_2, \ldots, R_k)$ respectively such that

$$\rho_i(x, y) \leq \mu(x) \wedge \mu(y) \forall x, y \in V$$

and $i = 1, 2, \ldots, k$. Then $\tilde{G} = (\mu, \rho_1, \rho_2, \ldots, \rho_k)$ is a fuzzy graph structure of G.

Convention: Throughout this paper, unless otherwise stated, G and $\tilde{G}$ will have the above meaning.

Example 2.2. Let

$$\begin{aligned} V &= \{x_0, x_1, x_2, x_3, x_4, x_5\}, \\ R_1 &= \{(x_0, x_1), (x_1, x_0), (x_0, x_2), (x_2, x_0), (x_3, x_4), (x_4, x_3)\}, \\ R_2 &= \{(x_1, x_2), (x_2, x_1), (x_4, x_5), (x_5, x_4)\}, R_3 = \{(x_2, x_3), (x_3, x_2)\} \end{aligned}$$

and $G = (V, R_1, R_2, R_3)$ be a graph structure. Let $\tilde{G} = (\mu, \rho_1, \rho_2, \rho_3)$ where

$$\mu(x_0) = 0.8, \mu(x_1) = 0.9, \mu(x_2) = 0.6, \mu(x_3) = 0.5, \mu(x_4) = 0.6, \mu(x_5) = 0.7$$

$$\begin{aligned} &\rho_1(x_0, x_1) = 0.8 = \rho_1(x_1, x_0), \rho_1(x_0, x_2) = 0.5 = \rho_1(x_2, x_0), \\ &\rho_1(x_3, x_4) = 0.4 = \rho_1(x_4, x_3), \rho_2(x_1, x_2) = 0.6 = \rho_2(x_2, x_1), \\ &\rho_2(x_4, x_5) = 0.5 = \rho_2(x_5, x_4), \rho_3(x_2, x_3) = 0.3 = \rho_3(x_3, x_2), \\ &\rho_3(x_0, x_5) = 0.5 = \rho_3(x_5, x_0) \end{aligned}$$

Here notice that $\rho_1(x_0, x_1) \leq \wedge(\mu(x_0), \mu(x_1))$ and so on. Therefore, $\tilde{G}$ is a fuzzy graph structure of G. Now we recall some basic notions of fuzzy graph structures. In all these, $i \in \{1, 2, \ldots, k\}$.

Definition 2.3. $\tilde{F} = (\mu, \tau_1, \tau_2, \ldots, \tau_k)$ is a partial fuzzy spanning subgraph structure of $\tilde{G} = (\mu, \rho_1, \rho_2, \ldots, \rho_k)$ if $\tau_r \subseteq \rho_r \; for \; r = 1, 2, \ldots, k$.

Definition 2.4. Let G be a graph structure and $\tilde{G}$ be a fuzzy graph structure of G. If $(x, y) \in supp(\rho_i)$, then (x,y) is said to be a $\rho_i - edge$ of $\tilde{G}$.

In example 2.2, $(x_0, x_1), (x_0, x_2), (x_3, x_4)$ are ρ_1-edges, $(x_1, x_2), (x_4, x_5)$ are ρ_2-edges and $(x_2, x_3), (x_0, x_5)$ are ρ_3-edges.

Definition 2.5. A ρ_i-path of a fuzzy graph structure $\tilde{G}$ is a sequence of vertices, x_0, $x_1, \ldots, x_n$ which are distinct (except possibly x_n, x_0) such that (x_{j-1}, x_j) is a ρ_i-edge for all $j \in \{1, 2, \ldots, n\}$.

In example 2.2, x_1, x_0, x_2 is a ρ_1-path.

Definition 2.6. Two vertices of a fuzzy graph structure $\tilde{G}$, joined by a ρ_i-path are said to be ρ_i-connected.

In example 2.2, x_1 and x_2 are ρ_1-connected and x_0 and x_5 are ρ_3-connected.

Definition 2.7. The strength of a ρ_i-path $x_0, x_1, \ldots, x_n$ of a fuzzy graph structure $\tilde{G}$ is $\bigwedge_{j=1}^{n} \rho_i(x_{j-1}, x_j)\ for\ i = 1, 2, \ldots, k.$

In example 2.2, strength of the ρ_1- path x_1, x_0, x_2 is 0.5.

Definition 2.8. In any fuzzy graph structure $\tilde{G}$,

$$\rho_i^2(x, y) = \rho_i \circ \rho_i(x, y) = \bigvee_z \{\rho_i(x, z) \wedge \rho_i(z, y)\}$$

and $\rho_i^j(x, y) = \rho_i^{j-1} \circ \rho_i(x, y),\ j = 2, 3, \ldots, m$ for any m ≥ 2. Also

$$\rho_i^{\infty}(x, y) = \vee\{\rho_i^j(x, y) : j = 1, 2, \ldots\}.$$

Definition 2.9. $\tilde{G}$ is a ρ_i-cycle iff (supp(μ), supp(ρ_1),supp(ρ_2),..., supp(ρ_k)) is an R_i-cycle (where an R_i-cycle is a sequence of vertices $x_0, x_1, \ldots, x_{n-1}, x_n = x_0$ in V such that each (x_{j-1}, x_j) is an R_i-edge for $j = 1, 2, \ldots, n$.

Example 2.10. Let $V = \{x_0, x_1, x_2, x_3, x_4, x_5\}$, $R_1 = \{(x_0, x_1), (x_1, x_0), (x_1, x_2), (x_2, x_1), (x_2, x_3), (x_3, x_2), (x_0, x_3)), (x_3, x_0))\}$, $R_2 = \{(x_0, x_4), (x_4, x_0)\}$, $R_3 = \{(x_3, x_4), (x_4, x_3), (x_4, x_5), (x_5, x_4), (x_5, x_0), (x_0, x_5), (x_6, x_7), (x_7, x_6)\}$ and $G = (V, R_1, R_2, R_3)$ be a graph structure. Let $\tilde{G} = (\mu, \rho_1, \rho_2, \rho_3)$ where

$$\mu(x_0) = 0.8, \mu(x_1) = 0.9, \mu(x_2) = 0.6, \mu(x_3) = 0.5, \mu(x_4) = 0.6,$$
$$\mu(x_5) = 0.7, \mu(x_6) = 0.6, \mu(x_7) = 0.5$$
$$\rho_1(x_0, x_1) = 0.8 = \rho_1(x_1, x_0), \rho_1(x_1, x_2) = 0.5 = \rho_1(x_2, x_1),$$
$$\rho_1(x_2, x_3) = 0.5 = \rho_1(x_3, x_2), \rho_1(x_0, x_3) = 0.5 = \rho_1(x_3, x_0),$$
$$\rho_2(x_0, x_4) = 0.6 = \rho_2(x_4, x_0), \rho_3(x_3, x_4) = 0.5 = \rho_3(x_4, x_3),$$
$$\rho_3(x_4, x_5) = 0.6 = \rho_3(x_5, x_4), \rho_3(x_5, x_0) = 0.4 = \rho_3(x_0, x_5),$$
$$\rho_3(x_6, x_7) = 0.5 = \rho_3(x_7, x_6)$$

In example 2.10, $(x_0, x_1), (x_1, x_2), (x_2, x_3), (x_0, x_3)$ is a ρ_1-cycle.

Definition 2.11. $\tilde{G}$is a fuzzy ρ_i-cycle iff (supp(μ),supp(ρ_1),supp(ρ_2),..., supp(ρ_k)) is an R_i-cycle and there exists no unique (x,y) in supp(ρ_i) such that $\rho_i(x, y) = \wedge\{\rho_i(u, v) | (u, v) \in supp(\rho_i)\}$.

In example 2.10, $(x_0, x_1), (x_1, x_2), (x_2, x_3), (x_0, x_3)$ is a fuzzy ρ_1-cycle.

Definition 2.12. $\tilde{G}$ is a ρ_i-forest for $i = 1, 2, \dots, k$ if its ρ_i-edges form an R_i-forest (where an R_i-forest is a Graph Structure which does not contain R_i-cycles).

In example 2.10, $(x_3, x_4), (x_4, x_5), (x_5, x_0), (x_6, x_7)$ is a ρ_3-forest.

Definition 2.13. $\tilde{G}$ is a ρ_i-tree if it is a ρ_i-connected ρ_i-forest.

In example 2.10, $(x_3, x_4), (x_4, x_5), (x_5, x_0)$ is a ρ_3-tree.

Definition 2.14. $\tilde{G}$ is a fuzzy ρ_i-forest if it has a partial fuzzy spanning subgraph structure $\tilde{F_i} = (\mu, \tau_1, \tau_2, \dots, \tau_k)$ which is a τ_i-forest where for all ρ_i-edges not in $\tilde{F_i}$, $\rho_i(x, y) < \tau_i^\infty(x, y)$.

Definition 2.15. $\tilde{G}$ is a fuzzy ρ_i-tree for $i = 1, 2, \dots, k$ if it has a partial fuzzy spanning subgraph structure $\tilde{F_i} = (\mu, \tau_1, \tau_2, \dots, \tau_k)$ which is a τ_i-tree where for all ρ_i-edges not in $\tilde{F_i}$, $\rho_i(x, y) < \tau_i^\infty(x, y)$.

Example 2.16. Let

$$\begin{aligned} V &= \{x_0, x_1, x_2, x_3, x_4, x_5\}, \\ R_1 &= \{(x_0, x_1), (x_1, x_0), (x_1, x_2), (x_2, x_1), (x_2, x_3), (x_3, x_2), (x_3, x_0), (x_0, x_3), \\ &\quad (x_4, x_5), (x_5, x_4)\}, R_2 = \{(x_0, x_4), (x_4, x_0)\} \end{aligned}$$

and $G = (V, R_1, R_2)$ be a graph structure.

Let $\tilde{G} = (\mu, \rho_1, \rho_2)$ where

$$\begin{aligned} \mu(x_0) &= 0.5, \mu(x_1) = 0.6, \\ \mu(x_2) &= 0.7, \mu(x_3) = 0.8, \mu(x_4) = 0.9, \mu(x_5) = 0.8 \\ \rho_1(x_0, x_1) &= 0.4 = \rho_1(x_1, x_0), \rho_1(x_1, x_2) = 0.5 = \rho_1(x_2, x_1), \\ \rho_1(x_2, x_3) &= 0.6 = \rho_1(x_3, x_2), \rho_1(x_0, x_3) = 0.3 = \rho_1(x_3, x_0), \\ \rho_1(x_4, x_5) &= 0.8 = \rho_1(x_5, x_4), \rho_2(x_0, x_4) = 0.5 = \rho_2(x_4, x_0) \end{aligned}$$

$\tilde{G}$ is a fuzzy graph structure of G. Let $\tilde{F_1} = (\mu, \tau_1, \tau_2)$ be a partial fuzzy spanning subgraph structure of $\tilde{G}$ defined by

$$\begin{aligned} \tau_1(x_0, x_1) &= 0.4 = \tau_1(x_1, x_0), \\ \tau_1(x_1, x_2) &= 0.5 = \tau_1(x_2, x_1), \tau_1(x_2, x_3) = 0.6 = \tau_1(x_3, x_2), \\ \tau_1(x_4, x_5) &= 0.8 = \tau_1(x_5, x_4), \tau_2(x_0, x_4) = 0.5 = \tau_2(x_4, x_0) \end{aligned}$$

and $\tau_1 = 0, \tau_2 = 0$ for all other ρ_1 and ρ_2 edges. Then $\tilde{F_1} = (\mu, \tau_1, \tau_2)$ is a τ_1-forest with $\rho_1(x, y) < \tau_1^\infty(x, y)$for all ρ_1-edges not in $\tilde{F_1}$. Hence $\tilde{G}$ is a fuzzy ρ_1-forest.

In the above example, $(x_0, x_1), (x_1, x_2), (x_2, x_3), (x_3, x_0)$ form a fuzzy ρ_1-tree.

3. ρ_i-bridges and ρ_i-cut vertices

We introduce some more new concepts like ρ_i-bridges and ρ_i-cut vertices of a fuzzy graph structure.

Definition 3.1. Let (x,y) be a ρ_i-edge of $\tilde{G}$. Let $(\mu, \rho_1', \rho_2', \ldots, \rho_i', \rho_{i+1}', \ldots, \rho_k')$ be a partial fuzzy spanning subgraph structure obtained by deleting (x,y) with $\rho_i'(x, y) = 0$ and $\rho_i'(x_1, y_1) = \rho_i(x_1, y_1)\,\forall\rho_i$-edge (x_1, y_1) other than (x,y). If $\rho_i'^{\infty}(u, v) < \rho_i^{\infty}(u, v)$ for some $(u, v) \in supp(\rho_i)$, then (u,v) is a ρ_i-bridge.

Now we move on to some results using the concept of ρ_i-bridges.

Theorem 3.2. Let $\tilde{G}$ be a fuzzy graph structure. If (x,y) is a ρ_i-bridge, then $\rho_i'^{\infty}(x, y) < \rho_i(x, y)$ where $(\mu, \rho_1', \rho_2', \ldots, \rho_k')$ is a partial fuzzy spanning subgraph structure obtained by deleting (x,y), for $i = 1, 2, \ldots, k$.

Proof. If possible, let $\rho_i'^{\infty}(x, y) \geq \rho_i(x, y)$ for some ρ_i-bridge (x,y). ie., there is a ρ_i-path of strength greater than $\rho_i(x, y)$from x to y which does not involve (x,y). Thus any ρ_i-path involving (x,y) can be replaced by a ρ_i-path not involving (x,y) without reducing strength.

This is a contradiction to the fact that (x,y) is a ρ_i-bridge. Hence $\rho_i^{\infty}(x, y) < \rho_i(x, y)$ for $i = 1, 2, \ldots, k$. ■

Remark 3.3. Converse of the above result also holds. ie., if $\rho_i'^{\infty}(x, y) < \rho_i(x, y)$, then (x, y) is a ρ_i-bridge.

Proof. If possible, let (x, y) be not a ρ_i-bridge. Then $\rho_i'^{\infty}(x, y) = \rho_i^{\infty}(x, y) \geq \rho_i(x, y)$ which is a contradiction to our assumption.

Therefore (x, y) is a ρ_i-bridge. ■

Theorem 3.4. Let $\tilde{G}$ be a fuzzy graph structure which is a fuzzy ρ_i-forest. Then the ρ_i-edges of the partial fuzzy spanning subgraph structure $\tilde{F}_i = (\mu, \tau_1, \tau_2, \ldots, \tau_k)$ which is a τ_i-forest, are the ρ_i-bridges of $\tilde{G}$.

Proof. ***Case 1:****(x,y) is a ρ_i-edge not in $\tilde{F}_i$*

By definition of a fuzzy ρ_i-forest, $\rho_i(x, y) < \tau_i^{\infty}(x, y) \leq \rho_i'^{\infty}(x, y)$

Therefore (x,y) is not a ρ_i-bridge by theorem 3.2.

Case 2:*(x,y) is a τ_i-edge of $\tilde{F}_i$*

If possible, let (x,y) be not a ρ_i-bridge. Then there exists a ρ_i-path P_i from x to y not involving (x,y) with strength greater than or equal to $\rho_i(x, y)$. P_i and $\tilde{F}_i$ form a ρ_i-cycle.

But $\tilde{F}_i$ does not contain τ_i-cycles. Therefore, P_i contains ρ_i-edges not in $\tilde{F}_i$.

Let (u_r, v_r) be such a ρ_i-edge of P_i.

This can be replaced by a τ_i-path P_{ir} in $\tilde{F}_i$ having strength greater than $\rho_i(u_r, v_r)$ by definition of a fuzzy ρ_i-forest.

Also $\rho_i(u_r, v_r) \geq \rho_i(x, y)$. All τ_i-edges of P_{ir} are stronger than $\rho_i(u_r, v_r)$ which is greater than or equal to $\rho_i(x, y)$.

Therefore P_{ir} does not contain (x, y). If it contains (x, y), its strength will be less than or equal to $\tau_i(x, y) \leq \rho_i(x, y)$.

Thus we have a τ_i-path in $\tilde{F}_i$ from x to y not involving(x, y). This gives a τ_i-cycle in $\tilde{F}_i$ and hence a ρ_i-cycle which is not possible. Hence (x,y) is a ρ_i-bridge. Thus the ρ_i-edges of $\tilde{F}_i$ are the ρ_i-bridges of $\tilde{G}$. ∎

Now, we define a ρ_i-cut vertex. For that first we define the partial subgraph strucure $(\mu', \rho_1', \rho_2', \ldots, \rho_k')$.

Definition 3.5. $\tilde{G}' = (\mu', \rho_1', \rho_2', \ldots, \rho_k')$ is the partial fuzzy subgraph structure obtained by the deletion of a vertex w of $\tilde{G}$. ie., $\mu'(w) = 0$ and $\mu'(u) = \mu(u) \forall u \neq w$ $\rho_i'(w, v) = 0 \forall v \in V$ and $\rho_i'(u, v) = \rho_i(u, v) \forall (u, v) \neq (w, v), i = 1, 2, \ldots, k$.

Definition 3.6. A vertex w of $\tilde{G}$ is a ρ_i-cut vertex if $\rho_i'^{\infty}(u, v) < \rho_i^{\infty}(u, v)$ for some u, v with $u \neq w \neq v$ where μ' and ρ_i' are as in definition 3.5.

Now we discuss some results on ρ_i-bridges and ρ_i-cut vertices.

Theorem 3.7. Let $\tilde{G}$ be a fuzzy graph structure with

$$\tilde{G}^* = (supp(\mu), supp(\rho_1), supp(\rho_2), \ldots, supp(\rho_k))$$

a fuzzy ρ_i-cycle. If a vertex of $\tilde{G}$ is a ρ_i-cut vertex of $\tilde{G}$, then it is a common vertex of two ρ_i-bridges.

Proof. Consider a ρ_i-cut vertex w of $\tilde{G}$. By the definition of a ρ_i-cut vertex, there exists two vertices u and v different from w such that w is on every strongest $u - v$ ρ_i-path.

Given that $\tilde{G}^*$ is a fuzzy ρ_i-cycle. Then there exists only one strongest ρ_i-path P_i from u to v containing w. In P_i, all ρ_i-edges are ρ_i-bridges.

Thus w is common to two ρ_i-bridges. ∎

Remark 3.8. Converse of the above result also holds as is evident from the next theorem.

Theorem 3.9. Let $\tilde{G}$ be a fuzzy graph structure. If w is common to at least two ρ_i-bridges of $\tilde{G}$, then w is a ρ_i-cut vertex.

Proof. Let (u_1, w) and (w, v_2) be two ρ_i-bridges with w as the common vertex. Since (u_1, w) is a ρ_i-bridge, it is on every strongest $u - v$ ρ_i-path for some u and v.

Case 1: $w \neq u, w \neq v$

In this case, w is on every strongest $u - v$ ρ_i-path for some u and v. Then w is a ρ_i-cut vertex.

Case 2: $Either\ w = u\ or\ w = v$

In this case either (u_1, w) is on every strongest $u - w$ ρ_i-path or (w, v_2) is on every strongest $w - v$ ρ_i-path. If possible, let w be not a ρ_i-cut vertex.

By definition of ρ_i-cut vertex, there exists a strongest ρ_i-path not containing w between any pair of vertices. Consider such a path P_i joining $u_1$and v_2. Then P_i, (u_1, w), (w, v_2) form a ρ_i-cycle.

a) Let u_1, w, v_2 be not a strongest ρ_i-path.

Then (u_1, w) or (w, v_2) or both become the weakest ρ_i-edges of the above ρ_i-cycle consisting of P_i, $(u_1, w)$and (w, v_2)since every ρ_i-edge of $P_i$will be stronger than (u_1, w) and (w, v_2).

This is not possible since (u_1, w) and (w, v_2) are ρ_i-bridges.

b) Let u_1, w, v_2 also be a strongest ρ_i-path joining u_1 and v_2.

Then $\rho_i^\infty(u_1, v_2) = \rho_i(u_1, w) \wedge \rho_i(w, v_2)$ i.e., either (u_1, w) or(w, v_2) or both are the weakest ρ_i-edges of the above ρ_i-cycle because P_i is as strong as u_1, w, v_2.

This is not possible because u_1, w, v_2 is a strongest ρ_i-path. Therefore, w is a ρ_i-cut vertex. ■

Now we prove that the internal vertices of a ρ_i-tree of a fuzzy ρ_i-tree are the ρ_i-cut vertices.

Theorem 3.10. Let $\tilde{G}$ be a fuzzy ρ_i-tree for which $\tilde{F}_i = (\mu, \tau_1, \tau_2, \ldots, \tau_k)$ is a partial fuzzy spanning subgraph structure which is a τ_i-tree and $\rho_i(x, y) < \tau_i^\infty(x, y) \forall (x, y)$ not in $\tilde{F}_i$. Then the internal vertices of $\tilde{F}_i$are precisely the ρ_i-cut vertices of $\tilde{G}$.

Proof. Consider a vertex w of $\tilde{F}_i$.

Case 1:*w is not an end vertex of $\tilde{F}_i$*

w is common to two τ_i-edges of $\tilde{F}_i$ at least and by theorem 3.4, they are ρ_i-bridges of $\tilde{G}$. Then by theorem 3.9, w is a ρ_i-cut vertex.

Case 2:*w is an end vertex of $\tilde{F}_i$*

If w is a ρ_i-cut vertex, it lies on every strongest ρ_i-path and hence τ_i-path joining u and v for some u and v in V. One of such τ_i-paths lies in $\tilde{F}_i$. But w is an end vertex of $\tilde{F}_i$. So this is not possible. So w is not a ρ_i-cut vertex. i.e., the internal vertices of $\tilde{F}_i$ are precisely the ρ_i-cut vertices of $\tilde{G}$. ■

The above theorem leads us to the following corollary.

Corollary 3.11. A ρ_i-cut vertex, of a fuzzy graph structure $\tilde{G}$ which is a fuzzy ρ_i-tree, is common to at least two ρ_i-bridges.

References

[1] Dinesh, T. & Ramakrishnan, T.V., On Generalised Fuzzy Graph Structures (Communicated).

[2] Harary, F., Graph Theory, Narosa Pub. House, 1995.

[3] Mordeson, J.N. & Nair, P.S., Fuzzy Graphs and Fuzzy Hypergraphs, Physica-verlag, 2000.

[4] Rosenfeld, A., Fuzzy Graphs in Fuzzy Sets and their Applications to Cognitive and Decision Processes (eds. L.A. Zadeh, K.S. Fu & M. Shimura), pp 77–95, Acad. Press, New York, 1975.

[5] Sampathkumar, E., Generalized Graph Structures, Bull. Kerala Math. Assoc., Vol 3, No.2 (Dec 2006), 65–123.