Boundary value problems with boundary jumps for singularly perturbed differential equations of conditionally stable type in the critical case

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Abstract:

In this article, we construct a solution of a singularly perturbed boundary value problem for a differential equation of conditionally stable type in the critical case. The issues of the limiting transition solution of the perturbed problem to the solution of the unperturbed problem as the small parameter approaches zero and the existence of the phenomenon of the boundary jump have been investigated. Analytical representation of the solution of the perturbed problem has been found using introduced initial and boundary functions. At the same time formulas for the boundary jumps, the orders of the jumps, asymptotic estimates for the solution of the considered boundary value problem have been found.

Keywords: asymptotic behavior, initial and boundary functions, boundary value problem, boundary jumps, degenerate problem, passage to the limit.

1.   Introduction

First studies devoted to the phenomena of the initial jumps of solutions of nonlinear singularly perturbed initial value problems with unbounded initial values aspiring of the small parameter to zero are the works of Vishik and Lyusternik [1], and of Kasymov [2]. These studies were continued in [3,4].

The phenomenon of the jump in applied problems is an important factor, which should be taken into account at the replacement of the perturbed problem by more simplified degenerate problem. The magnitude of the jump makes it possible to determine the range of the simplified problem applicability.

For example, new rationale of Painlevé paradox and the origin of jump phenomenon were proposed in the works of Neumark and Smirnov [5].

Mathematically, the jump phenomenon was investigated in [6-8]. In the papers [9-12] more general classes of singularly perturbed boundary value problems for linear differential equations of the n-th order, having a phenomenon of the initial jump under the condition of stability of rest points of the joint equation have been investigated.

The given condition consists in that the coefficient under the $n-1-$ derivative is different from zero on the interval [0,1]. In this case, phenomenon of the initial jump occurs only at one end of the segment [0,1].

In this paper, we will consider somewhat different problem for a singularly perturbed equation of the third order and it is not containing the second derivative.

Thus, condition for the stability of the joint equation's rest point is not performed for the considered equation.

2.   Statement of the Problem.

Thus, we consider a singularly perturbed boundary value problem

$$L_\varepsilon y \equiv \varepsilon y''' + B(t)y' + C(t)y = F(t),\tag{1}$$

$$y(0,\varepsilon) = a_0, \quad y'(0,\varepsilon) = a_1, \qquad y(1,\varepsilon) = b_0,\tag{2}$$

where $\varepsilon > 0$ is a small parameter, a_1, a_0, b_0 – are known constants.

Assume that:

(a) $B(t), C(t) \in C^3([0,1])$, $F(t) \in C^1([0,1])$;

(b) *a constant γ independent of ε, such that*

$$B(t) \le \gamma < 0, \ t \in [0,1];\tag{3}$$

(c) *we have the inequalities:*

$$a_1 B(0) + a_0 C(0) - F(0) \ne 0;$$

$$b_0 - a_0 \exp\left(-\int_0^1 \frac{C(x)}{B(x)}dx\right) - \int_0^1 \frac{F(s)}{B(s)}\exp\left(-\int_s^1 \frac{C(x)}{B(x)}dx\right)ds \ne 0.$$

Under condition (c), we obtain that additional characteristic equation $\mu^3 + B(t)\mu = 0$ has roots: $\mu_1 = 0$, $\mu_2 = -\sqrt{-B(t)}$, $\mu_3 = \sqrt{-B(t)}$.

This case is said to be conditionally stable of the critical case. Characteristic feature of this case is the presence of jumps simultaneously at the points: $t = 0, \ t = 1$.

Our aim is to establish the asymptotic estimates of solutions of singularly perturbed boundary value problem (1), (2), to prove the existence of the phenomenon of boundary jumps, determine the order of the boundary of the jump in the points $t = 0$ and $t = 1$, investigate the asymptotic behavior of the solution of the boundary value problem (1), (2) as $\varepsilon \to 0$.

The study will be conducted by a certain rule. In the first stage, on the basis of auxiliary functions we will construct the solution of the problem (1), (2). In the next step there will be investigated the asymptotic behavior of the solution of the problem (1), (2), phenomena of boundary jumps.

3. Fundamental Solution System.

At the same time with (1), we consider the homogeneous equation

$$L_\varepsilon y \equiv \varepsilon y''' + B(t) y' + C(t) y = 0. \tag{4}$$

Lemma 1. If *conditions (a) and (b) are satisfied, then, for sufficiently small* $\varepsilon > 0$, *the fundamental solution system* $y_i(t,\varepsilon), \quad i = \overline{1,3}$ *of the singularly perturbed Eq. (4) permits the asymptotic representations:*

$$y_1^{(q)}(t,\varepsilon) = u_1^{(q)}(t) + O(\varepsilon),$$

$$y_2^{(q)}(t,\varepsilon) = \frac{1}{\sqrt{\varepsilon}^q} e^{\frac{1}{\sqrt{\varepsilon}} \int_0^t \mu_2(x)dx} [u_2(t)\mu_1^q(t) + O(\sqrt{\varepsilon})], \tag{5}$$

$$y_3^{(q)}(t,\varepsilon) = \frac{1}{\sqrt{\varepsilon}^q} e^{-\frac{1}{\sqrt{\varepsilon}} \int_t^1 \mu_3(x)dx} [u_3(t)\mu_2^q(t) + O(\sqrt{\varepsilon})], \quad q = 0,1,2,$$

as $\varepsilon \to 0$, *where*

$$u_1(t) = \exp\left(-\int_0^t \frac{C(x)}{B(x)} dx\right); \ u_k(t) = \exp\left(-\int_0^t \frac{q_k(x)}{p_k(x)} dx\right) \neq 0, \ t \in [0,1], \ k = 2, 3,$$

$$p_k(t) = 2\mu_k^2(t) \neq 0, \ \mu_2 = -\sqrt{-B(t)}, \ \mu_3 = \sqrt{-B(t)}, \ k = 2,3,$$

$$q_k(t) = C(t) + 3\mu_k(t)\mu_k'(t), \ k = 2,3.$$

Proof. The proof of the lemma is readily obtained from the well-known theorems of Schlesinger [10] and Birkhoff [11] (for example, see [12, pp.29-34]).

For the Wronskian's determinant $W(t,\varepsilon)$ the fundamental solution system $y_i(t,\varepsilon), i = \overline{1,3}$ of Eq. (5), for sufficiently small $\varepsilon > 0$, we obtain

$$W(t,\varepsilon) = \frac{1}{\sqrt{\varepsilon}^3} u_1(t)u_2(t)u_3(t)\mu_3(t)\mu_2(t)(\mu_3(t) - \mu_2(t)) \times$$

$$\times \exp\left(\frac{1}{\sqrt{\varepsilon}} \int_0^t \mu_2(x)dx + \frac{1}{\sqrt{\varepsilon}} \int_1^t \mu_3(x)dx\right)(1 + O(\sqrt{\varepsilon})) \neq 0. \tag{6}$$

4. Construction of initial functions.

Following previous work [9], let us introduce the Cauchy function for the Eq. (4):

$$K(t,s,\varepsilon) = \frac{W(t,s,\varepsilon)}{W(s,\varepsilon)}, \ s,t \in [0,1], \tag{7}$$

where $W(t,s,\varepsilon)$ is the 3th-order determinant obtained from the determinant $W(s,\varepsilon)$ by replacing the 3th row of the fundamental solution system with $y_1(t,\varepsilon)$, $y_2(t,\varepsilon)$, $y_3(t,\varepsilon)$ of Eq. (4).

From the explicit expression (7) of Cauchy function $K(t,s,\varepsilon)$, defined at $0 \le s \le t \le 1$, it satisfies the homogeneous Eq. (4) with respect to the variable t and the initial conditions: $K^{(j)}(s,s,\varepsilon) = 0$, $j = \overline{0,1}$, $K''(s,s,\varepsilon) = 1$ and it does not depend on the choice of the fundamental system of solutions of the equation (4).

Now, we introduce the following functions:

$$K_0(t,s,\varepsilon) = \frac{P_0(t,s,\varepsilon)}{W(s,\varepsilon)}; \quad K_1(t,s,\varepsilon) = \frac{P_1(t,s,\varepsilon)}{W(s,\varepsilon)}. \tag{8}$$

Here $K_0(s,s,\varepsilon) + K_1(s,s,\varepsilon) = K(t,s,\varepsilon)$, $P_0(t,s,\varepsilon)$, $P_1(t,s,\varepsilon)$ is the 3^{d}-order determinants obtained from the determinant $W(s,\varepsilon)$ as a result of replacement of the 3-d row respectively:

$(y_1(t,\varepsilon)$, $y_2(t,\varepsilon)$, $0)$; $(0$, 0, $y_3(t,\varepsilon))$,

where $y_1(t,\varepsilon)$, $y_2(t,\varepsilon)$, $y_3(t,\varepsilon)$ is the fundamental system of solutions of Eq. (4).

Notice, that $K_0(t,s,\varepsilon)$, $K_1(t,s,\varepsilon)$ are continuous functions and, together with its derivatives up to third order inclusively, and as a functions of the variable t satisfy the homogeneous equation (4). The functions $K_0(t,s,\varepsilon)$, $K_1(t,s,\varepsilon)$ will be called the initial functions of the problem (1), (2).

Let conditions (a) and (b) be satisfied. Then, from (8) using estimates (5) and (6) for initial functions $K_0^{(q)}(t,s,\varepsilon)$ and $K_1^{(q)}(t,s,\varepsilon)$ for sufficiently small ε, we obtain the following asymptotic representations:

$$K_0^{(q)}(t,s,\varepsilon) =$$

$$= \varepsilon \left(\frac{u_1^{(q)}(t)}{u_1(s)B(s)} - \frac{1}{\sqrt{\varepsilon}^q} \frac{u_2(t)\mu_2^q(t)\exp\left(\frac{1}{\sqrt{\varepsilon}}\int_s^t \mu_2(x)dx\right)}{u_2(s)\mu_2(s)(\mu_3(s)-\mu_2(s))} + O\left(\sqrt{\varepsilon} + \frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}^q} e^{\frac{1}{\sqrt{\varepsilon}}\int_s^t \mu_2(x)dx}\right) \right), \tag{9}$$

$$K_1^{(q)}(t,s,\varepsilon) =$$

$$= \varepsilon \left(\frac{1}{\sqrt{\varepsilon}^q} \frac{u_3(t)\mu_3^q(t)}{u_3(s)\mu_3(s)(\mu_3(s)-\mu_2(s))} e^{-\frac{1}{\sqrt{\varepsilon}}\int_t^s \mu_3(x)dx} + O\left(\sqrt{\varepsilon} + \frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}^q} e^{-\frac{1}{\sqrt{\varepsilon}}\int_t^s \mu_3(x)dx}\right) \right).$$

5.　　Construction of boundary functions.

Consider the determinant of the third order

$$J(\varepsilon) = \begin{vmatrix} y_1(0,\varepsilon) & y_2(0,\varepsilon) & y_3(0,\varepsilon) \\ y_1'(0,\varepsilon) & y_2'(0,\varepsilon) & y_3'(0,\varepsilon) \\ y_1(1,\varepsilon) & y_2(1,\varepsilon) & y_3(1,\varepsilon) \end{vmatrix},$$

where $y_1(t,\varepsilon), y_2(t,\varepsilon), y_3(t,\varepsilon)$ is the fundamental system of solutions of Eq. (4). Expanding the determinant $J(\varepsilon)$ in elements of the 3-d row and taking into account (5), at $\varepsilon \to 0$ we obtain

$$J(\varepsilon) = \frac{1}{\sqrt{\varepsilon}} u_3(1) u_2(0) \mu_2(0) u_1(0)(1 + O(\sqrt{\varepsilon})) \neq 0. \tag{10}$$

Definition. Functions $\Phi_k(t,\varepsilon), k = \overline{1,3}$ are referred to as boundary functions of the perturbed problem (1) (2) if they satisfy the homogeneous Eq. (4) and boundary conditions

$$\Phi_1(0,\varepsilon) = 1, \Phi_k(0,\varepsilon) = 0, \ k = 2,3; \ \Phi_3(1,\varepsilon) = 1, \Phi_k(1,\varepsilon) = 0, \ k = 1,2;$$
$$\Phi_2'(0,\varepsilon) = 1, \Phi_k(0,\varepsilon) = 0, \ k = 1,3. \tag{11}$$

Theorem 1. *Let conditions (a), (b), and (c) be satisfied. Then, for sufficiently small $\varepsilon > 0$, the boundary functions $\Phi_k(t,\varepsilon), k = \overline{1,3}$ exist on the interval [0,1], are unique, and are given as follows:*

$$\Phi_k(t,\varepsilon) = \frac{J_k(t,\varepsilon)}{J(\varepsilon)}, \quad k = \overline{1,3}, \tag{12}$$

where $J_k(t,\varepsilon)$ is the determinant obtained from $J(\varepsilon)$ by substituting the kth row with the fundamental solution system $y_i(t,\varepsilon), i = \overline{1,3}$ of Eq. (4).

Let conditions (a), (b) and (c) be satisfied. Expanding the determinant $J_i(t,\varepsilon)$, in elements of the ith row, then from (12) and taking into account (5) and (10), at $\varepsilon \to 0$ we obtain the following asymptotic formula:

$$\Phi_1^{(q)}(t,\varepsilon) = \frac{u_1^q(t)}{u_1(0)} - \frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}^q} \frac{u_1'(0)}{u_1(0)} \frac{u_2(t)\mu_2^q(t)}{u_2(0)\mu_2(0)} \exp\left(\frac{1}{\sqrt{\varepsilon}} \int_0^t \mu_2(x)dx\right) -$$

$$- \frac{1}{\sqrt{\varepsilon}^q} \frac{u_1(1)}{u_1(0)} \frac{u_3(t)\mu_3^q(t)}{u_3(1)} \exp\left(-\frac{1}{\sqrt{\varepsilon}} \int_t^1 \mu_3(x)dx\right) +$$

$$+ O\left(\sqrt{\varepsilon} + \frac{\sqrt{\varepsilon}^2}{\sqrt{\varepsilon}^q} e^{\frac{1}{\varepsilon} \int_0^t \mu_2(x)dx} + \frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}^q} e^{-\frac{1}{\varepsilon} \int_t^1 \mu_3(x)dx}\right),$$

$$\Phi_2^{(q)}(t,\varepsilon) = \sqrt{\varepsilon}\left[-\frac{u_1^q(t)}{u_1(1)\mu_2(0)} + \frac{1}{\sqrt{\varepsilon}^q} \frac{u_2(t)\mu_2^q(t)}{u_2(0)\mu_2(0)} e^{\frac{1}{\sqrt{\varepsilon}} \int_0^t \mu_2(x)dx} - \right. \tag{13}$$

$$-\frac{1}{\sqrt{\varepsilon}^q}\frac{u_1(1)}{u_1(0)}\frac{u_3(t)\mu_3^q(t)}{u_3(1)\mu_2(0)}\mathrm{e}^{-\frac{1}{\sqrt{\varepsilon}}\int\limits_t^1\mu_3(x)dx}+O\left(\sqrt{\varepsilon}+\frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}^q}\mathrm{e}^{\frac{1}{\sqrt{\varepsilon}}\int\limits_0^t\mu_2(x)dx}+\frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}^q}\mathrm{e}^{-\frac{1}{\sqrt{\varepsilon}}\int\limits_t^1\mu_3(x)dx}\right)\Biggr]$$

$$\Phi_3^{(q)}(t,\varepsilon)=\frac{1}{\sqrt{\varepsilon}^q}\frac{u_3(t)\mu_3^q(t)}{u_3(1)\mu_2(0)}\mathrm{e}^{-\frac{1}{\sqrt{\varepsilon}}\int\limits_t^1\mu_3(x)dx}-$$

$$-\frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}^q}\frac{u_1'(0)u_3(t)\mu_3^q(t)}{u_1(0)u_3(1)\mu_2(0)}\mathrm{e}^{-\frac{1}{\varepsilon}\int\limits_t^1\mu_3(x)dx}+O\left(\sqrt{\varepsilon}+\frac{\sqrt{\varepsilon}^2}{\sqrt{\varepsilon}^q}\mathrm{e}^{-\frac{1}{\sqrt{\varepsilon}}\int\limits_t^1\mu_3(x)dx}\right).$$

6.   Analytic representation and asymptotic estimates of the solution.

Theorem 2. *Let conditions (a) and (b) be satisfied. Then the inhomogeneous boundary value problem (1), (2) has a unique solution and expressed by the formula*

$$y(t,\varepsilon)=a_0\Phi_1(t,\varepsilon)+a_1\Phi_2(t,\varepsilon)+b_0\Phi_3(t,\varepsilon)+\Phi_1(t,\varepsilon)\frac{1}{\varepsilon}\int\limits_0^1 K_1(0,s,\varepsilon)F(s)ds+$$

$$+\Phi_2(t,\varepsilon)\frac{1}{\varepsilon}\int\limits_0^1 K_1'(0,s,\varepsilon)F(s)ds-\Phi_3(t,\varepsilon)\frac{1}{\varepsilon}\int\limits_0^1 K_0(1,s,\varepsilon)F(s)ds+ \qquad (14)$$

$$+\frac{1}{\varepsilon}\int\limits_0^t K_0(t,s,\varepsilon)F(s)ds+\frac{1}{\varepsilon}\int\limits_1^t K_1(t,s,\varepsilon)F(s)ds.$$

Proof. It is sufficient to prove the theorem that the function defined by the formula (14) satisfies all the conditions determining the solutions of the problem (1), (2). Its uniqueness follows from the estimate (10). The theorem is proved.
Consider the formula (13). Using (9), (12), we obtain (13) on the segment $0\le t\le 1$ following asymptotic representation:

$$y^{(q)}(t,\varepsilon)=a_0\frac{u_1^{(q)}(t)}{u_1(0)}+\int\limits_0^t\frac{u_1^{(q)}(t)F(s)}{u_1(s)B(s)}ds+$$

$$+\frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}^q}\frac{u_2(t)\mu_2^q(t)}{u_2(0)\mu_2(0)}\exp\left(\frac{1}{\sqrt{\varepsilon}}\int\limits_0^t\mu_2 dx\right)\left[a_1-a_0\frac{u_1'(0)}{u_1(0)}-\frac{F(0)}{B(0)}\right]+$$

$$+\frac{1}{\sqrt{\varepsilon}^q}\frac{u_3(t)\mu_3^q(t)}{u_3(1)}\exp\left(-\frac{1}{\sqrt{\varepsilon}}\int\limits_t^1\mu_3(x)dx\right)\left[b_0-a_0\frac{u_1(1)}{u_1(0)}-u_1(1)\int\limits_0^1\frac{F(s)}{u_1(s)B(s)}ds\right]+$$

$$+\frac{\sqrt{\varepsilon}F(t)}{\sqrt{\varepsilon}^q(\mu_3(t)-\mu_2(t))}\left(\frac{\mu_3^q(t)}{\mu_3^2(t)}-\frac{\mu_2^q(t)}{\mu_2^2(t)}\right)+$$

$$+ O\left(\sqrt{\varepsilon} + \frac{\sqrt{\varepsilon}^2}{\sqrt{\varepsilon}^q} \exp\left(\frac{1}{\sqrt{\varepsilon}} \int_0^t \mu_2 dx \right) + \frac{\sqrt{\varepsilon}^2}{\sqrt{\varepsilon}^q} \exp\left(-\frac{1}{\sqrt{\varepsilon}} \int_t^1 \mu_3 dx \right) \right). \quad (15)$$

Now, we define a degenerate problem. From (15) we come to the conclusion that the initial condition for the degenerate solutions can be obtained from equation (2) in the form $\bar{y}(0) = a_0$. Thus, we obtain

$$L_0 \bar{y} \equiv B(t)\bar{y}' + C(t)\bar{y} = F(t), \quad \bar{y}(0) = a_0. \quad (16)$$

The solution of (16) can be represented in the form

$$\bar{y}(t) = a_0 \frac{u_1(t)}{u_1(0)} + \int_0^t \frac{u_1(t)F(s)}{u_1(s)B(s)} ds.$$

From this we have

$$\bar{y}'(t) = a_0 \frac{u_1'(t)}{u_1(1)} + \int_0^t \frac{u_1'(t)F(s)}{u_1(s)B(s)} ds + \frac{F(t)}{B(t)}.$$

Then, by condition (c) on the basis of (15) and (16) we obtain

$$\lim_{\varepsilon \to 0} y(t,\varepsilon) = \bar{y}(t), \quad 0 \le t < 1, \quad \lim_{\varepsilon \to 0} y'(t,\varepsilon) = \bar{y}'(t), \quad 0 < t < 1,$$

$$\Delta_1 = y(1,\varepsilon) - \bar{y}(1) = b_0 - a_0 e^{-\int_0^1 \frac{C(x)}{B(x)} dx} - \int_0^1 \frac{F(s)}{B(s)} e^{-\int_s^1 \frac{C(x)}{B(x)} dx} ds.$$

$$\Delta_0 = y'(0,\varepsilon) - \bar{y}'(0) = a_1 - \frac{F(0)}{B(0)} + \frac{C(0)}{B(0)} a_0,$$

$$y''(0,\varepsilon) = O\left(\frac{1}{\sqrt{\varepsilon}} \right), \quad y'(1,\varepsilon) = O\left(\frac{1}{\sqrt{\varepsilon}} \right), \quad y''(1,\varepsilon) = O\left(\frac{1}{\varepsilon} \right).$$

It follows from this and (16) that in points t=0, t=1 the solution of problem (1), (2) possesses a phenomenon of boundary jumps first and zero orders.

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