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A study on the Choquet integral with respect to a capacity and its applications

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Abstract

In this paper, we consider the Choquet integrals with respect to a capacity and the Choquet integral expected utility (CEU) represented by the preference functionals. In particular, by using the CEU preference functionals, we investigate the evaluation of Korea's animal exports with the trading partners examined.

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1. Introduction

Jang [5] and Zhang-Guo-Liu [12] have studied the interval-valued Choquet integrals with respect to capacity. Recently, Wood-Jang [11] introduced some imprecise market premium functionals that are related to the interval-valued Choquet integral and Chakravarty-Kelsey [2] who analyzed risk-sharing when individuals perceive ambiguity about future events. We note that the interval-valued Choquet-integral contributed to some representations of uncertainty, for example, uncertain risks, uncertain utility, and uncertain density functions etc. (see [1, 3, 4, 8, 9, 10, 12]).

In this paper, we consider the Choquet integrals with respect to a capacity and define the Choquet expected utility with respect to a capacity. We also consider two capacities

$\nu_k (k = 1, 2)$ and define the Choquet expected utility (CEU^{ν_k}) with respect to ν_k with a country's total animal product exports with the USA for years 2010–2013. Thus, we evaluate the rate of the CEU as representing the preference of total export amounts for each animal export category between Korea and the trading partners examined in this study. By using $CEUs$, we discuss the rate of Choquet expected utility of Korea's animal exports with the trading partner the USA for years 2010–2013.

2. The Choquet expected utility

In this section, we introduce a convex capacity Choquet integral, the degree of ambiguity of capacity ν, and the Choquet expected utility. Let S be a finite set of states of nature. The set of of outcomes is denoted by X. An act is a function from S to X.

Definition 2.1. A capacity on S is real-valued function ν on the subsets of S which satisfies

$$\begin{aligned} &\text{(i)} \quad A \subset B \Rightarrow \nu(A) \leq \nu(B); \\ &\text{(ii)} \quad \nu(\emptyset) = 0, \ \nu(S) = 1; \end{aligned} \tag{1}$$

Definition 2.2.

(1) The Choquet integrals with respect to ν of a function f is defined by

$$(C)\int f d\nu = \int_0^\infty \nu_f(\alpha) d\alpha, \tag{2}$$

where $\nu_f(\alpha) = \nu(\{x \in X | f(x) \geq \alpha\})$ and the integral on the right-hand side is an ordinary one.

(2) Let $S = \{s_1, s_2, \cdots, n\}$ be a finite set. The Choquet integral of f with respect to capacity ν is defined by

$$(C)\int f(s) d\nu(s) = \sum_{i=1}^{n} f(s^i)\left[\nu(A^i) - \nu(A^{i-1})\right], \tag{3}$$

where $A^i = \nu_f(f(s^i)) = \{s \in S | f(s) \geq f(s^i)\}$. By convention, let $A^0 = \emptyset$.

(3) A measurable function f is said to be Choquet integrable if $(C)\int f(s) d\nu(s)$.

Remark that if S is a finite set, then one can order the utility from a given act $a : S \to [0, \infty)$;

$$u(a^1) > u(a^2) > \cdots > u(a^n - 1) > u(a^n), \tag{4}$$

where $u(a^1), u(a^2), \ldots, u(a^n)$ are the possible utility levels yielded by act a, that is, $u(S) = \{a^1, a^2, \cdots, a^n\}$. We note that if beliefs are represented by a capacity ν on S, the expected utility of a given act can be found using the Choquet integral(Definition 2.3) as follows.

Definition 2.3. The Choquet expected utility (CEU) of u with respect to capacity ν is defined by

$$(C)\int u(a(s))d\nu(s) = \sum_{i=1}^{n} u(a^i)\left[\nu(A^i(a) - \nu(A^{i-1}(a)\right], \tag{5}$$

where $A^i(a) = \nu_{u(a)}(u(a^i)) = \{s \in S|u(a(s)) \geq u(a^i)\}$ is the set of states that yield a utility at least as high as a^i for $i = 1, 2, \ldots, n$. By convention, let $A^0(a) = \emptyset$.

3. Some applications

In this section, we define the Choquet expected utility (CEU) by using a country's total animal product exports with the rest of the world for the years 2010–2013. In conjunction with this analysis we also examined the total export amounts for these animal products with the USA; a significant trading partner for Korea. The export products examined were categorized into five groups using the online UN Comtrade trade database ([14]) and are defined as:

1. Live animals; animal products.
2. Meat and edible meat offal.
3. Fish and crustaceans, mollusks and other aquatic invertebrates.
4. Dairy produce; birds' eggs; natural honey; edible products of animal origin, not elsewhere specified or included.
5. Products of animal origin, not elsewhere specified or included.

In order to define the utility of trade value (USD) of commodity codes as being the products weighted value against all exports, we introduce the following Table 1.

Let TS be the sum of all trade values of Table 1. Then we get $TN = 392850046$. Thus, from Table 1, we define the utilities $u_l(a^i) = \dfrac{a^i}{TS}$ of trade value (USD) of commodity codes $i = 1, 2, 3, 4, 5$ as being the products weighted value against all exports for $l = 2010, 2011, 2012, 2013$ as follows:

$$\begin{aligned} &u_{2010}(a^1) = 0.0006, u_{2010}(a^2) = 0.0007, u_{2010}(a^3) = 0.0025, \\ &u_{2010}(a^4) = 0.0956, u_{2010}(a^5) = 0.1906, \end{aligned} \tag{6}$$

$$\begin{aligned} &u_{2011}(a^1) = 0.0008, u_{2011}(a^2) = 0.0009, u_{2011}(a^3) = 0.0010, \\ &u_{2011}(a^4) = 0.0110, u_{2011}(a^5) = 0.2435, \end{aligned} \tag{7}$$

Table 1: Total animal product exports between Korea and the USA for years 2010–2013.

Year	Trade Flow	Reporter ISO	Partner	Commodity Code	Trade Value (US$)
2010	Export	KOR	USA	1	$a(1) = 286,892 = a^2$
2010	Export	KOR	USA	2	$a(2) = 997,539 = a^4$
2010	Export	KOR	USA	3	$a(3) = 74,866,073 = a^5$
2010	Export	KOR	USA	4	$a(4) = 3,722,326 = a^3$
2010	Export	KOR	USA	5	$a(5) = 235,669 = a^1$
2011	Export	KOR	USA	1	$a(1) = 330,199 = a^1$
2011	Export	KOR	USA	2	$a(2) = 376,805 = a^3$
2011	Export	KOR	USA	3	$a(3) = 95,654,573 = a^5$
2011	Export	KOR	USA	4	$a(4) = 4,323,214 = a^4$
2011	Export	KOR	USA	5	$a(5) = 359,747 = a^2$
2012	Export	KOR	USA	1	$a(1) = 358,496 = a^3$
2012	Export	KOR	USA	2	$a(2) = 30,005 = a^1$
2012	Export	KOR	USA	3	$a(3) = 100,141,401 = a^5$
2012	Export	KOR	USA	4	$a(4) = 5,016,833 = a^4$
2012	Export	KOR	USA	5	$a(5) = 101,795 = a^2$
2013	Export	KOR	USA	1	$a(1) = 364,918 = a^3$
2013	Export	KOR	USA	2	$a(2) = 272,884 = a^1$
2013	Export	KOR	USA	3	$a(3) = 99,871,717 = a^5$
2013	Export	KOR	USA	4	$a(4) = 4,910,771 = a^4$
2013	Export	KOR	USA	5	$a(5) = 863,858 = a^2$

$$u_{2012}(a^1) = 0.0001, u_{2012}(a^2) = 0.0003, u_{2012}(a^3) = 0.0009,\\ u_{2012}(a^4) = 0.0128, u_{2012}(a^5) = 0.2549, \tag{8}$$

$$u_{2013}(a^1) = 0.00079, u_{2013}(a^2) = 0.0007, u_{2013}(a^3) = 0.0022,\\ u_{2013}(a^4) = 0.0025, u_{2013}(a^5) = 0.2542, \tag{9}$$

We also define two capacities ν^1, ν^2 as follows:

$$\nu^1(\{a^5\}) = 0.1, \nu^1(\{a^4, a^5\}) = 0.2, \nu^1(\{a^3, a^4, a^5\}) = 0.3,\\ \nu^1(\{a^2, a^3, a^4, a^5\}) = 0.7, \nu^1(\{a^1, a^2, a^3, a^4, a^5\}) = 1, \tag{10}$$

and

$$\nu^2(\{a^5\}) = 0.2, \nu^2(\{a^4, a^5\}) = 0.4, \nu^2(\{a^3, a^4, a^5\}) = 0.6,\\ \nu^2(\{a^2, a^3, a^4, a^5\}) = 0.8, \nu^1(\{a^1, a^2, a^3, a^4, a^5\}) = 1. \tag{11}$$

By using Definition 2.3, we define the $\mathrm{CEU}_l^{\nu^k}$ $(k = 1, 2)$ as representing the preference of total export amounts for each animal export category between Korea and its trading partner the USA for years $l = 2010, 2011, 2012, 2013$, where l is a year as following.

Definition 3.1. The Choquet expected utility $\mathrm{CEU}_l^{\nu^k}$ $(k = 1, 2)$ of u_l with respect to capacity ν^k is defined by

$$\mathrm{CEU}_l^{\nu^k} = \sum_{i=1}^{n} u_l(a^i) \left[\nu^k(A^i(a) - \nu(A^{i-1}(a)\right], \tag{12}$$

where $A^i(a) = \nu^k u_l(a)(u(a^i)) = \{s \in S | u_l(a(s)) \geq u_l(a^i)\}$ is the set of states that yield a utility at least as high as a^i. By convention, let $A^0(a) = \emptyset$.

Then, from Definition 3.1, we get

$$CEU_l^{\nu_1} = 0.1u_l(a^5) + 0.1u_l(a^4) + 0.1u_l(a^3) + 0.4u_l(a^2) + 0.3u_l(a^1), \tag{13}$$

and

$$CEU_l^{\nu_2} = 0.2\left(u_2(a^5) + u_2(a^4) + u_2(a^3) + u_2(a^2) + u_2(a^1)\right). \tag{14}$$

From (11), we get

$$\begin{aligned} CEU_2010^{\nu_1} &= 0.02072, CEU_2011^{\nu_1} = 0.02615, CEU_2012^{\nu_1} = 0.02697, \\ CEU_2013^{\nu_1} &= 0.02746, \end{aligned} \tag{15}$$

and

$$\begin{aligned} CEU_2010^{\nu_2} &= 0.04078, CEU_2011^{\nu_2} = 0.051144, CEU_2012^{\nu_2} = 0.05838, \\ CEU_2013^{\nu_2} &= 0.0541. \end{aligned} \tag{16}$$

Then we have the rate of the utility $CEU_l^{\nu^k}$ against $CEU_{l-1}^{\nu^k}$ for $k = 1, 2$ and $l = 2010, 2011, 2012, 2013$ as follows:

$$\frac{CEU_{2013}^{\nu^1}}{CUE_{2012}^{\nu^1}} = 1.0182 > 0.9267 = \frac{CEU_{2013}^{\nu^2}}{CUE_{2012}^{\nu^2}}, \tag{17}$$

$$\frac{CEU_{2012}^{\nu^1}}{CUE_{2011}^{\nu^1}} = 1.03136 < 1.1349 = \frac{CEU_{2012}^{\nu^2}}{CUE_{2011}^{\nu^2}}, \tag{18}$$

$$\frac{CEU_{2011}^{\nu^1}}{CUE_{2010}^{\nu^1}} = 1.2621 > 1.2614 = \frac{CEU_{2011}^{\nu^2}}{UE_{2010}^{\nu^2}}. \tag{19}$$

4. Conclusions

In this paper, by using the Choquet integral (see Definition 2.2), we obtained a new usage for the Choquet expected utility (see Definition 2.3). This CEU provides a useful tool for decision making purposes. We also considered two capacities which are different set functions. Firstly, a subjective measure (see Eq. (10)) that sees both the total volume of trade and the total number of traded products as being important for decision making purposes as each of the 5 animal products used in this study may have different levels of perceived importance. And secondly, an objective measure (see Eq. (11)) in which the country is only interested in the total trade volume it has with a particular trading partner because in this instance the weighted importance for each of the five products is the same.

In section 3, we obtained the Choquet expected utility (CEU^{ν_k}) with respect to ν_k with a country's total animal product exports from the USA for years 2010–2013 (see Definition 3.1). In particular, we gave the rate of the utility $CEU_l^{\nu^k}$ against $CEU_{l-1}^{\nu^k}$ for $k = 1, 2$ and $l = 2010, 2011, 2012, 2013$ (see Eq.s (17), (18), (19)).

In the future, we will study the interval-valued Choquet integrals with respect to a capacity of uncertain utility which is an interval-valued function. By using the uncertain utility and the interval-valued Choquet integral we can define the interval-valued Choquet expected utility.

From the interval-valued Choquet expected utility, we will discuss axioms for representing preferences by them and investigate some applications of the interval-valued Choquet expected utility. Our question will be, are there some useful applications of the interval valued Choquet expected utility (ICEU) with a country's total animal product exports with the rest of the world over a four year period?

Conflicts of interest

The authors declare that they have no Conflicts of interest.

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