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1-movable Independent Outer-connected Domination in Graphs

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Abstract

A nonempty subset S of $V(G)$ is a 1-*movable independent outer-connected dominating set* of G if S is an independent outer-connected dominating set of G and for every $v \in S$, there exists $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is an independent outer-connected dominating set of G. The cardinality of the smallest 1-movable independent outer-connected dominating set of G is called 1-*movable independent outer-connected domination number* of G denoted by $\widetilde{\gamma}^1_{mic}(G)$. A 1-movable independent outer-connected dominating set with cardinality equal to $\widetilde{\gamma}^1_{mic}(G)$ is called $\widetilde{\gamma}^1_{mic}$-*set* of G. This paper investigates the properties of the 1-movable independent outer-connected dominating set in graphs through characterization of those sets in the join and corona of graphs and the corresponding values or bounds of the parameter are determined.

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Keywords: Domination, independent domination, outer-connected domination, independent outer-connected domination, 1-movable independent outer-connected domination, join, corona.

1. Introduction

Let $G = (V(G), E(G))$ be a graph and $v \in V(G)$. The *open neighborhood* of v is the set $N_G(v) = N(v) = \{u \in V(G) : uv \in E(G)\}$ and the *closed neighborhood* of v is the set $N_G[v] = N[v] = N(v) \cup \{v\}$. If $S \subseteq V(G)$, then the *open neighborhood* of S is the set $N_G(S) = N(S) = \cup_{v \in S} N_G(v)$ and the *closed neighborhood* of S is the set $N_G[S] = N[S] = S \cup N(S)$.

A subset S of $V(G)$ is a *dominating set* of G if for every $v \in V(G) \backslash S$, there exists $u \in S$ such that $uv \in E(G)$, that is, $N_G[S] = V(G)$. The *domination number* of G denoted by $\gamma(G)$ is the cardinality of the smallest dominating set of G. A dominating set of G with cardinality equal to $\gamma(G)$ is called a γ-set of G. The join and corona of a dominating set were investigated in [4].

A subset S of $V(G)$ is an *independent set* of G if for every two elements $x, y \in S$, $xy \notin E(G)$. The *independence number* of G, denoted by $\beta(G)$, is the largest cardinality of an independent set in G. A nonempty subset S of $V(G)$ is an *independent dominating set* of G if S is both an independent set and a dominating set of G. The *independent domination number* of G, denoted by $\gamma_i(G)$, is the cardinality of the smallest independent dominating set of G. An independent dominating set of G with cardinality equal to $\gamma_i(G)$ is called a γ_i*-set* of G. Independent dominating sets and its variant were investigated in [2] and [13].

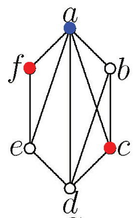
In 2007, Cyman defined outer-connected domination in graphs. A dominating set S is an *outer-connected dominating set* of G if the subgraph $\langle V(G) \setminus S \rangle$ is connected. The *outer-connected domination number* of G denoted by $\widetilde{\gamma}_c(G)$, is the cardinality of the smallest outer-connected dominating set of G. An outer-connected dominating set of G with cardinality equal to $\widetilde{\gamma}_c(G)$ is called $\widetilde{\gamma}_c(G)$-set of G. Outer-connected dominating sets in graphs were investigated in [3] and [5].

A dominating set S is an *independent outer-connected dominating set* of G if S is an independent dominating set of G and the subgraph $\langle V(G) \setminus S \rangle$ is connected. The *independent outer-connected domination number* of G denoted by $\widetilde{\gamma}_{ic}(G)$, is the cardinality of the smallest independent outer-connected dominating set of G. An independent outer-connected dominating set of G with cardinality equal to $\widetilde{\gamma}_{ic}(G)$ is called $\widetilde{\gamma}_{ic}$-set of G.

In 2011, Blair, Gera and Horton introduced 1-movable domination in graphs. A nonempty set $S \subseteq V(G)$ is a 1*-movable dominating set* of G if S is a dominating set of G and for every $v \in S$, $S \setminus \{v\}$ is a dominating set of G or there exists a vertex $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a dominating set of G. The 1*-movable domination number* of a graph G, denoted by $\gamma_m^1(G)$, is the cardinality of the smallest 1-movable dominating set of G. A 1-movable dominating set of G with cardinality equal to $\gamma_m^1(G)$ is called γ_m^1*-set* of G. This concept was investigated in [1] and [6]. Moreover, the movability of some variants of domination were investigated in [7], [8], [9], [10], [11] and [12].

A nonempty subset S of $V(G)$ is a 1*-movable independent outer-connected dominating set* of G if S is an independent outer-connected dominating set of G and for every $v \in S$, there exists $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is an independent outer-connected dominating set of G. The cardinality of the smallest 1-movable independent outer-connected dominating set of G is called 1-*movable independent outer-connected domination number* of G and is denoted by $\widetilde{\gamma}^1_{mic}(G)$. A 1-movable independent outer-connected dominating set with cardinality equal to $\widetilde{\gamma}^1_{mic}(G)$ is called $\widetilde{\gamma}^1_{mic}$-*set* of G.

The graph in the figure shows tha the set $S_1 = \{a\}$ of a graph G is a $\widetilde{\gamma}_{ic}$-set of G. Thus, $\widetilde{\gamma}_{ic}(G) = |S_1| = 1$. Moreover, the set $S_2 = \{c, f\}$ is a $\widetilde{\gamma}^1_{mic}$-set of G. Hence, $\widetilde{\gamma}^1_{mic}(G) = |S_2| = 2$.



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2. Results

A 1-movable independent outer-connected dominating set does not always exist in a connected nontrivial graph G. Here, we denote $\widetilde{\mathscr{R}}^1_{mic}$ be the family of all graphs with a 1-movable independent outer-connected dominating set. In this paper, it is considers all connected nontrivial graphs which belong to the family $\widetilde{\mathscr{R}}^1_{mic}$.

Remark 2.1. Let G be a connected nontrivial graph of order $n \geq 2$. Then $1 \leq \widetilde{\gamma}^1_{mic}(G) \leq \beta(G)$.

Theorem 2.2. Let G be a connected nontrivial graph of order $n \geq 2$. Then, $\widetilde{\gamma}^1_{mic}(G) = 1$ if and only if one of the following holds:

(i) G is a complete graph of order 2; or

(ii) there exists two adjacent vertices x, y and each dominates G.

Proof. Let G be a connected nontrivial graph of order $n \geq 2$. Assume that $\widetilde{\gamma}^1_{mic}(G) = 1$. Then the graph G has a 1-movable independent outer-connected dominating set of G, say S. If $|V(G)| = 2$, then G is a complete graph of order 2. Suppose $|V(G)| \geq 3$. Let $S = \{a\}$ be a 1-movable independent outer-connected dominating set of G. Since S is a 1-movable independent outer-connected dominating set of G, there exists $b \in (V(G) \setminus S) \cap N_G(a)$ such that $(S \setminus \{a\}) \cup \{b\} = \{b\}$ is an independent outer-connected dominating set of G. So, $ab \in E(G)$ and each vertex a and b dominates G.

For the converse, suppose that (i) holds. Then $G = K_2$. Let $V(K_2) = \{x, y\}$ and suppose $S = \{x\}$. Then S is an independent dominating set of G and $\langle V(G) \setminus S \rangle = \langle \{y\} \rangle$ is connected. Hence, S is an independent outer-connected dominating set of G. Moreover,

$S \setminus \{x\} \cup \{y\} = \{y\}$ is an independent dominating set and $\langle V(G) \setminus (S \setminus \{x\} \cup \{y\}) \rangle = \langle \{x\} \rangle$ is connected. Hence, $S \setminus \{x\} \cup \{y\} = \{y\}$ is an independent outer-connected dominating set of G. Thus, S is a 1-movable independent outer-connected dominating set of G. Since $|S| = 1$, S is a $\widetilde{\gamma}^1_{mic}$-set by Remark 2.1. Therefore, $\widetilde{\gamma}^1_{mic}(G) = |S| = 1$. Suppose (ii) holds. Let x, y be the vertices with $xy \in E(G)$ and each dominates G and suppose $S = \{x\}$. Then, S is an independent dominating set of G and $\langle V(G) \setminus S \rangle = \langle \{y\} \rangle + \langle V(G) \setminus \{x, y\} \rangle$ is connected. Hence, S is an independent outer-connected dominating set of G. Also, since y dominates G, $S \setminus \{x\} \cup \{y\} = \{y\}$ is an independent dominatng set of G and $\langle V(G) \setminus (S \setminus \{x\} \cup \{y\}) \rangle = \langle \{x\} \rangle + \langle V(G) \setminus \{x, y\} \rangle$ is connected. Hence, S is an independent outer-connected dominating set of G. Thus, S is a 1-movable independent outer-connected dominating set of G. Since $|S| = 1$, by Remark 2.1, $\widetilde{\gamma}^1_{mic}(G) = |S| = 1$. ■

Corollary 2.3. For any complete graph K_n of order $n \geq 2$, $\widetilde{\gamma}^1_{mic}(K_n) = 1$.

The *join* of two graphs G and H denoted by $G + H$ is the graph with vertex-set $V(G + H) = V(G) \dot{\cup} V(H)$ and edge-set $E(G + H) = E(G) \dot{\cup} E(H) \cup \{uv : u \in V(G), v \in V(H)\}$. The next results characterizes the 1-movable independent outer-connected dominating sets in the join of two connected graphs and the exact values or bounds of the parameter are determined.

Theorem 2.4. Let G and H be connected nontrivial graphs. A subset S of $V(G + H)$ is a 1-movable independent outer-connected dominating set of $G + H$ if and only if one of the following holds:

(i) S is an independent dominating set of G such that

(a) if $|S| = 1$, then either S is a 1-movable independent dominating set of G or there exists $z \in V(H)$ that dominates H; and

(b) if $|S| \geq 2$, then S is a 1-movable independent dominating set of G

(ii) S is an independent dominating set of H such that

(a) if $|S| = 1$, then either S is a 1-movable independent dominating set of H or there exists $w \in V(G)$ that dominates G; and

(b) if $|S| \geq 2$, then S is a 1-movable independent dominating set of H.

Proof. Assume that S is a 1-movable independent outer-connected dominating set of $G + H$. Then either $S \subseteq V(G)$ or $S \subseteq V(H)$. Suppose $S \subseteq V(G)$. Then S is an independent dominating set of G. Suppose $|S| = 1$, say $S = \{a\}$ for some $a \in V(G)$. Since S is a 1-movable outer-connected dominating set of $G + H$, there exists $u \in (V(G + H) \setminus S) \cap N(a)$ such that $(S \setminus \{a\}) \cup \{u\} = \{u\}$ is an independent outer-connected dominating set of $G + H$. If $u \in V(G)$, then S is a 1-movable independent dominating set of G. If $u \in V(H)$, then u dominates H. Suppose that $|S| \geq 2$ and let $v \in S$. Since S is an independent set, and $S \subseteq V(G)$. Since S is a 1-movable independent outer-connected dominating set of $G + H$, there exists $u \in (V(G + H) \setminus S) \cap N(v)$ such that $S \setminus \{v\} \cup \{u\}$ is an independent outer-connected dominating set of $G + H$. Moreover, S being an independent set, it follows that $u \in V(G) \setminus S$. Hence, S is a 1-movable independent dominating set of G. Thus, (i) holds. Similarly, (ii) holds if $S \subseteq V(H)$.

For the converse suppose (i) holds. Then S is an independent dominating set of $G + H$. Suppose $|S| = 1$, say $S = \{x\}$ for some $x \in V(G)$. Then $\langle V(G + H) \setminus S\rangle = \langle V(G) \setminus S\rangle + H$ is connected. Hence S is an independent outer-connected dominating set of $G+H$. Let $v \in S$. By assumption, suppose first that S is a 1-movable independent dominating set of G. Then there exists $u \in (V(G)\setminus S)\cap N_G(x)$ such that $S\setminus\{x\}\cup\{u\} = \{u\}$ is an independent dominating set of G and hence of $G + H$ and the subgraph $\langle V(G + H) \setminus \{u\}\rangle = \langle V(G) \setminus \{u\}\rangle + H$ is connected. Hence, $(S \setminus \{x\}) \cup \{u\} = \{u\}$ is an independent outer-connected dominating set of $G + H$. Suppose $|S| \geq 2$. Then $\langle V(G + H) \setminus S\rangle = \langle V(G) \setminus S\rangle + H$ is connected. Hence S is an independent outer-connected dominating set of $G + H$. Let $v \in S$. By assumption, there exists $u \in (V(G)\setminus S)\cap N_G(v)$ such that $(S\setminus\{v\})\cup\{u\}$ is an independent dominating set of $G+H$. Moreover, $\langle V(G + H) \setminus (S \setminus \{v\}) \cup \{u\}\rangle = \langle V(G) \setminus (S \setminus \{v\}) \cup \{u\}\rangle + H$ is connected. Thus, $(S\setminus\{v\})\cup\{u\}$ is an independent outer-connected dominating set of $G+H$. Hence, in either case, S is a 1-movable independent outer-connected dominating set of $G + H$. Similarly, if (ii) holds, then $S \subseteq V(H)$ is a 1-movable independent outer-connected dominating set of $G + H$. ■

Corollary 2.5. Let G and H be connected nontrivial graphs. Then

$$\widetilde{\gamma}^1_{mic}(G + H) = \begin{cases} 1 \ \ if \gamma(G) = 1 = \gamma(H) \text{ or } \gamma^1_{mi}(G) = 1 \text{ or } \gamma^1_{mi}(H) = 1 \\ min \left\{\gamma^1_{mi}(G), \gamma^1_{mi}(H)\right\} \ otherwise. \end{cases}$$

Proof. Let G and H be connected nontrivial graphs. Then consider the following cases:

Case 1: $\gamma(G) = 1 = \gamma(H)$.
Let $S = \{a\}$ be a γ-set of G and $\{z\}$ be a γ-set of H. By Theorem 2.4 (ia), S is a 1-movable independent outer-connected dominating set of $G+H$. So, $1 \leq \widetilde{\gamma}^1_{mic}(G+H) \leq |S| = 1$. Thus $\widetilde{\gamma}^1_{mic}(G + H) = 1$.

Case 2: $\gamma^1_{mi}(G) = 1$ or $\gamma^1_{mi}(H) = 1$.
Let $S = \{a\}$ be a γ^1_{mi}-set of G. By Theorem 2.4(ia), S is a 1-movable independent outer-connected dominating set of $G + H$. So, $1 \leq \widetilde{\gamma}^1_{mic}(G + H) \leq |S| = 1$. So, $\widetilde{\gamma}^1_{mic}(G + H) = 1$. Similarly, if $D = \{w\}$ is a $\widetilde{\gamma}^1_{mic}(H)$, then by Theorem 2.4$(iia)$, S is a 1-movable independent outer-connected dominating set of $G + H$. So, $1 \leq \widetilde{\gamma}^1_{mic}(G + H) \leq |S| = 1$. Thus, $\widetilde{\gamma}^1_{mic}(G + H) = 1$.

Case 3: $\gamma^1_{mi}(G) \geq 2$ and $\gamma^1_{mi}(H) \geq 2$.
Let S be a γ^1_{mi}-set of G with $|S| \geq 2$. By Theorem 2.4(ib), S is a 1-movable independent

outer-connected dominating set of $G+H$. Also, if S is a γ^1_{mi}-set of H, then by Theorem 2.4(iib), S is a 1-movable independent outer-connected dominating set of $G+H$. Hence, $\widetilde{\gamma}^1_{mic}(G+H) \leq min\left\{\gamma^1_{mi}(G), \gamma^1_{mi}(H)\right\}$. Suppose that S is a γ^1_{mic}-set of $G+H$. By Theorem 2.4(ib), S is a 1-movable independent dominating set of G or by Theorem 2.4(iib), S is a 1-movable independent dominating set of H. Hence, $\widetilde{\gamma}^1_{mic}(G+H) = |S| \geq min\left\{\gamma^1_{mi}(G), \gamma^1_{mi}(H)\right\}$. Therefore, $\widetilde{\gamma}^1_{mic}(G+H) = min\left\{\gamma^1_{mi}(G), \gamma^1_{mi}(H)\right\}$. ■

Theorem 2.6. Let H be a connected nontrivial graph. Then $S \subseteq V(K_1+H)$ is a 1-movable independent outer-connected dominating set of K_1+H if and only if one of the following holds:

(i) $S = V(K_1)$ and there exists $a \in V(H)$ that dominates H.

(ii) S is an independent dominating set of H such that if $|S| \geq 2$, then S is a 1-movable independent dominating set of H.

Proof. Suppose S is a 1-movable independent outer-connected dominating set in K_1+H and $S = V(K_1) = \{x\}$. Then there exists $a \in V(H)$ such that $(S \setminus V(K_1)) \cup \{a\} = \{a\}$ is an independent outer-connected dominating set of K_1+H. Thus, $a \in V(H)$ dominates H. Suppose $S \neq V(K_1)$. Then $S \subseteq V(H)$ and S is an independent dominating set in H. Suppose $|S| \geq 2$. Since S is a 1-movable independent outer-connected dominating set of K_1+H, for all $v \in S$, there exists $u \in (V(K_1+H) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is an independent outer-connected dominating set of K_1+H. Since S is an independent set, $u \neq x$. Hence $u \in (V(H) \setminus S) \cap N_G(v)$ for all $v \in S$. Therefore, S is a 1-movable independent dominating set of H.

For the converse, suppose (i) holds. Then $S = V(K_1) = \{x\}$ is an independent dominating set of K_1+H and $\langle V(K_1+H) \setminus V(K_1)\rangle = H$ is connected. Hence, S is an independent outer-connected dominating set of K_1+H. By assumption, there exists $a \in V(H)$ that dominates H. Thus, $(S \setminus \{x\}) \cup \{a\} = \{a\}$ is an independent dominating set of K_1+H and $\langle V(K_1+H) \setminus \{a\}\rangle = K_1 + \langle V(H) \setminus \{a\}\rangle$ is connected. Hence, $(S \setminus \{x\}) \cup \{a\} = \{a\}$ is an independent outer-connected dominating set of K_1+H. This concludes that S is a 1-movable independent outer-connected dominating set of K_1+H. Suppose (ii) holds. Then S is an independent dominating set of K_1+H and $\langle V(K_1+H) \setminus S\rangle = K_1 + \langle V(H) \setminus S\rangle$ is connected. Thus S is an independent outer-connected dominating set of K_1+H. Suppose $|S| = 1$, say $S = \{w\}$ for some $w \in V(H)$. Then $(S \setminus \{w\}) \cup \{x\} = \{x\}$ is an independent dominating set of K_1+H and $\langle V(K_1+H) \setminus \{x\}\rangle = H$ is connected. Thus, $(S \setminus \{w\}) \cup \{x\} = \{x\}$ is an independent outer-connected dominating set of K_1+H. Suppose $|S| \geq 2$. Let $v \in S$. By assumption, S is a 1-movable independent dominating set of H. Hence there exists $u \in (V(H) \setminus S) \cap N_H(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is an independent dominating set of H and hence of K_1+H and $\langle V(K_1+H) \setminus S\rangle = K_1 + \langle V(H) \setminus [(S \setminus \{v\}) \cup \{u\}]\rangle$ is connected. Therefore, S is a 1-movable independent outer-connected dominating set of K_1+H. ■

Corollary 2.7. Let H be a connected nontrivial graph. Then

$$\widetilde{\gamma}^1_{mic}(K_1 + H) = \begin{cases} 1 \ if \gamma(H) = 1 \\ \gamma^1_{mi}(H) \ if \gamma(H) \neq 1. \end{cases}$$

Proof. Consider the following cases:

Case 1: $\gamma(H) = 1$
Let $S = V(K_1)$. Since $\gamma(H) = 1$, there exists $a \in V(H)$ that dominates H. Hence S is a 1-movable independent outer-connected dominating set of $K_1 + H$ by Theorem 2.6(i). Hence,$1 \leq \widetilde{\gamma}^1_{mic}(K_1 + H) \leq |S| = 1$. So, $\widetilde{\gamma}^1_{mic}(K_1 + H) = 1$. Similarly, if $S = \{z\}$ is a γ-set of H, then S is a 1-movable independent outer-connected dominating set of K_1+H by Theorem 2.6(i). Thus, $1 \leq \widetilde{\gamma}^1_{mic}(K_1+H) \leq |S| = 1$. Therefore, $\gamma^1_{mic}(K_1+H) = 1$.

Case 2: $\gamma(H) \neq 1$
Let S be a $\widetilde{\gamma}^1_{mic}$-set of $K_1 + H$. Since $\gamma(H) \neq 1$, $|S| \geq 2$. By Theorem 2.6(ii), S is a 1-movable independent dominating set of H. Hence, $\widetilde{\gamma}^1_{mic}(K_1+H) = |S| \geq \gamma^1_{mi}(H)$. On the other hand, suppose that S is a γ^1_{mi}-set of H. By Theorem 2.6(ii), S is a 1-movable independent outer-connected dominating set of $K_1 + H$. Hence, $\widetilde{\gamma}^1_{mic}(K_1+H) \leq |S| = \gamma^1_{mi}(H)$. Therefore, $\widetilde{\gamma}^1_{mic}(K_1 + H) = \gamma^1_{mi}(H)$. ■

The *corona* of two graphs G and H, denoted by $G \circ H$, is the graph obtained by taking one copy of G of order n and n copies of H, and then joining the ith vertex of G to every vertex in the ith copy of H. For every $v \in V(G)$, we denote by H^v the copy of H whose vertices are joined or attached to the vertex v. The next results present the characterization of the 1-movable independent outer-connected dominating sets in the corona of two connected nontrivial graphs and the exact values or bounds of the corresponding parameter are determined.

Theorem 2.8. Let G and H be connected nontrivial graphs of orders $n \geq 2$ and $m \geq 2$, respectively. A subset C of $V(G \circ H)$ is a 1-movable independent outer-connected dominating set of $G \circ H$ if and only if $C = \bigcup_{v \in V(G)} D_v$ where D_v is a 1-movable independent dominating set of H for each $v \in V(G)$.

Proof. Assume that C is a 1-movable independent outer-connected dominating set of $G \circ H$. Suppose $C \bigcap V(G) \neq \phi$. Let $v \in C \bigcap V(G)$. Since C is an independent set, $w \notin C$ for all $w \in V(H^v)$. Hence, $\langle V(H^v) \rangle$ is a component subgraph of $\langle V(G \circ H) \setminus C \rangle$ which contradicts the assumption that $\langle V(G \circ H) \setminus C \rangle$ is connected. Hence, $C \bigcap V(G) = \phi$. This leads to a conclusion that $C \bigcap V(H^v) \neq \phi$ for all $v \in V(G)$. Let $D_v = C \bigcap V(H^v)$ for each $v \in V(G)$. Since C is a 1-movable independent outer-connected dominating set of $G \circ H$, by Theorem 2.6(ii), D_v is a 1-movable independent dominating set of H^v for each $v \in V(G)$.

For the converse, suppose $C = \bigcup_{v\in V(G)} D_v$ where D_v is a 1-movable independent dominating set of H^v for each $v \in V(G)$. Then, $C = \bigcup_{v\in V(G)} D_v$ is an independent dominating set of $G \circ H$ and $\langle V(G \circ H) \setminus C\rangle = G \circ \big\langle V(H^v) \setminus D_v\big\rangle$ is connected. Hence, $C = \bigcup_{v\in V(G)} D_v$ is an independent outer-connected dominating set of $G \circ H$. Let $w \in C$. Then $w \in D_v$ for all $v \in V(G)$. Since D_v is a 1-movable independent dominating set H^v, there exists $u \in V(H^v) \setminus D_v \cap N(w)$ such that $C \setminus \{w\} \cup \{u\}$ is an independent dominating set of $G \circ H$ and the subgraph $\langle V(G \circ H) \setminus (C \setminus \{w\} \cup \{u\})\rangle = G \circ \langle V(H^v) \setminus (D_v \setminus \{w\}) \cup \{u\}\rangle$ is connected. Thus, $C \setminus \{w\} \cup \{u\}$ is an independent outer-connected dominating set of $G \circ H$. Therefore, C is a 1-movable independent outer-connected dominating set of $G \circ H$. ■

Corollary 2.9. Let G and H be connected nontrivial graphs of orders $m \geq 2$ and $n \geq 2$, respectively. Then, $\widetilde{\gamma}^1_{mic}(G \circ H) = m\gamma^1_{mi}(H)$.

Proof. Suppose that C is a $\widetilde{\gamma}^1_{mic}$-set of $G \circ H$. By Theorem 2.8, $C = \bigcup_{v\in V(G)} D_v$ where D_v is a 1-movable independent dominating set of H^v for each $v \in V(G)$. Hence, $\widetilde{\gamma}^1_{mic}(G \circ H) = |C| = |\bigcup_{v\in V(G)} D_v| = |V(G)||D_v| \geq m\gamma^1_{mi}(H)$. Also, suppose D is a γ^1_{mi}-set of H. For each $v \in V(G)$, let $D_v \cong D$. Then, $C = \bigcup_{v\in V(G)} D_v$ is a 1-movable independent outer-connected dominating set of $G \circ H$. Hence, $\widetilde{\gamma}^1_{mic}(G \circ H) \leq |C| = \left|\bigcup_{v\in V(G)} D_v\right| = |V(G)|\,|D_v| = m\gamma^1_{mi}(H)$. Therefore, $\widetilde{\gamma}^1_{mic}(G \circ H) = m\gamma^1_{mi}(H)$. ■

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