

Structure and mechanisms of developing mathematical abilities of schoolchildren

Maret Balaudinovna Visitaeva and Achmed Magomedovich Gachaev

*Chechen Institute for Advanced Training of Education Workers
364037, Grozny, Butirina Str., 30, Russia
Chechen state university
364037, Grozny, Scheripova Str., 32, Russia*

Abstract

An attempt has been made to find out the structure of mathematical abilities of schoolchildren and phenomena aimed at their development, conducive to the development of the student's personality at large; the complex character of the mathematical activity is pointed out. *It is highlighted that* the skills development level based on the natural abilities (including mathematical ones) depends on the environment they find themselves in. The R. Sternberg's triarchic theory of successful intelligence is presented and its relation to the specific directions (cultural, applied, creative) adequate to the national educational system, that need to be considered when time comes for a student to choose profession. The author describes in detail abilities (including mathematical ones) which reveal, form and develop in the process of properly organized activities; the problem solution is considered in the article as a type of mathematical activity. The dependence of student natural inclinations' direction on created conditions adequate to the choice of his/her future profession is revealed. Prospects of further researches on the given problem (special abilities) are possibility to explore it in other subject fields such as physics, chemistry, geography etc.

Keywords: Structure of students' mathematical abilities; natural abilities; activity; development; R. Sternberg's theory of successful intelligence; cultural, applied and creative directions; national education system; choice of profession; environment.

Introduction

The questions of research of the student's personality structure, in the context of development of his mathematical abilities (MA) (Gusev, 2003), (Druzhinin, 2007),

(Krutetsky, 1968), (Lefransois, 2005), (Metelsky, 1977), (Sternberg, 1994), (Visitaeva, 2013), (Visitaeva, 2014 a), (Visitaeva, 2014 b), (Visitaeva and Vakilov, 2012), (Chudnovsky, 2006 etc) are the most topical in the modern paradigm of education. Relation of the abilities development level (including mathematical) to natural faculties (Visitaeva, 2013); dependence of the level of development of the abilities on the adequateness of their environment are under discussion (Sarantsev, 2012). The R. Sternberg's (Lefransois, 2005), (Sternberg, 1994) triarchic theory of successful intelligence is presented and its relation to the specific directions (cultural, applied, creative) (Boltyansky *et al.*, 1993).

The author is the first to present in the article relation of R. Sternberg's triarchic theory of successful intelligence and earlier revealed (by other scientists) MA and peculiarities of directions of mathematical education (cultural, applied, creative).

The paper is aimed to reveal relation of student's inclinations and conditions which are the most optimal for development of student's MA (as far as development of his/her personality), adequate choice of his/her future profession that fit in multilevel content of teaching.

Methods

We have used the following methods in the considered context: examination and analysis of the psychological, pedagogical and methodical literature on the problem of research, state educational standards, teaching programs, secondary school textbooks on mathematics; generalization of pedagogical experience of teachers of maths; supervision of the learning process, conduction of pedagogical measuring (questionnaires, interviews, analysis of the products of schoolchildren's activities) and statistical treatment of the results.

Requisites making up the **methodological foundation** of the research are given below:

- activity approach (V.V. Davydov, A.K. Markova (Davydov and Markova, 1981), S.L. Rubinstein (Rubinstein, 2008), G.I. Sarantsev (Sarantsev, 2012), C. Rogers (Rogers, 1983) etc);
- the structure of mathematical abilities in schoolchildren, and mechanisms of their formation and developing of psychologists (B.B. Aysmontas (Aysmontas, 2006), E.A. Golubeva (Golubeva, 2005), V.A. Krutetsky (Krutetsky, 1968), Guy Lefransois (Lefransois, 2005), R.Y. Sternberg (Sternberg, 1994), L.V. Cheremushkina, T.N. Asinine (Cheremushkina and Asinine, 2012), V.E. Chudnovsky (Chudnovsky, 2006), L.B. Schneider (Schneider, 2013), I.S. Yakimanskaya (Yakimanskaya, 2004) etc); mathematicians Methodists (V.A. Gusev (Gusev, 2003) etc);
- Methodological framework for the development of modern mathematical education in line with the following lines of development: liberalization and humanization of mathematical education, student-oriented learning mathematics (V.G. Boltyansky, G.D. Glaser, R.S. Cherkasov (Boltyansky *et al.*, 1993), V.A. Gusev (Gusev, 2003), N.V. Metelsky (Metelsky, 1977), G.I. Sarantsev (Sarantsev, 2012), I.S. Yakimanskaya (Yakimanskaya, 2004) etc).

The research was conducted between 2004 and 2014; its theoretical basis have been formed during courses at the Academy of Skills Upgrade and Professional Retraining of Educational workers (Moskov, 2005) joint UNESCO and Russian federation project «SUPPORT FOR THE RESTORATION AND DEVELOPMENT OF EDUCATION SYSTEM OF THE CHECHEN REPUBLIK».

The monitoring of individual dynamics and learners' progress based on the results of evaluation conducted in the classroom, which served the basis for creation of their individual educational path, has been carried out during the research.

In the process of education the teacher was not (according to C. Rogers (Rogers, 1983)) a deliverer of knowledge, but a facilitator (assistant).

Discussion and results

Natural inclinations as pre-condition for the development of abilities

The challenges related to development of abilities are tightly bound with the challenges linked to cultivating interest in different types of activities. It is on the basis of a long, deep and persistent interest in a specific subject that inclinations and abilities are developed. (A.A. Bodalev, 1984); V.A. Krutetsky (Krutetsky, 1968); in a number of works by S.L. Rubinstein and others).

Alongside with general abilities (needed to perform not just one but many types of activities, which provides for a person's capacity to achieve success simultaneously in different spheres) psychologists distinguish special abilities-abilities to carry out certain types of activities (Rubinstein, 2008), (Golubeva, 2003), (Druzhinin, 2007) and others). They can be defined as individual psychological peculiarities of a person corresponding to the requirements of this activity and conditioning its successful performance.

In psychology mathematical abilities imply individual psychological peculiarities of a person providing for successful performance of mathematical activity.

As practice shows, different kinds of special abilities may overlap "a more detailed qualitative group-by-group analysis of the students" of a mathematical form showed that at least half of them have significant literary abilities alongside with mathematical aptitude" (Golubeva, 2005, p. 327). A good example of this is S.V. Kovalevskaya, a talented mathematician, the winner of prizes in Paris and Sweden for her "Problem of rotation of a rigid body around a fixed point", the author of nine research works in mathematics, corresponding member of the Russian Academy of Science, also world-known for her fiction writing.

The teaching practice and research in the context of the observed problem show that abilities, including mathematical ones, are formed on the basis of native aptitudes (natural gifts) and whether these abilities find the necessary environment or not determine the level of their development in future. To the point, two kinds of natural faculties are defined: "first, faculties that have no specific orientation, like strength/weakness of nervous system, and second, aptitudes having narrow orientations" (Chudnovsky, 2006, p. 114), to which we can also refer mathematical aptitude.

An English philosopher Francis Bacon pointed out that “they are happy men whose natures sort with their vocations”. It seems that S.L. Rubinstein dwelled on the idea: “forming on the basis of instincts, abilities are still not the function of the instincts as they are, but of the development where natural gifts are a background, a pre-condition. Taking part in the development of an individual they develop themselves ...” (Rubinstein, 2008, p. 536) ...” (Rubinstein, 2008, p. 536).

The complex nature of abilities

The researches of psychologists (A. Blackwell, E.R. Dunkan, K. Dounker, V. Haecker, H.R. Hemley, V.A. Krutetsky, F.W. Mitchell, I.S. Yakimanskaya and others), mathematicians (Zh. Adamar, A. Puankare, A.N. Kolmogorov, A.I. Markushevich, A.Ya. Khinchin, etc.) and methodologists (V.A. Gusev, G.I. Sarantsev, S.I. Schwarzburd, etc) play an important role in understanding of the structure and mechanisms of developing “mathematical thinking”. Analysis of the works on the problems of structure and development of mathematical abilities of the mentioned authors and many other researchers suggests a complex nature of mathematical activity.

According to the mathematical abilities structure consists of the following (Krutetsky, 1968):

1. Receipt of mathematical information
2. Processing of mathematical information
3. Storage of mathematical information
4. General synthetic component.

The highlighted aspects are tightly bound and form together a united system, an integral structure, a kind of mathematical aptitude syndrome of a mathematical mindset.

Researchers of psychological and pedagogical fundamentals of teaching mathematics point to a number of deficiencies in the abovementioned scheme of mathematical abilities’ structure. They are, in particular, *abstract thinking ability*, *mathematical intuition*.

Mathematical intuition is the ability of a mathematician to put forward a hypothesis, quickly see the way to an optimal solution of a problem. Mathematical intuition is considered to be a complex ability “to anticipate results or the ways leading to goals in creative thinking in the mathematical field” (Metelsky, 1977, p. 41). French mathematician G. Adamar (1970), also spoke about a specific mathematical intuition inherent in mathematicians, about subconscious creative work and about specific character of mathematical thinking.

Isolation of the abovementioned (psychological) approaches to working out reasonably clear notions about the structure and development of “mathematical thinking” from mathematical methods of research is a significant drawback.

Various authors consider different structural and functional elements of the latter as the most important. Such general operations of thinking as comparison, deduction, analysis and synthesis are given an obligatory status. The following abilities may be called specifically mathematical: 1) manipulation of spatial objects, ability to imagine

spatially; 2) manipulation of ideas and notions in an abstract form without concrete support; 3) classification; 4) understanding of symbols and using them; 5) intellectual curiosity; 6) memory, extraordinary memory; 7) strong visualization; 8) ability to reason logically; 9) ability to apply knowledge to a new situation; 10) quick thinking; 11) ability to find similarities in different spheres; 12) dominance of logical scheme of reasoning; 13) inductive and deductive thinking skills; 14) exactness, brevity and clearness of the verbal expression of thought, quite strong skills of mathematical speech, habit of complete argumentation, etc.

The structure of mathematical abilities contains memory which is defined by Guy Lefrancois as the “results of the influence of experience allegedly practiced upon our intelligence. The term stands for the storage of the results of this influence” (Lefrancois, 2005, p. 131). The new researches (Cheremushkina and Asinine, 2012) show that memory (recollection) is not just a photograph, it is a dynamic process, and according to F. Barlette, recollection is an active process which is not just a reproduction but reconstruction of past experience in relation to new tasks” (Schneider, 2013, p. 158). Still earlier, in her publications starting since 1985, based on researches of S.L. Rubinstein and others, I.S. Yakimanskaya points out that “creation of an image begins with perception (or with reproduction in memory) of the three drawing views (front, top and left-side views)... image of an object created according to a drawing is not the image of the drawing itself (of its three projections), it is not a simple sum, it doesn't appear as a result of overlapping of three views (similarly to the principle of photograph) ...The same happens when creation of the image is based on a picture, scheme, graphic, etc that is on the basis of any visuals” (Yakimanskaya, 2004, p. 91).

Let us point out the existence of this thesis despite the opinion reflected in the psychological and pedagogical literature where “memory” is considered not as a dynamic, but reproductive process mainly. It is interesting (not only for the theoretical, but for the practical teaching of mathematics in particular) to note in this regard the data of American researchers that allow regulating the process of memorizing: relation of memorizing information to its position on the blackboard (in the upper left corner-28%, in the lower left corner-16%, in the upper right corner-33% and in the lower right corner-23%) (Aysmontas, 2006, p. 63).

R. Sternberg's theory of successful intelligence

Returning to R. Sternberg's triarchic theory of successful intelligence. (Lefrancois, 2005), (Sternberg, 1994) *analytical, creative and practical skills* are to be mentioned here. Successful intelligence requires strong skills of the mentioned types.

Analytical skills are necessary for selection and evaluation of variants, follow-up of one's rises and falls and for working out a strategy. Analytical skills comprise *judgment, estimation, contraposition, comparison, analysis, etc.*

Creative skills allow an individual to generate ideas and test new methods of selecting and forming environment and adapting to it.

Creative skills manifest themselves in such activities as *discovery, imagination, invention, forecasting, etc.*

Practical skills allow an individual to realize chosen options and implement skills and forms of behavior that are part of the process of selection and formation of the

environment and adaptation to it. Practical skills often require tacit knowledge, and here, it should be noted that what is difficult to express verbally, may be sometimes better explained through visual tools (drawing, painting etc).

Successful intelligence requires: establishing balance between selection of conditions of the environment, forming these conditions and adaptation to them; ability to evaluate one's own strong and weak point. Or, which happens more often, people simply *choose such environmental conditions under which their weaknesses are hardly perceptible or they adjust some aspects of the surroundings.*

It is necessary to consider strong and weak points of the learner in the process of training. Activities aimed at selection, forming and adjusting environment and adaptation to it are topical in terms of vocational training. R. Sternberg's model of successful intelligence has a special value for the educational sphere: it highlights the fact that *intelligence is an information processing activity.*

All the above-mentioned confirms advisability of rethinking the problems of improvement of the students' mathematical skills in the process of studying mathematics in the context of the present-day educational approaches to the analysis of functioning of learning activities regulation mechanisms and developing a unified theoretical position in relation to the possibilities of solving this problem. Some aspects of the structure, development and forming (including appropriate conditions), evaluation of mathematical abilities of students are described in the works (Golubeva, 2005), (Gusev, 2003), (Druzhinin, 2007), (Krutetsky, 1968), (Lefransois, 2005), (Metelsky, 1977), (Sternberg, 1994), (Visitaeva, 2013), (Visitaeva, 2014 a), (Visitaeva, 2014 b), (Visitaeva and Vakilov, 2012), (Cheremushkina and Asinine, 2012), (Chudnovsky, 2006), (Schneider, 2013), (Yakimanskaya, 2004 etc).

Thus, in our view, the work on forming students' mathematical skills should be conducted in two directions: *1) creating background within the process of learning needed for cultivating interest for mathematics; 2) work with the students who show interest and skills.*

The concept of general mathematical education

It is advisable for the schoolchildren "with whom mathematics is just a part of general education and who will implement mathematics in their future profession to a little extend" (Boltyansky *et al.*, 1993, p. 235) that they should master a general mathematical culture. For example, different representatives of culture. "Students for whom mathematics will play an important role in their future profession, and who will regularly implemented it solving operational issues" will comprise the second group [same source]. They are, for example, economical experts. It is necessary *to create preconditions within the process of learning of these two groups needed to arouse interest to mathematics.* The second direction is comprised of the students belonging to the third group, "those, who will choose mathematics (or fields of knowledge close to it) as a basis for their future occupation. The students of this group show high interest to studying maths and must creatively learn its fundamentals" (Boltyansky *et al.*, 1993, p. 236). Of course, the types of abilities determined by R. Sternberg may be found in each of the direction, for example, creativity, allowing a student to transform ideas and implement new methods of selection, forming of the environment and

adaptation to it, is necessary for the group of students showing interest to studying mathematics and must be creative in learning its fundamentals.

Promoting the concept of general mathematical education the authors (Boltyansky *et al.*, 1993) point out that the levels of knowledge of mathematics of these three groups may be relatively classified according to the three following levels: cultural, applied and creative, and speak about the necessity to create three mathematics textbooks corresponding to these levels. They have recently started to publish mathematical textbooks and manuals addressed to definite categories of students (for example, a set of teaching materials by A.G. Mordkovich, applying the concept of multistage scheme of school course of mathematics for the principal and senior schools; or “Geometry for 10-11 forms” (Smirnova and Smirnov, 2008) (Base and profile levels) by I.M. Smirnova, V.A. Smirnov). Of course, the levels presenting the concept of general mathematical education may be modified and refined according to the condition of the national mathematical education system, labour market demands, etc.

Development of the individual student software multilevel content of teaching mathematics

The system of geometric problems allows a teacher to reveal the process of student’s work on creation of a geometric image, drawing in particular. To make a drawing for some students may become a starting point for solving a problem and for the others it may become a result of transformations carried out in mind (Visitaeva and Vakilov, 2012). We should point out in this regard that working on one and the same material one student would successfully use the verbal form, the other-would transform it into a visual form (a drawing, a draft, etc), and we have produced a complex of problems in this connection.

Abilities, including mathematical ones, show up, reveal and develop in the process of intendedly and correctly arranged activities, and solving problems is one of the kinds of mathematical activities. Practical experience is for us the main criterion to find out whether or not the students have mathematical abilities. Examining the issue of teaching mathematics and developing the students through problems (V.A. Gusev, M.I. Zaikin, S.E. Kanin, Yu.M. Kolyagin, M.I. Rodionov, G.I. Sarantsev, L.M. Friedmann, P.M. Erdniev, B.P. Erdniev, and others) we have come to understand that to develop his individuality the student must be engaged into this activity through different mathematical problems.

In the considered research context, a pupil starts his way to learning through “discoveries” to himself, structuring and then applying what was dynamically perceived at the previous stage to solving the problems the life sets in front of him. It should be highlighted that each activity is “bipolar: directed to creation of objective values and to develop the individual himself” (Yakimanskaya, 2004, p. 91). We assume that each activity is bipolar: directed to creation of objective values, and to develop the individual himself, *including their dynamic perception*. O.V. Gushin (2004) points out that the dynamic perception is an attempt to see not the object as it is, but perception itself in its internal dynamics, that constantly refreshes objects located within his sight and perceived by him.

We should point out in this regard that working on one and the same material one student would successfully use the verbal form, the other would transform it into a visual form (a drawing, a draft, etc), we have produced a complex of problems in this connection. According to G.I. Sarantsev “writing problems with the use of a prepared picture doesn’t only improve students’ ability to work with a problem, prove, but is also a good tool of their intellectual development” (Sarantsev, 2012, p. 65). It is known that the system of geometric problems allows a teacher to reveal the process of a pupil’s work on a geometric image, a draft in particular. Making a draft for some students may be a starting point for solving a problem and for the others it may become a result of transformations carried out in mind.

Of course, a young person will have to learn to develop his abilities independently, search, select, analyze and use correct information, have high communication skills, developed abilities of constructive interrelation with people of different views and values. He must also have positive attitude towards innovation, leadership skills, and ability to create, collaborate, make non-standard decisions, overcome out-of-date stereotypes of thinking, behaving and communicating (<http://www.standart.edu.ru/>, 2012). Thus, where his natural talent will be directed and whether it will be implemented in the activity that favours its manifestation most depends on whether the conditions created to meet the choice of his profession in future are adequate to this choice. It is likely that these conditions include ensuring various levels of learning contents, mathematics in particular, conducive to development of MA and students’ personality in general.

It is also important to consider how much the environment the teacher creates during a lesson and outside of it is conducive to the development of a child’s capabilities, how it ensures the self-actualization of his personal potential and urges to reach his own results in education. Creation of such dynamic environment is achieved, in particular, through a scientifically grounded use of problems in mathematical studies (Visitaeva and Vakilov, 2012).

Thus, directions (created taking into account the abovementioned substantive characteristics and presenting the concept of general mathematical education) may be modified and specified according to the situation with the national system of mathematical education, labour market demands, etc. (e.g., a schoolchild in Israel can choose the level of the Unified State Examination (USE) he will take; this level is surely correlated with his right to choose an appropriate high school). For the first time will be (2014-2015 school year) a two-level certification exam on mathematics (basic and vocational levels) and in Russian schools.

Experience in experimental work

Some aspects of theoretical grounds of the experiment

More than 10 years of experience in the experimental examination of development of schoolchildren’s MA has shown feasibility of their MA development, in particular improvement of their geometrical vision (to visually embrace the drawing, mentally transport and reconstruct its elements), improvement of flexibility of their thinking processes (diversity of aspects in solving problems), switching over from the direct

flow of thoughts to a reverse inductive and deductive ways of solving problems, easy orientation under new conditions, etc.

Moreover, the students of forms where the mentioned methods were implemented differ from the others by their developed spatial representations (“ideas of form, position, size, direction and other spatial relations of real world objects”) and spatial imagination (ability to manipulate imaginary objects), which produces positive influence upon studying of joint subjects (geography, physics, astronomy, etc).

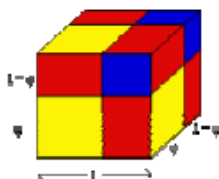
Didactic value of problem solutions based on use of visual methods expresses in the fact of their promotion of students’ aimed teaching to methods of work modification analysis (drawing, real model etc) , its transformation: including one and the same element in various figures, reexamination of drawing’s elements, verbal description of the situation (drawn at the picture, drawing) basing on perception. In the process of development spatial representations and spatial imagination valuable role has verbal description of a situation, not based on perception.

Practical experiment

Let’s take some problem, taken in the process of my experimental work.

Problem 1.

Look attentively at the picture 1. How many cubes are there in the picture?



Pic. 1

Visual configurations, presented in the picture 2 could be a source for making a problem.

Visual configurations, presented in the picture 2 could be a source for making a problem.

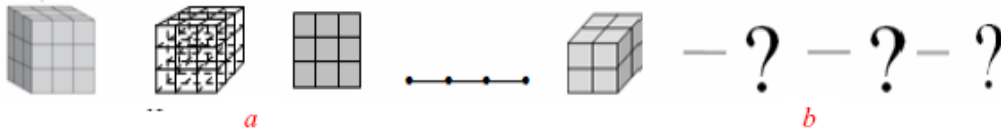
Problem 2.

- a) *explain the situation, presented in the picture 2/*
- b) *make up several problems, conditions for which you could see in the picture 2, problem 3 could act as an example (problems 1, 3, 4 were presented to the school students of the middle classes), problem 5 was given to the school students of the higher classes, while problem 2 could be modified for students of all ages depending on subject content).*

Problem 3

(for imagination development, based on perception).

1. Look attentively at picture 2a. How many interpenetrating figures are drawn there in every figure?
2. Think and explain the way the first four figures connected (pic. 2a), build the corresponding three figures connected with the first one in the same way (pic. 2b).



Pic. 2

Problem 4

(for imagination development not based on perception).

The length of an edge of painted wooden cube is 2, 3, 4, 5, 6 cm. The cube was sawn to cubes with the edge length 1 cm. How many cubes were made up which had 3, 2, 1, 0 painted face(s)?

This problem connected with some other problems. We can make up these problems if we make it clear in the text that some parts of the big cube were painted before sawing

- 1) One cube's face
- 2) Two adjacent cube's face
- 3) Two opposite faces
- 4) Three faces with common corner
- 5) Four faces, three out of which have common corner
- 6) Four faces, three out of which have no common corner
- 7) Five faces

In the given problem we implement open approach in teaching geometry, the problem where can be received several results has several questions.

The given problems could be a beginning of students' research activity being modified and detailed in various ways.

Problem 5.

- a) Has the equation $x^2 - 25y^2 = 13$ a solution in integers?
- b) draw a curve line specified by the equation; b) make up and solve similar problem.

In accordance with the concept of general mathematic education (2.4): problem 1 can be referred to cultural level, problem 2-to applied level, problems 3 and 5-to creative level (other variations are also possible, it depends on conditions, for example problem 4 could be referred to both applied level and to creative one) of the mathematic education.

We generalized definition 'interpenetrating figures' (given by I.S Yakimanskaya for flat two-dimensional figures) as for flat (one-dimensional and two-dimensional) so for

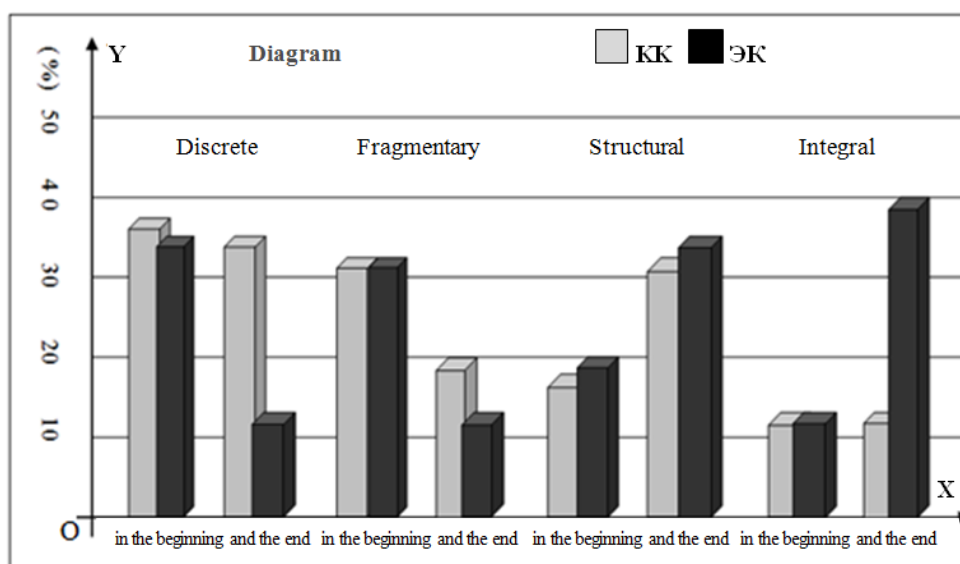
spatial (three-dimensional) figures. ‘Interpenetrating figures’ are figures which have part of common length (area or volume): by some of their parts they overlay each other, by others they do not match.

Presentation of research results

Let’s present criterion of middle class students MA development, which has four following levels: discrete, fragmentary, structural, integral (comprehensive criterion of students’ MA development basing on the four levels is grounded and described in source [18].

- discrete level (K_1); student’s MA are developed, if he/she can make definite mathematic operations without any succession, recognizes earlier perceived example of actions using algorithmic order;
- fragmentary level (K_2); student can implement in general all mathematic operations, but he/she is not fully conscious of his/her actions. He/she implements actions in standard problem situation basing on obvious examples, not following any algorithm.
- structural level (K_3); student’s MA are developed if she/he is able to show corresponding mathematic abilities in a new nonstandard problem situation using earlier presented examples.
- integral level (K_4); if a student can present developed MA in nonstandard problem situation not using any obvious examples.

The result of implemented experimental work is presented in the picture 3, where axis OX has levels of student’s MA development in the beginning and the end of study in the experimental and control classes; and axis OY has their per cent data.



Pic. 3. Dynamics levels of student’s MA development in the beginning and the end of study in the experimental and control classes.

Conclusion

1. Realization is needed of natural abilities in the activities that are favouring their exercise best, and at the same time it is necessary to create conditions (adequate to the choice of his/her profession in future), providing learning material of different levels.
2. It is necessary to organize multilevel material, and create in this regard, within the learning process preconditions offering an incentive of interest in mathematics as well as ensure a proper direction of work with the students showing interest and abilities.
3. The types of abilities defined by R. Sternberg may be found in each of the presented directions (creative abilities are needed, allowing a schoolchild to transform ideas and test new ways of selection and forming of the environment and adaptation to it, etc).
4. The conducted research may be implemented in practical training of mathematics in the national system of secondary education, in working out learning material, including its diagnostics, etc.

REFERENCES

1. Aysmontas, B.B. (2006). *Educational psychology: circuit tests*. Moscow: VLADOS PRESS.
2. Boltyansky, V.G., G.D. Glaser, R.S. Cherkasov. (1993). On restructuring of general mathematical education. Improving of mathematical training at school. *Improving the teaching of mathematics in school: Bk. Teacher Teacher*. Comp. G.D. Glaser. Metelsky, N.V. (pp. 231-238). Moscow: Education.
3. Federal state educational standard of medium (complete) general education (Approved by the Ministry of Education and Science of the Russian Federation of the RF on 17.05.2012. # 413) / <http://www.standart.edu.ru/>
4. Golubeva, E.A. (2005). *Ability. Personality. Individuality*. Dubna: Phoenix.
5. Gusev, V.A. (2003). *Psychological and pedagogical foundations of teaching mathematics*. Moscow: IzDAT. center "Academy".
6. Davydov, V.V., A.K. Markova. (1981). The concept of learning activities of students. *Questions of psychology*, 6: (13-26).
7. Druzhinin, V.N. (2007). *Psychology of general abilities*. Masters Psychology. 3 ed.-St. Petersburg.-Peter.
8. Krutetsky, V.A. (1968). *Psychology of mathematical abilities of students*. Moscow: Education.
9. Lefrancois Guy. (2000/2005). *Pedagogical psycho-techniques* (V. Volochoncki, N. Mironov, C. Rysev, A. Tunik, Trans.). St. Petersburg: PRAYM, EURO-SIGN (The main textbook of the project).
10. Metelsky, N.V. (1977). *Psychological and pedagogical foundations of mathematics*. Minsk: Graduate School.
11. Rogers, C. *Freedom to learn for the 80' s*. (1983): Columbus-Totonto-Sydney: Ch. E. Merril Publ. Company, A. Bell & Howell Company.

12. Rubinstein, S.L. (2008). *Fundamentals of general psychology*. St. Petersburg: Peter.
13. Sarantsev, G.I. (2012). *Methodology of teaching mathematics: methodology and theory*. Kazan: Centre of innovative technologies.
14. Sternberg, R.Y. (1994). *Encyclopedia of human intelligence*. New York: Macmillan.
15. Smirnov, I.M., Smirnov, V.A. (2008). *Textbook: Geometry. 10-11 class: for students of general institutions (Base and profile levels)-5th ed., Corr. and supplementary*. Moscow: Mnemosyne.
16. Visitaev, M.B. (2013). The structure of conception "mathematical abilities" // *XXXIII International Scientific and Practical Conference "Trends in the development of science,"* May 16-17, 2013.-Ukraine, Horlivka. Academy of Sciences, 104-107. [http://conference.org/load/2013/xxxiii ... naucho ... nauki/55](http://conference.org/load/2013/xxxiii...nauchno...nauki/55)
17. Visitaeva, M.B. (2014 A) How to Develop Multiform Geometric Vision in Schoolchildren. *Profil'naya shkola*. V. 2, I. 1: (p. 13-21). DOI: 10.12737/3075.
18. Visitaeva, M.B. (2014 b). Assessing the development of mathematical abilities of students. *Pedagogy*, 4: (53-57).
19. Visitaeva, M.B. Sch.M. Vakilov. (2011/2012). *Methodological aspects of formation of mathematical abilities of pupils of 5-6 classes in the study of geometrical material: (Textbook)*. Education and Science of the Russian Federation, Medical University "Dagestan State Pedagogical University". (2nd ed.). Ext. Makhachkala: Aleph.
20. Cheremushkina, L.V., T.N. Asinine. (2012). *Memory: the laws of reproduction of educational material*. Moscow: Prometheus.
21. Chudnovsky, V.E. (2006). On three scientific ideas of B.M. Teplov. *Questions of psychology*, 6: (112-119).
22. Schneider, L.B. (2013). Psychology of memory: new research playback educational material. *Questions of psychology*, 4: (158-160).
23. Yakimanskaya, I.S. (2004). *Psychological bases of mathematical education: Moscow: Izdat. center "Academy"*.

