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Relativistic Electronic Broadening of the Degenerated Isolated Spectral Line in Plasma

A. Naam[a*], L. Benmebrouk[b] and M. T. Meftah[b]

[a]Univ kasdi Merbah Ouargla, Fac. Faculty of Applied Science, Lab. Rayonnement et Plasmas et Physique de Surface, Ouargla 30 000, Algeria.

[b]Univ kasdi Merbah Ouargla, Fac. Faculty of Mathematics and Matter Sciences, Lab. Rayonnement et Plasmas et Physique de Surface, Ouargla 30 000, Algeria

The impact collision operator for the relativistic electron broadening of degenerate lines in plasmas, are calculated in the semiclassical approximation. The impact approximation is used to treat the elastic interaction between the radiating or absorbing hydrogen atom or hydrogenic ion) and the relativistic perturbing electron. In this work, we considered that the trajectory of electrons is a straight line, taking into account the relativistic effects. On the other hand, we included the relativistic effects in the velocity distribution taking those of Maxwell-Juttner. The results are compared with the non relativistic case.

The spectroscopy Spectral line Stark broadening in dense plasma is primarily due to interactions of emitting atoms, ions, or molecule with the perturbing plasma constituents. For accurate spectroscopic observations, it is necessary to studies this phenomenon, it can in addition yield useful information about the concentrations and conditions in the plasma. In astrophysics plasmas, the alternative methods are not possible, the line shapes and shifts affords a important and sensitive diagnostic for determining plasma characteristics such as particle density. According to Anderson, Baranger, Griem, and Kolb,[1-3] the electron contribution to the broadening can be calculated by using a recently developed impact theory. The perturber electrons can be treated as point charges moving along their classical trajectories (classical path approximation).

In high-temperature ionized gases (number of hot astrophysical plasmas: Stellar interior, Solar photosphere ...) where pressure broadening could dominate, the much thought has given to using this pressure as a tool for measuring temperatures and densities inside the plasma. In the other hand, the perturber electrons may become relativistic due to the extreme temperatures. Then, it must taken account the modification to the pressure broadening by relativistic effects. The relativistic collision operator calculated by[4] where the trajectory of perturbed electron is considered hyperbola.

Instead of the hyperbolic trajectories, we can use the straight classical path for perturber electron to calculate the impact broadening of hydrogenic ion lines.[5] In the present work, the trajectory of the perturber electron is taken straight lines. The influence of the external field (due to the ion or atom) on the perturber electron trajectory is negligible.

In this study, we use the region corresponding to the particular condition of plasma: very high temperature and high density. In this region, the elastic electron- emitter collision will be binary, and the dynamics of perturber electrons will be treated relativistically.

We developed the electron collision operator in the impact approximation. We revisit the standard semiclassical collision of the degenerate isolated line shapes and take into account relativ-istic effects.

We consider a radiant hydrogen atom (or hydrogenic ion), motionless in the middle of the gas formed by perturber electrons. Electrons are fast particle have very low masses, the collision delay τ is too small then the time interval with which the correlation function are calculated.[2,6] So they are treated by the impact approximation. The validity condition of the impact approximation[2,6-8] can be written as:

$$\omega\tau \ll 1 \qquad (1)$$

where ω is the line width.

By using the estimation of elastic collision delay τ and the line width ω, the validity condition of the impact theory[2] is written as:

$$\frac{2Z^3(2\pi m_e K_B T)^{3/2}}{N_e h^3 n^6} \gg 1 \qquad (2)$$

where m_e is the rest mass of electron, K_B is the Boltzmann constant.

This condition is well satisfied for the high density and high plasma temperature. In this range of densities and temperatures, we shall use the statistical classical mechanics (not the quantum statistical mechanics as in the Fermi-Dirac distribution) because the De Broglie λ_{th} thermal length is inferior to $N_e^{-1/3}$:

$$\lambda_{th} = \frac{h}{\sqrt{2\pi m_e K_B T}} < N_e^{-1/3} \qquad (3)$$

We consider a system of plane polar coordinates (r, α), the perturper electron movement is plane. The vector of the angular momentum of the system M is constant; its modulus is given by:

$$M = mr^2\dot{\alpha} \qquad (4)$$

In our case, the emitter is neutral or ion neglecting the influence of its field on the perturber electron trajectory. The perturber electron trajectory is considered to be a straight path. Then, the total energy E of the system is maintained:

$$\frac{mv_e^2}{2} = E = \frac{mv^2}{2} \qquad (5)$$

where v_e the velocity of the perturber electron, defined by:

$$v_e = \sqrt{\dot{r}^2 + r^2\dot{\alpha}^2} \qquad (6)$$

and:

$$v_e = v \qquad (7)$$

with v is the initial velocity of the perturber electron.

By using expressions of energy and kinetic moment conservations (4) and (5), we get the time t and the polar angle α :

$$t = \int \frac{dr}{\sqrt{\frac{2}{m}E - \frac{M^2}{m^2r^2}}} + Cte \qquad (8)$$

$$\alpha = \int \frac{\frac{M}{r^2}\,dr}{\sqrt{2mE - \frac{M^2}{r^2}}} + Cte \qquad (9)$$

We perform an elementary integration, we get:

$$\alpha = \arccos\frac{M}{mvr} \qquad (10)$$

One can choose the origin of the angle so that the constant is zero and introduce the formula of angular momentum:

$$M = M_0 = mv\rho \qquad (11)$$

where ρ is impact parameter.

Including the formula of the straight path of perturber electron can be written as:

$$r\cos\alpha = \rho \qquad (12)$$

One can also express the parametric equations and the Cartesian coordinates (X,Y) of the electron is obtained:

$$r = \rho\cosh x \qquad (13)$$

$$t = \frac{\rho}{v}\sinh x \qquad (14)$$

$$r\cos\alpha = \rho = X \qquad (15)$$

$$Y = \rho\sinh x \qquad (16)$$

with x real parameter, varies between $-\infty$ and ∞.

Within the impact approximation, electronic collision width of the isolated spectral line can be expressed in terms of the operator matrix elements [3,9,10]:

$$\phi(0) = -\frac{2\pi N_e e^2}{\hbar^2}\int_0^\infty v f(v)\,dv \int_{\rho_{\min}}^{\rho_{\max}} \rho d\rho$$

$$\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt\,\overleftrightarrow{E(t_1)} \times \overleftrightarrow{E(t_2)} \qquad (17)$$

where N_e is the electron density, $\overrightarrow{E(t)}$ is the electric micr-

owave field which can be defined as:

$$\overrightarrow{E(t)} = \frac{e}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3} \qquad (18)$$

$f(v)$ is the Maxwell distribution of velocity:

$$f(v) = 4\pi\left(\frac{m}{2\pi K_B T}\right)^{3/2} v^2 \exp(-\frac{mv^2}{2K_B T}) \qquad (19)$$

Using parametric equations polar r and Cartesian X and Y coordinate, the broadening operator for degenerate is-olated lines is written as (Sahal-Bréchot,1969a):

$$\phi(0) = -\frac{2\pi N_e e^4}{3\hbar^2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \int_0^\infty \frac{f(v)}{v} dv \int_{\rho_{\min}}^{\rho_{\max}} \frac{d\rho}{\rho}$$

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{t_1} dx_2 \frac{1+\sinh x_1 \sinh x_2}{\cosh^2 x_1 \cosh^2 x_2} \qquad (20)$$

We can consider two functions $G_1(0)$, $G_2(0)$:

$$G_1(0) = \int_{-\infty}^{\infty} dx \frac{1}{\cosh^2 x} \qquad (21)$$

$$G_2(0) = \int_{-\infty}^{\infty} dx \frac{\sinh x}{\cosh^2 x} \qquad (22)$$

Then the formula (20) can be written as:

$$\phi(0) = -\frac{2\pi N_e e^4}{3\hbar^2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \int_0^\infty \frac{f(v)}{v} dv$$

$$\int_{\rho_{\min}}^{\rho_{\max}} \frac{d\rho}{\rho}[G_1^2(0)+G_2^2(0)] \qquad (23)$$

After integration over the parameter x, we find functions $G_1(0)$, $G_2(0)$:

$$G_1(0)=\int_{-\infty}^{\infty} dx \frac{1}{\cosh^2 x}=\left[\frac{\sinh x}{\cosh x}\right]_{-\infty}^{+\infty}=2 \qquad (24)$$

$$G_2(0)=\int_{-\infty}^{\infty} dx \frac{\sinh x}{\cosh^2 x}=0 \qquad (25)$$

what we allow to deduce the following formula:

$$\phi(0)=-\frac{8\pi N_e \exp(4)}{3\hbar^2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \int_0^{\infty}\frac{f(v)}{v}dv \int_{\rho_{\min}}^{\rho_{\max}}\frac{d\rho}{\rho} \qquad (26)$$

Now integrating over the impact parameter, we get:

$$\phi(0)=-\frac{8\pi N_e \exp(4)}{3\hbar^2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \int_0^{\infty}\frac{f(v)}{v}dv \ln\frac{\rho_{\max}}{\rho_{\min}} \qquad (27)$$

Replacing the Maxwell distribution of velocity by its formula, we obtain[11]:

$$\phi(0)=-\frac{8\pi N_e e^4}{3\hbar^2}\frac{1}{(4\pi\varepsilon_0)^2}\sqrt{\frac{2}{\pi}}\left(\frac{m}{K_B T}\right)^{3/2}$$

$$\int_0^{\infty} v \exp\left(-\frac{mv^2}{2K_B T}\right)dv \ln\frac{\rho_{\max}}{\rho_{\min}} \qquad (28)$$

It should normally use the Debye length λ_D to choose the maximum impact parameter $\rho_{\max}$. The maximum impact parameter is introduced to account for Debye shielding[3] reported a similar cut to $\rho_{\max}=1.1\lambda_D$.

On the other hand,S. Alexiou[12] uses the value $\rho_{\max}=0.68\lambda_D$. One can also choose:

$$\rho_{\max}=\lambda_D \qquad (29)$$

In this work, we use a simplified determination of $\rho_{\min}(v)$ based on the average velocity v of the perturber electron[12]:

$$\rho_{\min}(v)=\frac{\hbar}{mZv} \tag{30}$$

where Z is the atomic number of the emitter.

Bearing the expression (29) and (30) into (28) we have:

$$\phi(0)=-\frac{8\pi N_e e^4}{3\hbar^2}\frac{1}{(4\pi\varepsilon_0)^2}\sqrt{\frac{2}{\pi}}\left(\frac{m}{K_B T}\right)^{3/2}$$

$$\int_0^\infty v\exp\left(-\frac{mv^2}{2K_B T}\right)dv\ln\frac{\lambda_D mZ}{\hbar}v \tag{31}$$

By integrating over the initial electron velocity v, we find the formula electronics collision operator: $\phi(0)=-\frac{4\pi N_e e^4}{3\hbar^2}\frac{1}{(4\pi\varepsilon_0)^2}\sqrt{\frac{2m}{\pi K_B T}}$

$$\left[\ln\left(\frac{2K_B T m\lambda_D^2 Z^2}{\hbar^2}\right)-\gamma_e\right] \tag{32}$$

with γ_e the Euler-Mascheroni constant:

$$\gamma_e=0.577 \tag{33}$$

Note that this operator depends on the electron temperature T and the electron density N_e, and also related to the atomic number of the emitter Z.

In this section, we take into account the relativistic effect of perturber electron in the calculation of the electronic collision operator. It is assumed that all the electron-electron interactions are neglected. Then we are in the image of a set of binary collisions (atom- electron and ion-electron). To establish the expression of the relativistic electronic collision operator, It is necessary to study the relativistic motion of the electron.

Just as the problem in the non-relativistic case, we consider a system of plane polar coordinates (r^* , α^*). It is easy, expressed in terms of polar coordinates the relativistic kinetic momentum M^* :

$$M^* = mr^{*2}\dot{\alpha}^*\gamma \qquad (34)$$

with:

$$\gamma = \frac{1}{\sqrt{1-\beta_e^2}} \qquad (35)$$

and:

$$\beta_e = v_e / c \qquad (36)$$

where c the velocity of the light in vaccum.

We neglect the influence of the emitter on the trajectory of perturber electron, introducing the conservation laws of relativistic total energy E^* and relativistic momentum M^* of the system:

$$mc^2(\gamma - 1) = E^* = mc^2(\gamma_0 - 1) \qquad (37)$$

and:

$$M^* = mr^{*2}\dot{\alpha}^*\gamma = M_0^* = mv\rho\gamma_0 \qquad (38)$$

then:

$$\gamma_0 = \gamma, v_e = v \qquad (39)$$

Using the equations (37) and (38), we obtain:

$$\dot{r}^* = \frac{c\sqrt{(1+\frac{E^*}{mc^2})^2 - 1 - \frac{M^{*2}}{m^2c^2r^{*2}}}}{(1+\frac{E^*}{mc^2})} \qquad (40)$$

from where:

$$t^* = \int \frac{(1+\frac{E^*}{mc^2})dr^*}{c\sqrt{(1+\frac{E^*}{mc^2})^2 - 1 - \frac{M^{*2}}{m^2c^2r^{*2}}}} + Cte \qquad (41)$$

$$\alpha^* = \int \frac{\frac{M^*}{cmr^{*2}}dr^*}{\sqrt{(1+\frac{E^*}{mc^2})^2 - 1 - \frac{M^{*2}}{m^2c^2r^{*2}}}} + Cte \qquad (42)$$

By substituting equation (42), we find:

$$\alpha^* = \arccos \frac{1}{r^*\rho} \qquad (43)$$

So the formula of relativistic trajectory of the perturber elect-ron is:

$$r^* \cos\alpha^* = \rho \qquad (44)$$

Note that in the relativistic case, the perturber electron describes the same non-relativistic straight path. Relativity does not affect the straight path. Relativity does not affect the straight path, therefore, the Cartesian and the polar coordinates of the electron are the same in the expressions (13), (14), (15) and (16).

Now, we use the path formula and parametric equations of linear motion to get the relativistic electron collisions operator. The calculations are developed as part of the semi-classical theory. The average effect of disturbing electrons is calculated using the relativistic Maxwell- Juttner distribution of velocity electrons:

$$f(\beta)d\beta = \frac{\beta^2\gamma_0^5}{\theta K_2(1/\theta)} \exp(-\frac{mc^2}{kT}\gamma_0)d\beta \qquad (45)$$

with:

$$\theta = \frac{K_B T}{mc^2} \qquad (46)$$

where $K_2(x)$ is the modified Bessel function of t-he second kind of imaginary order.

Determining the influence of relativistic correction in the relativistic distribution, on the electronic collision operator, the formula (27) be-comes:

$$\phi(0) = -\frac{8\pi N_e e^4}{3\hbar^2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{1}{c\theta K_2(1/\theta)}$$

$$\int_0^1 \beta\gamma_0^5 \exp(-\gamma_0/\theta)\, d\beta \ln\frac{\lambda_D mZc}{\hbar}\beta \qquad (47)$$

Using the approximation $\gamma_0 \approx 1+\frac{\beta^2}{2}$, the formula (47) can be written as:

$$\phi(0) = -\frac{8\pi N_e e^4}{3\hbar^2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{\exp(-\frac{1}{\theta})}{c\theta K_2(1/\theta)} \int_0^1 \beta\gamma_0^5 \exp\left(-\frac{\beta^2}{2\theta}\right) d\beta \ln a\beta \qquad (48)$$

We introduce the following notations:

$$b = \frac{1}{2\theta}, a = \frac{\lambda_D mZc}{\hbar} \qquad (49)$$

We obtain:

$$\phi(0) = -\frac{8\pi N_e e^4}{3\hbar^2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{\exp(-\frac{1}{\theta})}{c\theta K_2(1/\theta)}$$

$$\int_0^1 \beta\gamma_0^5 \exp\left(-\frac{\beta^2}{2\theta}\right) d\beta \ln a\beta \qquad (50)$$

To get the expression of the relativistic electron collision operator, we calculate the integral over β:

$$\phi^*(0) = \frac{\pi N_e e^4}{48\hbar^2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{\exp(-\frac{1}{\theta})}{c\theta K_2(1/\theta)}$$

$$[-274b^{-6} - 500b^{-5} - 440b^{-4} - 240b^{-3} - 80b^{-2}$$
$$+8(15b^{-6} + 30b^{-5} + 30b^{-4} + 20b^{-3} + 10b^{-2}$$
$$+4b^{-1})\left(\gamma_e + \ln b + \mathrm{Ei}(1,b) - 2\ln a\right) + 6(40b^{-6} +$$
$$120b^{-5} + 180b^{-4} + 180b^{-3} + 135b^{-2} + 81b^{-1})$$
$$\ln a \exp(-b) + (274b^{-6} + 654b^{-5} + 747b^{-4} +$$
$$519b^{-3} + 211b^{-2})\exp(-b)] \qquad (51)$$

It is clear that the expression of the relativistic electron collision operator depends on the temperature T , electron density N_e , and the velocity of light c .

To validate our results we take the limit $\beta \to \infty$:

$$\lim_{\beta \to 0} \phi^*(0) = -\frac{4\pi N_e e^4}{3\hbar^2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \sqrt{\frac{2m}{\pi KT}} \left(\ln\frac{2KTm\lambda_D^2 Z^2}{\hbar^2} - \gamma_e\right) = \phi(0) \qquad (52)$$

This is the case where the collision operator relativistic reduces to the non-relativistic case given by the formula (32). From a mathematical point of view, the expression in the non-relativistic case is the limited development of second order β of relativistic expression.

The fig. (1) presents the variation of the operator of electron relativistic and non-relativistic collision, as a function of the electron temperature. Note that due to the shorter collision duration for fast collision, both operators decrease with increasing temperature, with relativistic operator exhibiting a slower decrease.

In fig. (2) shows the variation of $\Delta\phi(0)/\phi(0)$ ($\Delta\phi(0)$ is the difference between the relativistic and non-relativistic $\phi^*(0) - \phi(0)$) as a function on the temperature in the case of Lyman-α transition with electron density 10^{20} cm^{-3} . We remark that $\Delta\phi(0)/\phi(0)$ increases to reach 35.91% at a temperature of 10^9 K .

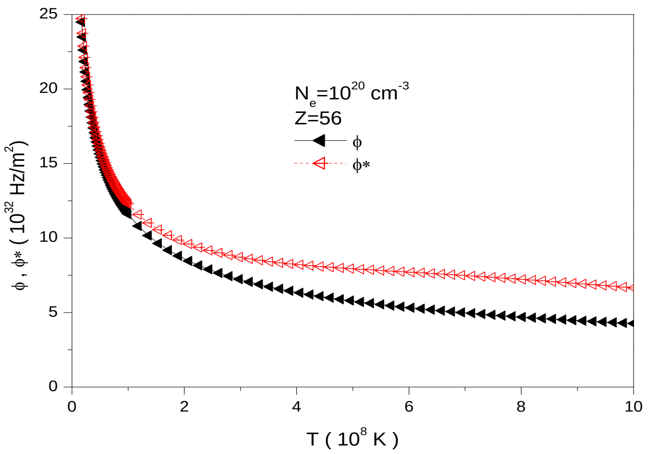

Figure 1. Relativistic and non-relativistic collision operators versus temperature for Lyman-α transition in the case of $Z = 56$ (Barium).

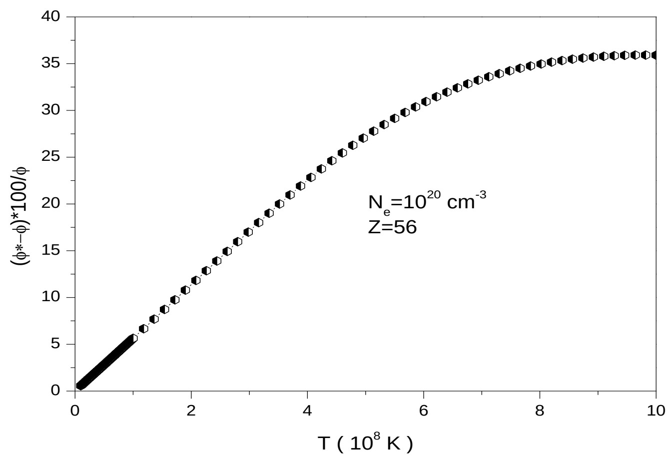

Figure 2. Variation of $\Delta\phi(0)/\phi(0)$ versus temperature for Lyman-α transition in the case of $Z=56$ (Barium).

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