

Study of Blood Flow with Suspension of Nanoparticles through Tapered Stenosed Artery

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Abstract

This article presents the effect of heat and mass transfer on the blood flow through a tapered stenosed artery assuming blood as a Jeffrey fluid model. The equations governing the blood flow is modeled in cylindrical co-ordinates. Analytical solutions are constructed for the velocity, temperature, concentration and flux by solving flow governing non-linear coupled equations using Homotopy Perturbation Method. The important characteristics of blood flow such concentration and temperature are found by using Homotopy Perturbation Method and these solutions are used to find exact solution for velocity profile. Variation in velocity profile, temperature profile, concentration profile and flux profile for different values of thermophoresis and Brownian motion parameter are discussed. Homotopy Perturbation Method technique is used to calculate these expressions and Matlab programming is used to find computational results. And then computational results are presented graphically. The significance of the present model over the existing models has been pointed out by comparing the result with other theories both analytically and numerically. Here in this article we have discussed some important phenomena raised in biotechnology and medicine at the nanoscale. So this article about nanoparticles behavior could be useful in the development of new diagnosis tools for many diseases in medical field, biotechnology as well as in medicine at the nanoscale.

Keywords: Arteriosclerosis, Jeffrey fluid model, Nanoparticles, Homotopy Perturbation Method, temperature profile, Concentration profile, Velocity profile, Flux profile.

INTRODUCTION

Cardiovascular disease is the number one killer in this world. Each year about 1.1 million human suffer acute myocardial infarction (AMI), i.e., heart attack, and almost half of them die from it, accounting for 1 in 5 deaths in the United States. A vast majority of these cardiac events occur suddenly in patients with little known history of coronary artery disease. This disorder is known to be responsible for over seventy five percent of all deaths and “Arteriosclerosis” is one of the major causes for these. Additionally, health care expenditure associated with cardiovascular diseases is increasing rapidly [5]. Therefore, there is an overwhelming need to develop methods for early diagnosis and acute treatment of atherosclerosis—the root cause of cardiovascular diseases, and prevent its progression to heart attack. Heart attacks are caused by blockages in the coronary arteries that supply oxygen rich blood to the heart’s muscular wall (the myocardium). A model for abnormal growth in the lumen of artery is called “Stenosis”, which is developed due to intravascular plaques. As the disease advances, it affects severely on the coronary flow rate and perfusion [2]. There is evidence that cardiovascular stenotic flows as well as vascular wall deformability play important roles in the development and progression of arterial stenosis, one of the most widespread diseases in human beings, leading to the malfunction of the cardiovascular system. Some researchers have the opinion that the damage to the inner coating of the artery, the intima, is responsible for the initial formation of stenosis and also for re-growth of the stenosis after balloon angiography [1, 3, 12]. It has been established that once a mild stenosis is developed, the resulting flow disorder further influences the development of the disease and arterial deformability, and changes the regional blood rheology, as well [4, 13]. Because of the fact that real atherosclerotic lesions are asymmetric and irregular [1, 17], we need to have better understanding factor controlling blood flow in such constricted geometries. In general, the surface irregularities of the stenosis add complexity to experimental and numerical simulations of the flow phenomena. Keeping in view such complexities, a great amount of scientific effort has already been invested on investigating the flow characteristics of blood through occluded vessels [5–12]. Johnston and Kilpatrick [14] obtained in their findings that the highest pressure drop was noted across the cosine-shaped stenosis rather than the irregularly-shaped stenosis throughout the entire range of Reynolds numbers and concluded that the smoothness factor was the prime cause for such behaviour. Several researchers have studied the flow of blood through arteries.

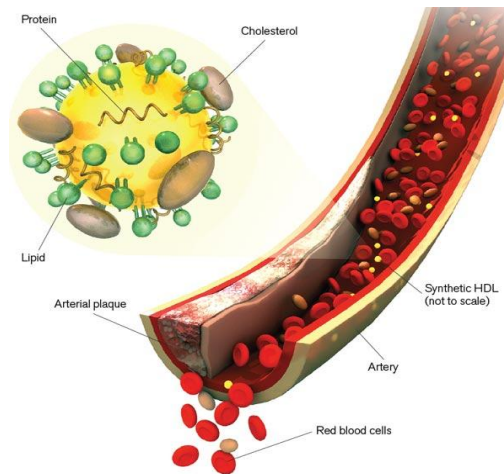


Fig.1: Stenosed artery with red cell

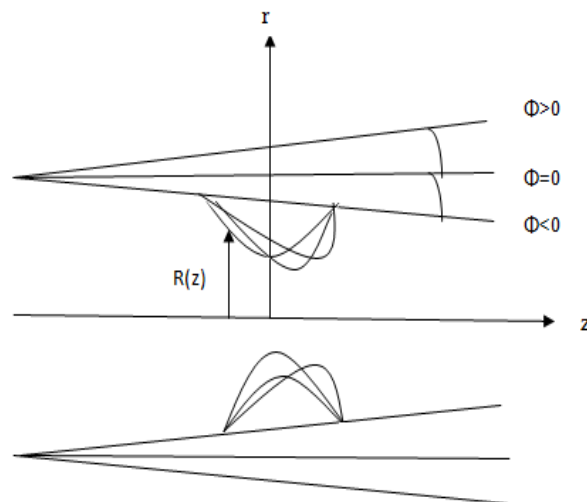


Fig.2: Schematic diagram of blood flow in a stenosed tapered artery

In 2013, [16] studied the behavior of non-Newtonian fluid flow of the blood in a stenosed artery. In 2014, [18] investigated the influence of body acceleration and slip velocity at the wall by considering the blood as a two-fluid model. The multiple stenoses effects with viscosity variation for power law fluid model [15, 20]. In 2010, Srivastava et. al., and Kumar et. Al.,[19, 20] developed a model and studied the multiple stenoses influence through an inclined tube of non-uniform cross section. To better interpret and analyze the experimental data on blood it is helpful to turn to the literature on the rheology of blood as particle suspensions. For blood flow in presence of stenosis, a vast amount of published literature exists. However, the study of suspensions of multiple, interacting, highly deformable particles, nanoparticles in

blood, has received less attention and presents a challenge for both theoretical and computational fluid dynamicists. In this section we present a brief overview of the rheological properties of blood with the help of mathematical model of Jeffrey fluid vai nanoparticles in tapered stenose artery, including its most relevant characteristics and discuss constitutive other models introduced to capture one or more of these properties.

FORMULATION OF THE PROBLEM:

Consider blood is treated as an incompressible Jeffrey fluid with nano-particles, having constant viscosity μ and density ρ , through a tube shaped artery of radius R_0 and length L [12]. The cylindrical coordinate system (r, θ, z) is chosen such that the velocity components in r and z directions is u and v respectively. Here $r=0$ is selected the axis of the symmetry of the tube. Heat and mass transfer phenomenon is calculated by assigning temperature T_1 and concentration C_1 to the wall of the tube .The geometry of the arterial wall with overlapping stenosis is given as [5]:

$$\frac{R(z)}{D(z)} = [1 - \psi(L_0^{n-1}(z - d_0) - (z - d_0)^n)] ; \quad d_0 < z \leq d_0 + L_0$$

$$\frac{R(z)}{D(z)} = 1 , \quad \text{otherwise} \quad (1)$$

With

$$\psi = \frac{(\delta)^{n-1}}{R_0 L_0^n (n-1)} \quad (2)$$

$$d(z) = R_0 + \xi z, \quad (3)$$

in which δ denotes the maximum height of the stenosis located at

$$z = d_0 + \frac{L_0}{n} \quad (4)$$

Here R_0 is the radius of the non-tapered artery in the non-stenotic region, $d(z)$ is the radius of the tapered arterial segment in the stenotic region, ξ is the tapering parameter, L_0 is the length of the stenosis, $n(\geq 2)$ is a parameter determining the shape of the constriction profile, referred to as the stenosis shape parameter for which the symmetric stenosis is found for $n=2$ and d_0 indicates location of stenosis, as shown in Fig. [1,2]. The equations governing the flow are:

$$\frac{1}{r} \frac{\partial(rv)}{\partial r} + \frac{\partial u}{\partial z} = 0 \quad (5)$$

$$\rho \left(v \frac{\partial v}{\partial r} + u \frac{\partial v}{\partial z} \right) = - \frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (r S_{rr}) + \frac{\partial}{\partial z} (S_{rz}) - \frac{1}{r} (S_{\theta\theta}), \quad (6)$$

$$\rho \left(v \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r S_{rz}) + \frac{\partial}{\partial z} (S_{zz}) + \rho g \alpha_1 (T - T_1) + \rho g \alpha_1 (C - C_1) \quad (7)$$

$$\begin{aligned} \left(v \frac{\partial T}{\partial r} + u \frac{\partial T}{\partial z} \right) &= \alpha_1 \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) \\ &+ \tau \left[D_B \left(\frac{\partial C}{\partial r} \frac{\partial T}{\partial r} + \frac{\partial C}{\partial z} \frac{\partial T}{\partial z} \right) + \frac{D_T}{T_0} \left(\left(\frac{\partial T}{\partial r} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 \right) \right] \end{aligned} \quad (8)$$

$$\left(v \frac{\partial C}{\partial r} + u \frac{\partial C}{\partial z} \right) = D_B \left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right) + \frac{D_T}{T_0} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) \quad (9)$$

where p is pressure, g – is the acceleration due to gravity, T -is temperature, C - is concentration, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ is the ratio between the effective heat capacity of the nanoparticle and heat capacity of the fluid. The ambient values of T and C as r tent to R are denoted by T_1 and C_1 , D_B is the Browning diffusion coefficient and D_T is the thermospheric diffusion coefficient.

$$\begin{aligned} S_{rr} &= \frac{2\mu}{1 + \lambda_1} \left(1 + \lambda_2 \left(v \frac{\partial}{\partial r} + u \frac{\partial}{\partial z} \right) \right) \frac{\partial v}{\partial r}, \\ S_{rz} &= \frac{\mu}{1 + \lambda_1} \left(1 + \lambda_2 \left(v \frac{\partial}{\partial r} + u \frac{\partial}{\partial z} \right) \right) \left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r} \right), \\ S_{zz} &= \frac{2\mu}{1 + \lambda_1} \left(1 + \lambda_2 \left(v \frac{\partial}{\partial r} + u \frac{\partial}{\partial z} \right) \right) \frac{\partial u}{\partial z}, \end{aligned}$$

where λ_1 – is the ratio between relaxation to retardation times ,and λ_2 – is the retardation time . Defining;

$$\begin{aligned} r' &= \frac{r}{R_0}; & z' &= \frac{z}{L_0}; & v' &= \frac{L_0}{\delta U}; & u' &= \frac{u}{U}; \\ R' &= \frac{R}{R_0}; & p' &= \frac{R_0^2}{U \mu L_0} p; & \varphi &= \frac{T - T_1}{T_0 - T_1}; & \sigma &= \frac{C - C_1}{C_0 - C_1}; \\ G_r &= \frac{\rho g \alpha_1 R_0^3}{\mu} (T_0 - T_1); & B_r &= \frac{\rho g \alpha_1 R_0^3}{\mu} (C_0 - C_1); & R_e &= \frac{\rho U R_0}{\mu}; \\ N_t &= \frac{(\rho c)_p D_T T_0}{(\rho c)_f \alpha_1}; & N_b &= \frac{(\rho c)_p D_B C_0}{(\rho c)_f \alpha_1}; \end{aligned} \quad (10)$$

where R_e –is the Reynolds number , N_t –is the thermophoresis parameter , N_b –is the Brownian motion parameter , G_r –is the local temperature Grashof number , B_r –is the local Grashof number. Using the non-dimensional variables in equation (10) along with the additional boundary conditions.

$$1). \quad \frac{R_e \delta n^{\frac{1}{n-1}}}{L_0} \ll 1 ,$$

$$2). \quad \delta^* = \frac{R_0 n^{\frac{1}{n-1}}}{L_0} \sim o(1) ,$$

and for mild stenosis $\left(\frac{\delta}{R_0} \ll 1\right)$ in equation (5) to (9) , after dropping the dashes take the form

$$\delta^* \left(\frac{\partial v}{\partial r} + \frac{v}{r} \right) + \frac{\partial u}{\partial z} = 0 , \quad (11)$$

$$\frac{\partial p}{\partial r} = 0 , \quad (12)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{r}{1+\lambda_1} \left(\frac{\partial u}{\partial r} \right) \right) + G_r \varphi + B_r \sigma , \quad (13)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \varphi}{\partial r} \right) + N_b \frac{\partial \sigma}{\partial r} \frac{\partial \varphi}{\partial r} + N_t \left(\frac{\partial \varphi}{\partial r} \right)^2 = 0 , \quad (14)$$

$$N_b \frac{\partial}{\partial r} \left(r \frac{\partial \sigma}{\partial r} \right) + N_t \frac{\partial}{\partial r} \left(r \frac{\partial \varphi}{\partial r} \right) = 0 , \quad (15)$$

The boundary conditions are as follows:

$$\frac{\partial u}{\partial r} = 0, \quad \frac{\partial \varphi}{\partial r} = 0, \quad \frac{\partial \sigma}{\partial r} = 0 \quad \text{at } r=0$$

$$w=0, \quad \varphi = 0, \quad \sigma = 0 \quad \text{at } r=R(z),$$

where

$$\frac{R(z)}{1 + \xi_1 z} = [1 - \psi_1 ((z - d_0^*) - (z - d_0^*)^n)], \quad d_0^* < z \leq d_0^* + 1 ,$$

$$\frac{R(z)}{1 + \xi_1 z} = 1, \quad \text{otherwise,} \quad (16)$$

where

$$d_0^* = \frac{d_0}{L_0}, \quad \xi_1 = \frac{\xi L_0}{R_0}, \quad \psi_1 = \delta^* \frac{n^{\frac{n}{n-1}}}{(n-1)} . \quad (17)$$

SOLUTION OF THE PROBLEM:

The solution of the coupled equations (14) and (15) are calculated by homotopy perturbation method as

$$H(k, \varphi) = (1 - k)[L(\varphi) - L(\varphi)_{10}] + k \left[L(\varphi) + N_b \frac{\partial \sigma}{\partial r} \frac{\partial \varphi}{\partial r} + N_t \left(\frac{\partial \varphi}{\partial r} \right)^2 \right], \quad (18)$$

$$H(k, \sigma) = (1 - k)[L(\sigma) - L(\sigma)_{10}] + k \left[L(\sigma) + \frac{N_t}{N_b} \frac{\partial}{\partial r} \left(r \frac{\partial \varphi}{\partial r} \right) \right], \quad (19)$$

where k is the embedding parameter, which has the range $0 \leq k \leq 1$, $L = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right)$, is a linear operator.

Taking the following initial guesses

$$\varphi_{10}(r, z) = - \left(\frac{r^2 - R^2}{4} \right) \quad (20)$$

$$\sigma_{10}(r, z) = - \left(\frac{r^2 - R^2}{4} \right) \quad (21)$$

Define

$$\varphi = \varphi_0 + k\varphi_1 + k^2\varphi_2 + o(k)^3 \quad (22)$$

$$\sigma = \sigma_0 + k\sigma_1 + k^2\sigma_2 + o(k)^3 \quad (23)$$

Putting equations (22) and (23) in equation (14) and (15) respectively, and taking $k \rightarrow 1$, the following for temperature and concentration profile is written as follows:

$$\varphi(r, z) = (2N_t + N_b) \left(\frac{r^4 - R^4}{64} \right) - \left(\frac{r^6 - R^6}{1152} \right) (2N_t + N_b)(N_t + N_b), \quad (24)$$

$$\sigma(r, z) = (N_t + N_b) \frac{N_t}{N_b} \left(\frac{r^4 - R^4}{64} \right), \quad (25)$$

Substituting these equations (24) and (25) into equation (13), the exact solution for velocity is obtained as:

$$u(r, z) = \frac{r^2}{2} (1 + \lambda_1) \frac{dp}{dz} + Gr(1 + \lambda_1)(2N_t + N_b) \left[\left(\frac{r^6 - 3r^2R^4}{2304} \right) - \left(\frac{r^8 - 16r^2R^6}{73728} \right) (N_t + N_b) \right] + B_r(1 + \lambda_1)(N_t + N_b) \frac{N_t}{N_b} \left(\frac{r^6 - 3r^2R^4}{2304} \right), \quad (26)$$

The volume flow rate Q is expressed as:

$$Q = \int_0^R r u dr$$

Using equation number (26), we have the expression for Q as:

$$Q = \frac{R^4}{4} (1 + \lambda_1) \frac{dp}{dz} + G_r (1 + \lambda_1) (2N_t + N_b) \left[\frac{R^8 - 6R^4 h^4}{9216} - \left(\frac{R^{10} - 40R^4 h^6}{368640} \right) (N_t + N_b) \right] + B_r (1 + \lambda_1) \frac{N_t}{N_b} (N_t + N_b) \left(\frac{R^8 - 6R^4 h^4}{9216} \right) \quad (27)$$

GRAPHICAL RESULTS AND DISCUSSIONS:

In developing a complex model that involves incorporating a number of physiological phenomena, a few practical issues need to be considered. In the present study, the behavior of blood in arteries as non-Newtonian fluid is investigated analytically and the data has been taken from practical issues. Homotopy Perturbation Method is applied to solve the governing equations. A significant influence of different prominent flow parameters on the blood flow with metallic nanoparticles are taking into account by the graphs of temperature profile (φ), concentration profile (σ), velocity profile $u(r, z)$, flux (Q). The graphs of flux (Q), velocity profile $u(r, z)$, temperature profile (φ), concentration profile (σ) are discussed for different values of thermophoresis parameter (N_t) and Brownian motion parameter (N_b). Figure (3-4) shows the variation of temperature profile with radius of the artery for different values of thermophoresis parameter (N_t) and Brownian motion parameter (N_b).

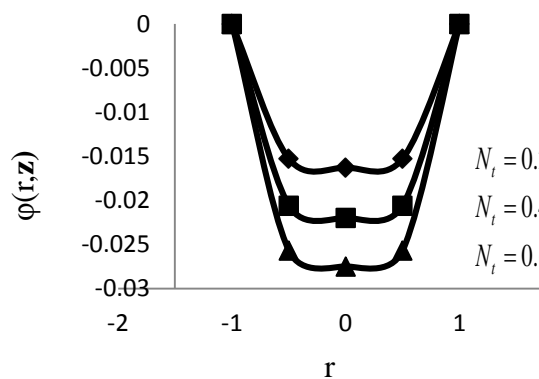


Fig.3 Variation of temperature profile (φ) with (r) for different values of N_t

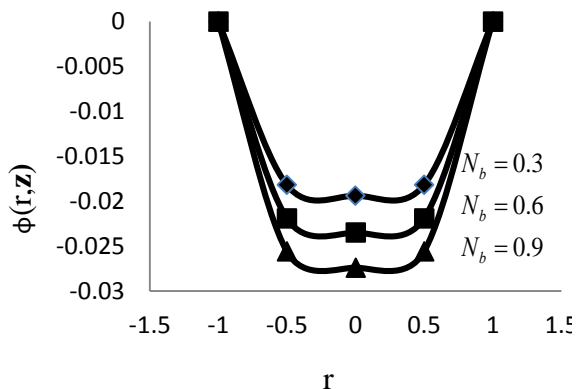


Fig.4 Variation of temperature profile (φ) for with (r) different values of N_b

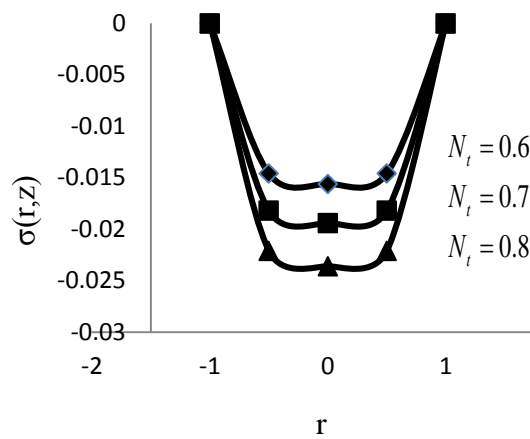


Fig.5 Variation of concentration(σ)profile with (r) for different values of N_t

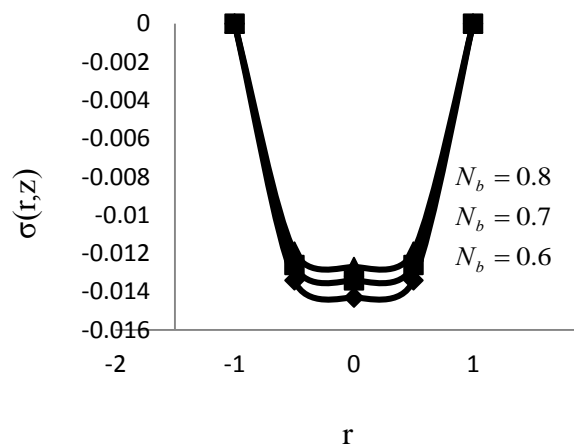


Fig.6 Variation of concentration profile(σ) for with (r) different value of N_b

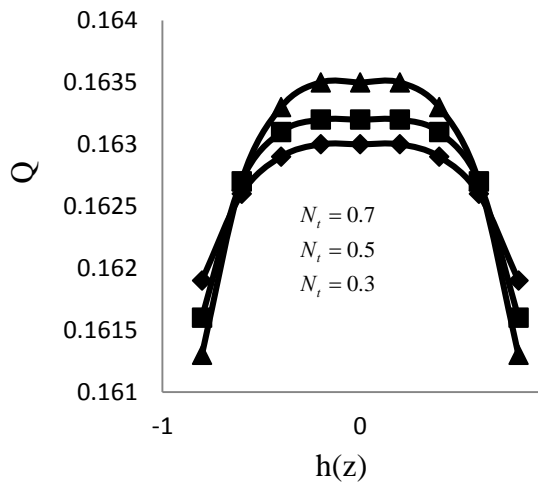


Fig.7 Variation in flux 'Q' for different values of N_t

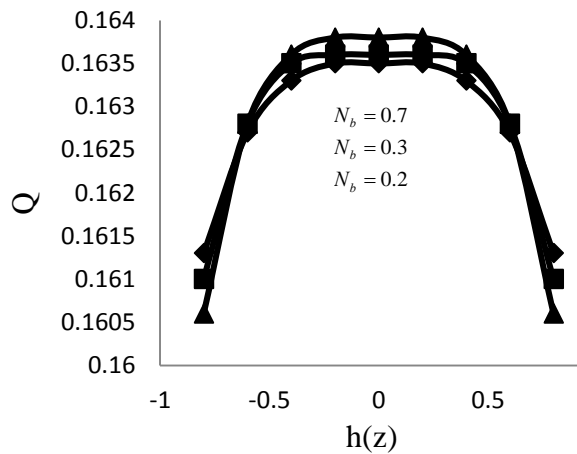


Fig.8 Variation in flux (Q) for different values of N_b

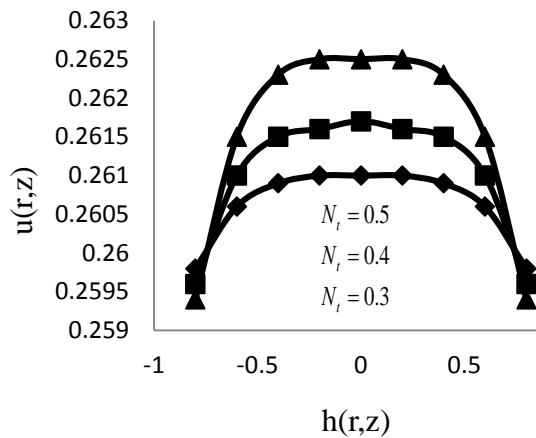


Fig.9 Variation of velocity for different values of N_t

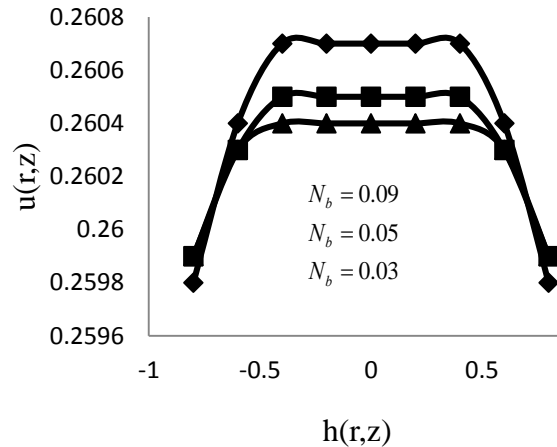


Fig.10 Variation of velocity for different values of N_b

Where Figure (3) shows the variation of temperature profile with radius of the artery for different values of thermophoresis parameter (N_t). It is observed in this figure that with an increase in the thermophoresis parameter (N_t), the temperature profile decreases. And Figure (4) shows the variation of temperature profile with radius of the artery for different values of Brownian motion parameter (N_b). It has been seen in this figure that with an increase in the Brownian motion parameter (N_b), have shown a decrease in the temperature profile. Hence the similar results have been found with an increase in the thermophoresis parameter (N_t) and Brownian motion parameter (N_b), for temperature profile. i.e., temperature profile decreases with the increase in thermophoresis parameter (N_t) and Brownian motion parameter (N_b). Figure (5-6) shows the variation of concentration profile (σ) with radius of the artery for different values of thermophoresis parameter (N_t) and Brownian motion parameter (N_b). Where Figure (5) shows the variation of concentration profile with radius of the artery for different values of thermophoresis parameter (N_t). It is observed in this figure that with an increase in the thermophoresis parameter (N_t), the concentration profile decreases. And Figure (6) shows the variation of concentration profile with radius of the artery for different values of Brownian motion parameter (N_b). It has been seen in this figure that with an increase in the Brownian motion parameter (N_b), have shown an increase in the concentration profile. Hence, it is found that concentration profile decreases with increase in the thermophoresis parameter (N_t) and interestingly concentration profile increase with an increase in the Brownian motion parameter (N_b). Figure (7-8) depict the variation in flux (Q) for different values of the Brownian motion parameter (N_b) and thermophoresis parameter (N_t). In figure (7), it is observed that with an increase in the thermophoresis parameter (N_t), the flux graph increases.

And it is also observed in figure (8) that an increase in the Brownian motion parameter (N_b), results in an increase in flux. Hence the similar results have been found with an increase in the thermophoresis parameter (N_t) and Brownian motion parameter (N_b), for flux profile. i.e., flux profile increases with the increase in thermophoresis parameter (N_t) and Brownian motion parameter (N_b). Figure (9-10) depicts the variation of velocity profile $u(r, z)$ for different values of Brownian motion parameter (N_b) and thermophoresis parameter (N_t). It is shown in figure (9), that with an increase in the thermophoresis parameter (N_t) the velocity profile increases. And, it is also observed in figure (10) that with an increase in the Brownian motion parameter (N_b), the velocity profile decreases. Hence, it is found that velocity profile increases with an increase in the thermophoresis parameter (N_t) and interestingly concentration profile decreases with an increase in the Brownian motion parameter (N_b).

CONCLUSION

In the present study, metallic nanoparticles analysis through an axisymmetric mild stenosis, with blood is considered as non-Newtonian fluid is investigated. The heat and mass transfer via nanoparticles are also taken into account. This process relies on understanding the detail of blood flow through arteries. The flow governing equations are solved using Homotopy Perturbation Method. The main points of the performed analysis are as follows:-

- 1) The temperature profile (φ) decreases with an increase in the thermophoresis parameter (N_t) and Brownian motion parameter (N_b).
- 2) The concentration profile (σ) decreases with an increase in the thermophoresis parameter (N_t) and increases with an increase in Brownian motion parameter (N_b).
- 3) The flux profile (Q) increases with an increase in the thermophoresis parameter (N_t) and Brownian motion parameter (N_b).
- 4) The velocity profile $u(r, z)$ increases with an increase in thermophoresis parameter (N_t) and decreases with an increase in Brownian motion parameter (N_b).

Hence from all the above discussions we can conclude that a careful choice of the fluid model will affect the flow characteristics and can be utilized for engineering, biotechnology applications as well as in medicine field at the nanoscale.

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REFERENCES

- [1] Akram, S., Nadeem, S., “Influence of induced magnetic field and heat transfer on the peristaltic motion of a Jeffrey fluid in an asymmetric channel: closed form solutions”, *J. Magn. Magn. Mater.*, 328 (2013) 11–20.
- [2] Changdar, S., Soumen D., “Analysis of non linear pulsatile blood flow in artery through a generalized multiple stenosis”, *Arab. J. Math.*, (2016) 5:51-61.
- [3] Fang, B., Zhu, L., Fok, P., Lu, X., “Simulation of a pulsatile non-Newtonian flow past a stenosed 2D artery with atherosclerosis”, *Comput. Bio. Med.* (2013) 43, 1098-1113 (2013).
- [4] Haldar K., “Effects of the shape of stenosis on the resistance of blood flow through an artery”, *Bulletin of Mathematical Biology.* (1985), 47(4): 545–550.
- [5] He, J. H., “Homotopy perturbation method for solving boundary value problems”, *Phys. Lett. A*, 350 (2006) 87–88.
- [6] He, J. H., “Homotopy perturbation technique”, *Comput. Methods Appl. Mech. Eng.*, 178 (1999) 257–262.
- [7] Jain, N., Singh, S., and M. Gupta, “Steady flow of blood through an atherosclerotic artery: A non-Newtonian model”, *International Journal of Applied Mathematics and Mechanics*, (2012), Vol.8, pp. 52-63.
- [8] Ling, S., C., Atabek, H., B., “A nonlinear analysis of pulsatile flow in arteries”, *J. Fluid. Mech.* (1972), 55, 493-511.
- [9] N., Mustafa, Mandal, P. K., Abdullah, I., Amin, N. S., Hayat, T., “Numerical Simulation of generalized Newtonian blood flow past a couple of irregular arterial stenosis” *Numer. Meth. Partial Diff. Eqs.* (2011) 27, 960–981.
- [10] Nadeem, S., Ijaz, N.S. Akbar, “Nanoparticle analysis for blood flow of Prandtl fluid model with stenosis”, *Int. Nano Lett.* (2013).
- [11] Nadeem, S., Ijaz, S., “Theoretical analysis of metallic nanoparticles on blood flow through stenosed artery with permeable walls”, *Physics Letter A*, 379(2015) 542-554.
- [12] Nadeem, S., Lee, C., “Boundary layer flow of nanofluid over an exponentially stretching surface”, *Nanoscale Res. Lett.*, (2012) 7, 94.
- [13] Rahbari, A., Fakour, M., Hamzehnezhad, A., Akbari Vakilabadi, M., Ganji, D. D., “Heat transfer and fluid flow of blood with nanoparticles through

- porous vessels in a magnetic field: A quasi-one dimensional analytical approach”, *Mathematical Biosciences* 283 (2017) 38–47.
- [14] Reddy J. V., Srikanth D., Murthy S. V. “Mathematical modeling of pulsatile flow of blood through catheterized unsymmetrical stenosed artery: Effects of tapering angle and slip velocity”, *European Journal of Mechanics B: Fluids*, (2014), 48:236–244.
- [15] S. Nadeem, Sher Akbar, N., “Jeffrey fluid model for blood flow through a tapered artery with a stenosis”, *J. Mech. Med. Biol.* (2011) 11, 529–545.
- [16] Sheikholeslami, M., Gorji-Bandpy, Soheil Soleimani, “Two phase simulation of nanofluid flow and heat transfer using heatline analysis”, *Int. Commun. Heat Mass Transf.* 47 (2013) 73–81.
- [17] Sher Akbar, N., Hayat, T., Nadeem, S., Awatif Hendi, A., “Influence of mixed convection on blood flow through of Jeffrey fluid through a tapered stenosed artery”, *Thermal Science*, (2013),.17(2), 533-546.
- [18] Shit, G. C., Roy M., and Sinha A., “Mathematical modelling of blood flow through a tapered overlapping stenosed artery with variable viscosity”, *Applied Bionics and Biomechanics* 11 (2014) 185–195.
- [19] Srivastava V. P., Vishnoi R., Sinha P., “Particulate suspension blood flow through a stenosed catheterized artery”, *Applications and Applied Mathematics*, (2010), 5(10),1352–1368.
- [20] Varshney, G., Katiyar, V. K., Kumar, S., “Effect of magnetic field on the blood flow in artery having multiple stenosis: a numerical study”, *Int. J. Eng. Sci. Tech* (2). (2010), 67–82.

