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Fuzzy fixed point of multivalued Ciric type fuzzy contraction mappings in b-metric spaces

Anju Panwar and Anita

*Department of Mathematics, Maharshi Dayanand University,
Rohtak (Haryana)-124001, India*

Abstract:-

The purpose of our paper is to study the existence of α-fuzzy fixed point for multivalued fuzzy contraction mapping under Ciric type contractive condition in the setting of complete b-metric spaces and an example is given to support the main result.

Keywords: fuzzy set, fuzzy mappings, α-fuzzy fixed point, b-metric spaces.

1. Introduction:-

The theory of multivalued mappings has an important role in various branches of pure and applied mathematics because of its many applications, for instance, in real and complex analysis as well as in optimal control problems. In 1928, Von Neumann [2] was the first person to introduce the idea of fixed point for multivalued mappings. The development of geometric fixed point theory for multivalued functions was initiated by Nadler [7].

The concept of fuzzy set was initiated by Zadeh [3]. Many researches were conducted on the generalization of the concept of a fuzzy set. Heilpern [9] introduced the concept of fuzzy contraction mappings which maps from an arbitrary set to a certain subfamily of fuzzy sets in a metric linear space X. He also proved the existence of a fuzzy fixed point theorem which is a generalization of Nadler's [7] fixed point theorem for multivalued mappings. Frigon and Regan [5] generalized the Heilpern theorem under a contractive condition for 1-level sets (i.e $[Tx]_1$) of a fuzzy contraction T on a complete metric space.

In 1993, Czerwik [12] introduced the notion of b-metric spaces which generalized the concept of metric spaces and observed a characterization of the celebrated Banach fixed point theorem [8] in the context of complete b-metric spaces. Subsequently, several other authors studied the fixed point theorem for single valued to multivalued mappings in b-metric spaces (see [1, 6, 11]).

In 2015, S. Phiangsungnoen [14] introduced the new concept of multivalued fuzzy contraction mappings in b-metric space and gave sufficient conditions for the existence of α-fuzzy fixed point for this class of mappings. The aim of this paper is to obtain α-fuzzy fixed point for fuzzy contraction mapping T on a b-metric space under a contractive condition of Ciric type for α-level sets (i.e $[Tx]_\alpha$) of T in connection with Hausdorff metric. This results generalize the various known results proved by S. Phiangsungnoen [14]. At last, we observe relation between multivalued mappings and fuzzy mappings which can be useful to obtain fixed point for multivalued mappings.

Definition 1.1:-Let X and Y be non empty sets. T is said to be a multivalued mapping from X to Y if T is a function from X to the power set of Y. We denote a multivalued mapping by $X \to 2^y$.
A point $x \in X$ is said to be a fixed point of multivalued mapping T if $x \in Tx$. We denote the set of fixed points of T by Fix(T).

Definition 1.2[12]:-Let X be a non empty set and let $s \geq 1$ be a given real number. A function d: $X \times X \to R^+$ is said to be a b-metric if and only if for all x, y, z $\in$ X the following conditions are satisfied:
(i) d(x, y) = 0 if and only if x = y;
(ii) d(x, y) = d(y, x) for all x, y $\in$ X;
(iii) $d(x, y) \leq s[d(x, z) + d(z, y)]$ for all x, y, z $\in$ X.
Then (X, d) is called a b-metric space.

Note that a (usual) metric space is evidently b-metric space. However, Czerwik [10, 11, 12] has shown that a b-metric on X need not be a metric on X. In following example, Singh and Prasad [13] proved that a b-metric on X need not be a metric on X.

Example 1.3:-Consider the set X = [0, 1] endowed with the function d: $X \times X \to R^+$ defined by $d(x, y) = |x - y|^2$ for all x, y $\in$ X. Clearly, (X, d) is a b-metric space with s = 2, but it is not a metric space.

Example 1.4:-Let X = {a, b, c} and d(a, c) = d(c, a) = m $\geq$ 2, d(a, b) = d(b, c) = d(b, a) = d(c, b) = 1 and d(a, a) = d(b, b) = d(c, c) = 0. Then,

$$d(x, y) \leq \frac{m}{2}[d(x, z) + d(z, y)]$$

for all x, y, z $\in$ X. If m > 2, the triangle inequality does not hold.

Definition 1.5[6]:-Let (X, d) be a b-metric space. Then a sequence $\{x_n\}$ in X is called:
(i) Convergent if and only if there exist x $\in$ X such that $d(x_n, x) \to 0$ as $n \to \infty$.
(ii) Cauchy if and only if $d(x_n, x_m) \to 0$ as n, m $\to \infty$.
(iii) Complete if and only if every Cauchy sequence is convergent.

Let (X, d) be a b-metric space, denote CB(X) the collection of non empty closed bounded subsets of X. For A, B $\in$ CB(X) and x $\in$ X, define the function H: CB(X) $\times$ CB(X) $\to R^+$ by

$$H(A, B) = \max\{\delta(A, B), \delta(B, A)\},$$

where $\delta(A, B) = \sup\{d(a, B): a \in A\}$, $\delta(B, A) = \sup\{d(b, A): b \in B\}$, with $d(x, A) = \inf\{d(x, a), a \in A\}$. Note that H is called the Hausdorff b-metric induced by the b-metric d.

Remark 1.6 [10]:-The function H: CL(X) $\times$ CL(X) $\to R^+$ is a generalized Hausdorff b-metric, that is H(A, B) = $+\infty$ if $\max\{\delta(A, B), \delta(B, A)\}$ do not exist.

Let (X, d) be a b-metric space. We cite the following lemma from Singh and Prasad [13].

Lemma 1.7[13]:-Let (X, d) be a b-metric space. For any A, B, C $\in$ CB(X) and any x, y $\in$ X, we have the following:

(i) $d(x, B) \le d(x, b)$ for any $b \in B$,
(ii) $d(x, B) \le H(A, B)$ for all $x \in A$,
(iii) $\delta(A, B) \le H(A, B)$,
(iv) $H(A, A) = 0$,
(v) $H(A, B) = H(B, A)$,
(vi) $H(A, C) \le s(H(A, B) + H(B, C))$,
(vii) $d(x, A) \le s(d(x, y) + d(y, A))$.

Lemma 1.8[13]:-Let (X, d) be a b-metric space. For any A $\in$ CB(X) and x $\in$ X, then we have $d(x, A) = 0 \Leftrightarrow x \in \overline{A} = A$, where $\overline{A}$ denotes the closure of the set A.

Let Ψ_b be a set of strictly increasing functions in b-metric space, $\psi: [0, \infty) \to [0, \infty)$ such that

$$\sum_{n=0}^{\infty} s^n \psi^n(t) < +\infty \text{ for each } t > 0,$$

where ψ^n denotes n-th iterate of the function ψ. It is well known that $\psi(t) < t$ for all $t > 0$ and $\psi(0) = 0$ for $t = 0$.

Now we introduced the concept of fuzzy set, fuzzy mappings and α-fuzzy fixed point in b-metric space.

Let (X, d) be a b-metric space. A fuzzy set in X is a function with domain X and value [0, 1]. If A is a fuzzy set and x $\in$ X, then the function value A(x) is called the grade of membership of x in A. The collection of all fuzzy sets in X is denoted by $\mathcal{F}(X)$. Let A $\in \mathcal{F}(X)$ and $\alpha \in [0, 1]$. The α-level set of A, denoted by $[A]_\alpha$ is defined by

$$[A]_\alpha = \{x: A(x) \ge \alpha\} \text{ if } \alpha \in [0, 1],$$

$$[A]_0 = \overline{\{x: A(x) > 0\}},$$

whenever $\overline{B}$ is a closure of set B in X.
For A, B $\in \mathcal{F}(X)$, a fuzzy set A is said to be more accurate than a fuzzy set B (denoted by $A \subset B$) if and only if $Ax \leq Bx$ for each x in X, where A(x) and B(x) denote the membership function of A and B respectively. Now, for $x \in X$, $A, B \in \mathcal{F}(X)$, $\alpha \in [0, 1]$ and $[A]_\alpha, [B]_\alpha \in CB(X)$, we define

$$d(x, S) = \inf\{d(x, a);\ a \in S\},$$
$$p_\alpha(x, A) = \inf\{d(x, a);\ a \in [A]_\alpha\},$$
$$p_\alpha(A, B) = \inf\{d(a, b);\ a \in [A]_\alpha, b \in [B]_\alpha\},$$
$$p(A, B) = \sup p_\alpha(A, B), \text{ and}$$
$$H([A]_\alpha, [B]_\alpha) = \max\left\{\sup_{a \in [A]_\alpha} d(a, [B]_\alpha),\ \sup_{b \in [B]_\alpha} d(b, [A]_\alpha)\right\}.$$

Here $H([A]_\alpha, [B]_\alpha)$ is called Hausdorff fuzzy b-metric.

Then the function H: $CL(X) \times CL(X) \to \mathcal{F}(X)$ is a generalized Hausdorff fuzzy b-metric induced by d is defined as

$$H(A, B) = \begin{cases} \max\{\delta(A, B),\ \delta(B, A)\}, & \textit{if the maximum exists;} \\ +\infty & \textit{otherwise,} \end{cases}$$

for all A, B $\in$ CL(X).

Definition 1.9[4]:-Let (X, d) be a metric space, a self mapping T: X $\to$X is called Ciric type contraction if and only if for all x, y $\in$X, there exist h < 1 and

$$d(Tx, Ty) \leq h \max\{d(x, y), d(x, Tx), d(y, Ty), \frac{1}{2}[d(x, Ty), d(y, Tx)]\},$$

Definition 1.10[14]:-Let X be a nonempty set and Y be a b-metric space. A mapping T is said to be a fuzzy mapping if T is a mapping from the set X into $\mathcal{F}(Y)$. The function value (Tx)(y) is the grade of membership of y in Tx.

Definition 1.11[14]:-Let (X, d) be a b-metric space and T be a fuzzy mapping from X into $\mathcal{F}(X)$. A point z in X is called an α-fuzzy fixed point of T if $z \in [Tz]_{\alpha(z)}$.

2. Main Results:-

Theorem 2.1:-Let (X, d) be a complete b-metric space with coefficient $s \geq 1$. Let T: $X \to \mathcal{F}(X)$, α: $X \to (0, 1]$ such that $[Tx]_{\alpha(x)}$ is a non empty closed subset of X for all $x \in X$ and $\psi \in \Psi_b$, such that

$$H([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \leq \psi(M(x, y)) \tag{2.1}$$

where $M(x, y) = \max\{d(x, y), d(x, [Tx]_{\alpha(x)}), d(y, [Ty]_{\alpha(y)}),$

$$\frac{1}{2s}[d(x, [Ty]_{\alpha(y)}), d(y, [Tx]_{\alpha(x)})]\},$$

for all x, y$\in$X. Then T has an α-fuzzy fixed point.

Proof:-Let x_0 be an arbitrary point in X. Suppose that there exist $x_1 \in [Tx_0]_{\alpha(x_0)}$. Since $[Tx_1]_{\alpha(x_1)}$ is a nonempty closed subset of X. Clearly, if $x_0 = x_1$ or $x_1 \in [Tx_1]_{\alpha(x_1)}$, so x_1 is an α-fuzzy fixed point T. Hence, the proof is completed. Thus, throughout the proof, we assume that $x_0 \neq x_1$ and $x_1 \notin [Tx_1]_{\alpha(x_1)}$. Hence $d(x_1, [Tx_1]_{\alpha(x_1)}) > 0$, by condition (2.1) and $\psi \in \Psi_b$, we have

$$\begin{aligned}
0 < d(x_1, [Tx_1]_{\alpha(x_1)}) &\leq H([Tx_0]_{\alpha(x_0)}, [Tx_1]_{\alpha(x_1)}) \\
&\leq \psi(M(x_0, x_1)) \\
&= \psi(\max\{d(x_0, x_1),\ d(x_0, [Tx_0]_{\alpha(x_0)}),\ d(x_1, [Tx_1]_{\alpha(x_1)}), \\
&\qquad \frac{1}{2s}[d(x_0, [Tx_1]_{\alpha(x_1)}),\ d(x_1, [Tx_0]_{\alpha(x_0)})]\}) \\
&\leq \psi(\max\{d(x_0, x_1), d(x_0, x_1),\ d(x_1, [Tx_1]_{\alpha(x_1)}), \\
&\qquad \frac{1}{2s}[s(d(x_0, x_1) + d(x_1, [Tx_1]_{\alpha(x_1)})) + d(x_1, x_1)]\}) \\
&= \psi(\max\{d(x_0, x_1), d(x_0, x_1),\ d(x_1, [Tx_1]_{\alpha(x_1)}), \\
&\qquad \frac{1}{2}[(d(x_0, x_1) + d(x_1, [Tx_1]_{\alpha(x_1)})]\}) \\
&= \psi(\max\{d(x_0, x_1),\ d(x_1, [Tx_1]_{\alpha(x_1)})\}).
\end{aligned}$$

If $\max\{d(x_0, x_1),\ d(x_1, [Tx_1]_{\alpha(x_1)})\} = d(x_1, [Tx_1]_{\alpha(x_1)})$, then we have

$$0 < d(x_1, [Tx_1]_{\alpha(x_1)}) \leq \psi(d(x_1, [Tx_1]_{\alpha(x_1)})) < d(x_1, [Tx_1]_{\alpha(x_1)})$$

which is a contradiction. Thus,

$$\max\{d(x_0, x_1),\ d(x_1, [Tx_1]_{\alpha(x_1)})\} = d(x_0, x_1)$$

and since ψ is a strictly increasing, we have

$$0 < d(x_1, [Tx_1]_{\alpha(x_1)}) \leq \psi(d(x_0, x_1)) < \psi(rd(x_0, x_1)),$$

where $r > 1$ is a real number. This ensures that there exist $x_2 \in [Tx_1]_{\alpha(x_1)}$ and $x_1 \neq x_2$ such that

$$0 < d(x_1, x_2) \leq \psi(d(x_0, x_1)) < \psi(rd(x_0, x_1)).$$

Since, $[Tx_2]_{\alpha(x_2)})$ is a nonempty closed subset of X. We assume that $x_2 \notin [Tx_2]_{\alpha(x_2)})$ then, $d(x_2, [Tx_2]_{\alpha(x_2)}) > 0$ by condition (2.1) and $\psi \in \Psi_b$, we also have

$$\begin{aligned}
0 < d(x_2, [Tx_2]_{\alpha(x_2)}) &\leq H([Tx_1]_{\alpha(x_1)}, [Tx_2]_{\alpha(x_2)}) \\
&\leq \psi(\max\{d(x_1, x_2),\ d(x_1, [Tx_1]_{\alpha(x_1)}),\ d(x_2, [Tx_2]_{\alpha(x_2)}), \\
&\qquad \frac{1}{2s}[d(x_1, [Tx_2]_{\alpha(x_2)}),\ d(x_2, [Tx_1]_{\alpha(x_1)})]\}) \\
&= \psi(\max\{d(x_1, x_2),\ d(x_2, [Tx_2]_{\alpha(x_2)})\}).
\end{aligned}$$

If $\max\{d(x_1, x_2),\ d(x_2, [Tx_2]_{\alpha(x_2)})\} = d(x_2, [Tx_2]_{\alpha(x_2)})$, then we have

$$0 < d(x_2, [Tx_2]_{\alpha(x_2)}) \leq \psi(d(x_2, [Tx_2]_{\alpha(x_2)})) < d(x_2, [Tx_2]_{\alpha(x_2)})$$

which is a contradiction. Thus,

$$\max\{d(x_1, x_2), d(x_2, [Tx_2]_{\alpha(x_2)})\} = d(x_1, x_2)$$

and since ψ is strictly increasing, we have

$$0 < d(x_2, [Tx_2]_{\alpha(x_2)}) \leq \psi(d(x_1, x_2)) < \psi^2(rd(x_0, x_1)).$$

Suppose that there exists $x_3 \in [Tx_2]_{\alpha(x_2)}$ and $x_2 \neq x_3$ such that

$$0 < d(x_2, x_3) \leq \psi(d(x_1, x_2)) < \psi^2(rd(x_0, x_1)).$$

By induction, we can construct the sequence $\{x_n\}$ in X such that $x_n \notin [Tx_n]_{\alpha(x_n)}$, $x_{n+1} \in [Tx_n]_{\alpha(x_n)}$ and

$$\begin{aligned} 0 < d(x_n, [Tx_n]_{\alpha(x_n)}) &\leq d(x_n, x_{n+1}) \\ &\leq \psi(d(x_{n-1}, x_n)) \\ &< \psi^n(rd(x_0, x_1)) \end{aligned}$$

for all $n \in N$. For m, $n \in N$ with $m > n$, we have

$$\begin{aligned} d(x_n, x_m) &\leq s[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)] \\ &= sd(x_n, x_{n+1}) + sd(x_{n+1}, x_m) \\ &\leq sd(x_n, x_{n+1}) + s^2[d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_m)] \\ &= sd(x_n, x_{n+1}) + s^2 d(x_{n+1}, x_{n+2}) + s^2 d(x_{n+2}, x_m) \\ &\leq sd(x_n, x_{n+1}) + s^2 d(x_{n+1}, x_{n+2}) + \ldots + s^{m-n-1} d(x_{m-2}, x_{m-1}) \\ &\quad + s^{m-n} d(x_{m-1}, x_m) \\ &\leq s\,\psi^n(rd(x_0, x_1)) + s^2\,\psi^{n+1}(rd(x_0, x_1)) + \ldots + s^{m-n}\,\psi^{m-1}(rd(x_0, x_1)) \\ &\leq \frac{1}{s^{n-1}}[s^n\,\psi^n(rd(x_0, x_1)) + s^{n+1}\,\psi^{n+1}(rd(x_0, x_1)) + \ldots \\ &\quad + s^{m-1}\,\psi^{m-1}(rd(x_0, x_1))]. \end{aligned}$$

Since $\psi \in \Psi_b$, we know that the series $\sum_{i=0}^{\infty} s^i\,\psi^i(rd(x_0, x_1))$ converges. So $\{x_n\}$ is a Cauchy sequence in X. By the completeness of X, there exists $x^* \in X$ such that $\lim_{n\to+\infty} x_n = x^*$. Now, we claim that $x^* \in [Tx^*]_{\alpha(x^*)}$.

By condition (iii) of b-metric space, we have

$$\begin{aligned} d(x^*, [Tx^*]_{\alpha(x^*)}) &\leq s[d(x^*, x_{n+1}) + d(x_{n+1}, [Tx^*]_{\alpha(x^*)})] \\ &\leq s[d(x^*, x_{n+1}) + H([Tx_n]_{\alpha(x_n)}, [Tx^*]_{\alpha(x^*)})] \\ &\leq s[d(x^*, x_{n+1}) + \psi(d(x_n, x^*))]. \end{aligned}$$

Letting $n \to \infty$ and $\psi(0) = 0$, we have $d(x^*, [Tx^*]_{\alpha(x^*)}) = 0$. Since $[Tx^*]_{\alpha(x^*)}$ is closed we obtain that $x^* \in [Tx^*]_{\alpha(x^*)}$. Therefore, x^* is an α-fuzzy fixed point of T. This complete the proof.

Example 2.2:-Let $X = \{0, 1, 2\}$ and define $d: X \times X \to R$ by

$$d(x, y) = \begin{cases} 0 & x = y \\ \frac{1}{4} & x \neq y \text{ and } x, y \in \{0,1\} \\ \frac{1}{2} & x \neq y \text{ and } x, y \in \{0,2\} \\ 1 & x \neq y \text{ and } x, y \in \{1,2\}. \end{cases}$$

Here (X, d) is a complete b-metric space with the coefficient s = $\frac{4}{3}$. Define fuzzy mapping T: X $\to \mathcal{F}$(X) by

$$(T0)(t) = (T1)(t) = \begin{cases} \frac{1}{2} & t = 0 \\ 0 & t = 1, 2 \end{cases} \quad \text{and}$$

$$(T2)(t) = \begin{cases} 0 & t = 0, 2 \\ \frac{1}{2} & t = 1 \end{cases}$$

Define α: X $\to$(0, 1] by α(x) = $\frac{1}{2}$ for all x $\in$ X. Now we obtain that

$$[Tx]_{1/2} = \begin{cases} \{0\} & x = 0,1 \\ \{1\} & x = 2 \end{cases}$$

for x, y $\in$ X we get

$$H([T0]_{1/2}, [T2]_{1/2}), H([T1]_{1/2}, [T2]_{1/2}) = H(\{0\}, \{1\}) = \frac{1}{4}.$$

$$H([T0]_{1/2}, [T1]_{1/2}) = H(\{0\}, \{0\}) = 0.$$

And we know that

$$M(x, y) = \max\{d(x, y), d(x, [Tx]_{\alpha(x)}), d(y, [Ty]_{\alpha(y)}), \frac{1}{2s}[d(x, [Ty]_{\alpha(y)}), d(y, [Tx]_{\alpha(x)})]\}.$$

For M(0, 1), we first calculate

$d(0, 1) = \frac{1}{4}$, $d(x, [Tx]_{\alpha(x)}) = d(0, [T0]_{1/2}) = d(0, 0) = 0$,

$d(y, [Ty]_{\alpha(y)}) = d(1, [T1]_{1/2}) = d(1, 0) = \frac{1}{4}$, $d(x, [Ty]_{\alpha(y)}) = d(0, [T1]_{1/2}) = d(0, 0) = 0$,

and $d(y, [Tx]_{\alpha(x)})] = d(1, [T0]_{1/2}) = d(1, 0) = \frac{1}{4}$.

Thus $M(0, 1) = \frac{1}{4}$.

Similarly we have M(0, 2) = 1 and M(1, 2) = 1.

Define ψ: [0, ∞) $\to$ [0, ∞) by

$\psi(t) = \frac{1}{2}t$ for all t > 0. Thus we have

$$H([T0]_{1/2}, [T1]_{1/2}) = 0 < \frac{1}{2}M(0, 1),$$

$$H([T0]_{1/2}, [T2]_{1/2}) = \frac{1}{4} < \frac{1}{2}.1 = \frac{1}{2}M(0, 2),$$

and $$H([T0]_{1/2}, [T2]_{1/2}) = \frac{1}{4} < \frac{1}{2}.1 = \frac{1}{2}M(1, 2),$$

for all x, y $\in$ X. Therefore all conditions of Theorem 2.1 hold and there exist a point $0 \in X$ such that $0 \in [T0]_{1/2}$ is α-fuzzy fixed point of T.

Remark 2.2:-If $\max\{d(x, y), d(x, [Tx]_{\alpha(x)}), d(y, [Ty]_{\alpha(y)}),$

$$\frac{1}{2s}[d(x, [Ty]_{\alpha(y)}), d(y, [Tx]_{\alpha(x)})]\} = d(x, y),$$

in theorem 2.1, then we find the main result Theorem 3.1of [14]. Hence theorem 2.1 is an extension of the results [14].

Corollary 2.3(Theorem 3.1,[14])**:-**Let (X, d) be a complete b-metric space with coefficient s ≥ 1. Let T: $X \to \mathcal{F}(X)$, α: $X \to (0, 1]$ such that $[Tx]_{\alpha(x)}$ is a non empty closed subset of X for all $x \in X$ and $\psi \in \Psi_b$, such that

$$H([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \leq \psi(d(x, y)) \tag{2.2}$$

for all x, y $\in$ X. Then T has an α-fuzzy fixed point.

By substituting $\psi(t) = ct$ where $c \in (0, 1)$, in Theorem 2.1 and corollary 2.3, we get the following results.

Corollary 2.4:-Let (X, d) be a complete b-metric space with coefficient s ≥ 1. Let T: $X \to \mathcal{F}(X)$, α: $X \to (0, 1]$ such that $[Tx]_{\alpha(x)}$ is a non empty closed subset of X for all $x \in X$, such that

$$H([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \leq k(M(x, y)) \tag{2.3}$$

where $M(x, y) = \max\{d(x, y), d(x, [Tx]_{\alpha(x)}), d(y, [Ty]_{\alpha(y)}),$

$$\frac{1}{2s}[d(x, [Ty]_{\alpha(y)}), d(y, [Tx]_{\alpha(x)})]\}$$

for all x, y $\in$ X, where 0 < k < 1. Assume that $k < \frac{1}{s}$, then T has an α-fuzzy fixed point.

Corollary 2.5[14]:-Let (X, d) be a complete b-metric space with coefficient s ≥ 1. Let T: $X \to \mathcal{F}(X)$, α: $X \to (0, 1]$ such that $[Tx]_{\alpha(x)}$ is a non empty closed subset of X for all $x \in X$, such that

$$H([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \leq k(d(x, y)) \tag{2.4}$$

for all x, y $\in$ X, where 0 < k < 1. Assume that $k < \frac{1}{s}$, then T has an α-fuzzy fixed point.

Remark 2.6:-If we set s = 1 in Corollary 2.5 and $[Tx]_{\alpha(x)} \in CB(X)$, we get the following result.

Corollary 2.7[9]:-Let (X, d) be a complete metric space. T is a fuzzy mapping from X to $\mathcal{F}(X)$ and α: $X \to (0, 1]$ be a mapping such that $[Tx]_{\alpha(x)}$ is a non empty closed subset of X for all $x \in X$, such that

$$H([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \leq k(d(x, y)) \tag{2.5}$$

for all $x, y \in X$, where $0 \leq k < 1$, then T has an α-fuzzy fixed point.

Here, we study some relation of multivalued mappings and fuzzy mappings. Indeed, we indicate that Corollary 2.4 can be utilized to derive fixed point for multivalued mapping.

Corollary 2.8:-Let (X, d) be a complete b-metric space with coefficient $s \geq 1$ and such that S: $X \to CL(X)$ be multivalued mapping such that

$$H(Sx, Sy) \leq k(M(x, y)) \tag{2.6}$$

where $M(x, y) = \max\{d(x, y), d(x, Sx), d(y, Sy), \frac{1}{2s}[d(x, Sy), d(y, Sx)]\}$

for all $x, y \in X$, where $0 < k < 1$. Assume that $k < \frac{1}{s}$, then there exist $u \in X$ such that $u \in Su$.

Proof:-Let α: $X \to (0, 1]$ be an arbitrary mapping and T: $X \to \mathcal{F}(X)$ defined by

$$(Tx)(t) = \begin{cases} \alpha(x) & t \in Sx \\ 0 & t \notin Sx \end{cases}$$

By a routine calculation, we obtain that

$$[Tx]_{\alpha(x)} = \{t: (Tx)(t) \geq \alpha(x)\} = Sx.$$

Now condition (2.6) become condition (2.3). Therefore, Corollary 2.4 can be applied to obtain $u \in X$ such that $u \in [Tu]_{\alpha(x)} = Su$. This implies that multivalued mapping S have a fixed point. This complete the proof.

By taking M(x, y) = d(x, y) in Corollary 2.8, we get the following result.

Corollary 2.9[10]:-Let (X, d) be a complete b-metric space with coefficient $s \geq 1$ and such that S: $X \to CL(X)$ be multivalued mapping such that

$$H(Sx, Sy) \leq k(d(x, y)) \tag{2.7}$$

or all $x, y \in X$, where $0 < k < 1$. Assume that $k < \frac{1}{s}$, then there exist $u \in X$ such that $u \in Su$.

References:-

(1) H. Aydi, M. F. Bota, E. Karapinar and S. Mitrovic, “ A fixed point theorem for set valued quasi contractions in b-metric spaces”, Fixed Point Theory Appl., 2012, (2012),1-8.

(2) J. Von Neumann, “Zur theorie der gesellschaftsspiele”, Math. Annalen, 100, (1928), 295–320.

(3) L. A. Zadeh, “Fuzzy sets”, Inform and Control, 8, (1965), 338-353.

(4) Lj. B. Ciric, “A generalization of Banach’s contraction principle”, Proc. Amer. Math. Soc., 45, (1974), 267-273.

(5) M. Frigon and D. O. Regan, “Fuzzy contractive maps and fuzzy fixed points”. Fuzzy Sets Syst., 129, (2002), 39-45.

(6) M. Boriceanu, M. Bota and A. Petru, “ Multivalued fractals in b-metric spaces”, Central European J. Math., 8, (2010), 367-377.

(7) S. B. Nadler, “Multivalued contraction mapping”, Pacific J. Math., 30, (1969), 475–488.

(8) S. Banach, “Sur les operations dans les ensembles abstraits et leur Application aux equations integrals", Fundam. Math., 3, (1922), 133-181.

(9) S. Heilpern, “Fuzzy mappings and fixed point theorems”, J. Math. Anal. Appl., 83, (1981), 566–569.

(10) S. Czerwik,, “Nonlinear set-valued contraction mappings in b-metric spaces”, Atti Semin. Mat. Fis. Univ. Modena, 46(2), (1998), 263-276.

(11) S. Czerwik, K. Dlutek and SL. Singh, “Round-off stability of iteration procedures for operators in b-metric spaces”, J. Natur. Phys. Sci., 11, (1997), 87-94

(12) S. Czerwik, “Contraction mappings in b-metric spaces”, Acta Math. Inform. Univ. Ostrav., 1, (1993), 5-11.

(13) SL. Singh and B. Prasad, “Some coincidence theorems and stability of iterative procedures”, Comput. Math. Appl., 55, (2008), 2512-2520.

(14) S. Phiangsungneon and P. Kumam, “Fuzzy fixed point theorems for Multivalued fuzzy contractions in b-metric spaces”, J. Nonlinear Sci. App., 8, (2015), 55-63.